

Task 1

		Answers	Notes	Total
1	a	• $x = 4$		1
	b	• $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$	Accept (4,4) or $p = 4$ and $q = 4$	1
	c		Must be correct within tolerance of half a square Accept no label on D	1
	d	• ¹ $4 \times -4 + 4 \times 4$ or vector format $\begin{pmatrix} 4 \\ 4 \end{pmatrix} \begin{pmatrix} -4 \\ 4 \end{pmatrix}$ • ² 0	• ¹ Their answer (b) and $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$ which is given OE • ² ECF for correct multiplication of any two vectors	2
	e	• explanation that mentions 90° or perpendicular	ECF could be awarded for a correct comment on their product e.g. correctly state the angle is acute/obtuse. For example the candidate says : “It allows you to find the angle between the vectors” “it is the cosine of the angle between the vectors” “the angle is X i.e. they correctly find their angle”	1

2	a	<ul style="list-style-type: none"> •¹ 1st mark for two from the points below •² 2nd mark for four from the points below <p>Correct comment comparing median or LQ or UQ or min or max Correct comment comparing IQR or range Correct values of median or LQ or UQ or min or max Correct values of IQR or range</p>	<p>A comment for comparison would have higher or lower or same in both countries or WTTE</p> <p>Any reference to mode e.g. “most/usually shoe sizes” is incorrect Accept use of average for the median Accept values of IQR given as (LQ-UQ) Accept values of range given from min to max</p> <p>SC If there is not enough comments or correct values but the candidate states: that the shoes sizes in Brazil are bigger than in Egypt WTTE: award 1 mark</p>	2
	b	<ul style="list-style-type: none"> •¹ one multiplication correct •² second multiplication and addition shown •³ correct answer 0.08 	<ul style="list-style-type: none"> •¹ OE 6 % or 0.06 seen or 2 % or 0.02 •² OE 0.06+0.02 •³ OE 0.08 without working award 2 marks 	3
	c	<ul style="list-style-type: none"> •¹ $P(B/D) = \frac{P(B \cap D)}{P(D)}$ •² their 0.08 •³ $= \frac{1}{4}$ OE e.g. 0.25 	<ul style="list-style-type: none"> •¹ Use of conditional probability formula •² Accept their 0.02/their 0.08 ECF Note: Formula at •¹ can be implied All probabilities must be less than 1 •³ $\frac{1}{4}$ OE without working: award 2 marks 	3

3	a	<p>•¹ $f \circ g(x) = \frac{x+3-2}{3(x+3)-11}$</p> <p>•² $= \frac{x+3-2}{3x+9-11}$</p> <p>$= \frac{(x+1)}{(3x-2)}$ AG</p>	<p>•¹ Substitutes $x + 3$</p> <p>•² Opens out bracket</p> <p>Note: if they use $g \circ f$: award 0 marks</p>	2
	b	<p>•¹ $3yx - 2y = x + 1$</p> <p>•² $3yx - x = 2y + 1$</p> <p>•³ $x(3y - 1) = 2y + 1$</p> <p>•⁴ $x = \frac{2y + 1}{3y - 1}$</p> <p>•⁵ $(f \circ g)^{-1}(x) = \frac{2x + 1}{3x - 1},$</p> <p>•⁶ $x \neq \frac{1}{3}$</p>	<p>•¹ Changes $f \circ g$ to $g \circ f$ and cross multiplies</p> <p>•² x terms to same side</p> <p>•³ Taking x (or y if candidate exchanged them) as common factor</p> <p>•⁴ Writing x in terms of y</p> <p>•⁵ Exchanging x and y may be seen earlier</p> <p>•⁶ The domain can be written differently as $x \in \mathbf{R} \neq \left\{\frac{1}{3}\right\}$ or $\mathbf{R} - \{1/3\}$</p> <p>Note: Some working may be implied. Note: If an error makes the working easier full ECF cannot be awarded</p> <p>$(f \circ g)^{-1}(x) = \frac{2x + 1}{3x - 1},$ with no working : award 4 of first 5 marks</p>	6

Alternative method

3	b	<ul style="list-style-type: none"> •¹ $y = \frac{x+1}{3x-2}$ •² $\frac{x+1}{3x-2}$ •³ $h = \frac{2}{3}$ and $k = \frac{1}{3}$ or $y = \frac{5/9}{x-2/3} + \frac{1}{3}$ •⁴ new $h = \frac{1}{3}$ •⁵ placing $1/3$ for h or $y = \frac{5/9}{x-1/3} + \frac{2}{3}$ •⁶ Dom : $x \neq \frac{1}{3}$ 	<ul style="list-style-type: none"> •¹ Changes $(f \circ g)(x)$ to y •² Divides of the terms •³ Recognises h and k •⁴ Puts h instead of k •⁵ Puts k instead of h •⁶ Domain can be written differently 	
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4

•1 $r^2 = 2^2 + 2^2$ or $\sin 45 = 2/r$ **OE**

•2 $(r =) \sqrt{8}$

•3 height of cone 4

•4 $(V_{cylinder} =) \pi \times \text{their } r^2 \times 26$

•5 $(v =) 208\pi$

•6 $(V_{cone} =) \frac{1}{3} \pi \times \text{their } r^2 \times 4$

•7 $(V_{cone} =) \frac{32}{3} \pi$

•8 **their** cylinder + **their** cone

•9 $\frac{656\pi}{3}$

Accept the use of decimals in their working. The final answer must be exact. See note at mark •9

•1 Correct use of Pythagoras or trigonometry

•2 $\sqrt{8}$ SEEN Without working award 1 mark from •1 and •2

•3 finds height of cone (may be seen at any stage or implied)

•4 Substitutes in formula

•5 208π SEEN Without working award 1 mark from •4 and •5

•6 Substitutes in formula

•7 $32\pi/3$ SEEN Without working award 1 mark from •6 and •7

•8 Adding **their** two volumes. May be implied.

•9 Answer must be given in terms of π . Do not award •9 for an answer that is not exact.

SC: for the candidates that use $r = 2$ correctly: award 7 marks
See below answers for this **SC**

$(V_{cylinder} =) 208\pi$

$(V_{cone} =) \frac{32}{3} \pi$

$(Total V =) \frac{656\pi}{3}$

9

5	a	<ul style="list-style-type: none"> •¹ crosses at 0 and 30 or a is a distance travelled along the x-axis •² $x = 0$ or $x = a$ 	<ul style="list-style-type: none"> •¹ or when the ball is at 30 the height is 0 OE e.g. substitutes (30, 0) 	2
	b	<ul style="list-style-type: none"> •¹ the maximum height occurs at $x=15$ •² $h(15)=9$ m 	<ul style="list-style-type: none"> •¹ Can be seen as substitution in the equation •² 9 with no working award 1 mark 	2
	c	<ul style="list-style-type: none"> •¹ substitute $x = 26$ •² $h = 4.16$ •³ $4.16 > 3$ m therefore Score or yes 	<ul style="list-style-type: none"> •² 4.16 without working: award 1 mark •³ $4.16 > 3$ m, without working award 2 marks 	3
	d	<ul style="list-style-type: none"> •¹ $\frac{x(30-x)}{25}$ •² $x(30-x) > 75$ or $x^2 - 30x - 75 < 0$ this should be $+75$ •³ $x = \frac{30 \pm \sqrt{(-30)^2 - 4(1)(75)}}{2}$ •⁴ $(x=)27.2$ and $(x=)2.8$ •⁵ $27.2 - 2.8 = 24.4$ 	<ul style="list-style-type: none"> •¹ sets up inequation •² cross multiplies is minimum for the mark •³ substitutes into quadratic formula •⁴ finds 2 roots or estimates from the graph in the range $2 < x < 4$ and $26 < x < 28$ award: 1 mark from the first four bullets •⁵ subtracts to find the distance or ECF correctly subtracts their estimated values 	5

Task 2

Answers			Notes	Total
6	a	<ul style="list-style-type: none"> the values double 	or $r = 2$	1
	b	<ul style="list-style-type: none"> ¹ evaluates 3520 ² because the values are above 20 and below 20 000 	3520 must be seen Accept "because $20 < \text{their } A < 20\,000$ " OE Accept their 3520 is less than 20 000 Award 2 marks	2
	c	<ul style="list-style-type: none"> ¹ 1 term correct ² both terms correct and multiplied $(F =) 27.5^2 \times 2^n$	<ul style="list-style-type: none"> ¹ $F_n = F_0 \times 2^n$ award 1 mark ² Allow alternative fully correct formula i.e. $55 \times 2^{n-1}$ 	2
	d	<ul style="list-style-type: none"> ¹ $\frac{F}{27.5} = 2^n$ ² $\log_2 \frac{F}{27.5} = \log_2 2^n$ ³ $n = \log_2 \frac{F}{27.5}$ 	<ul style="list-style-type: none"> ¹ Rearrange ² Award 1 mark for $\log \frac{F_n}{27.5}$ ³ Award 2 marks for $\frac{\log \frac{F_n}{27.5}}{\log_2 2}$ OE ³ Correct answer without working: award 2 marks 	3
	e	<ul style="list-style-type: none"> ¹ $n = \log_2 \frac{28160}{27.5}$ ² $n = \log_2 1024 = 10$ yes as a whole value 	Alternative method <ul style="list-style-type: none"> ¹ Continue the sequence ² 10 seen and answer yes 	2

f	This table gives some key values which might be seen – the table goes over two pages		10	
	Aspect	1 mark		2 marks
	IR: Identification of relevant information	One numerical factor mentioned from: <ul style="list-style-type: none"> – Octave (either width 194.5mm or number of octaves 9) – Keys (either width 23mm or number 63-66) – Arm span 1.65m – Width of the piano as shown 148cm – Room/door size (eg. average room 3x4) – Human hearing 20HZ to 20 000HZ 		More than one numerical factor mentioned
	CM: Calculation of measurements including width	Relevant calculation without mentioning gaps or extra width Examples: Calculating the number of keys ie $52 + 2 \times 7$ Accept answer is in the range 63-66 OR Calculating the width of the piano keys $(63-66) \times 23$ ACCEPT any reasonable value		Calculating width including gap or adding extra width Examples: 9×194.5 9 octaves and includes gap between keys gap width: $(194.5 - 8 \times 23) / 7$ or 8 gaps = 1.5 / ----mm Key + its gap = $23 + 1.5 = 24.5$ Hence width = $66 \times 24.5 + 1.5$ (1 extra gap at the end) = 1618.5 mm OR Calculating extra width at the end in the original piano: In normal piano of width 1480mm there are 52 keys so extra width both sides is $1480 - (52 \times 24.5 + 1.5) = 204.5$ mm So the 66 keys piano needs to be $1618.5 \text{ mm} + 204.5 = 1823 \text{ mm}$
	JD: Justification of degree of accuracy	Rounding used in any element		Justifies their choice of rounding
PD: Practicality of new design	Limited argument: Some examples: Not acceptable because people are used to smaller pianos Acceptable as it will include all octaves that people can hear	Justified argument Some examples: Referring to price in anyway: Not acceptable because the added octaves will increase the price of the machine and at the same time 2 octaves will not add much Referring to the difference in width between normal piano and this one in any way: Acceptable because the added width will be only about 30 cm which can fit in a standard room width		
QD: Quality of overall discussion (Holistic judgement on the whole response)	Limited discussion	Balanced discussion		

7	a	$\cdot^1 \cos(73) = \frac{1.19}{s}$ $\cdot^2 s = \frac{1.19}{\cos(73)} (= 4.07 \text{ m}) \text{ AG}$		2
	b	$\cdot^1 \tan 73 = \frac{h}{1.19}$ $\cdot^2 (h =) 3.89... (2314616 \text{ cm})$	OE with sine ratio $\cdot^2 \text{ Correct answer without working award 1 mark only}$	2
	c	$\cdot^1 \text{ calculates angle } 20$ $\frac{x}{\sin 20} = \frac{4.07}{\sin 53}$ $\cdot^2 \sin 20 = \frac{4.07}{\sin 53}$ $\cdot^3 (x) = 1.74... (3000362 \text{ cm})$ $\cdot^4 w_2 = 55.3 - 2 \times (\text{their } 1.74 + 1.19) \text{ or their } 2.93 \text{ from } \cdot^3$ $\cdot^5 w_2 = 49.44 (\text{ cm})$	$\cdot^2 \text{ Use of sine rule}$ $\cdot^3 \text{ Calculate } x$ <p>Accept correct method using other lengths Alternative method for $\cdot^1 \cdot^2 \cdot^3$</p> $\cdot^1 \text{ TAN} = h/y \text{ uses TOH in right angle triangle}$ $\cdot^2 y = \tan/h \text{ cross multiply } y = \tan 53/3.89 \text{ their } 3.89$ $\cdot^3 y = 2.93 \text{ calculate}$	5
	d	$\cdot^1 \text{ attempt to work out another width ECF answer from (c)}$ $\cdot^2 w_3 = 55.3 - 2(8) \times (\text{their } 1.74 + 1.19)$ $\cdot^3 w_3 = 8.42 (\text{ cm})$	$\cdot^1 w_3 = 49.44 - 2 \times (\text{their } 1.74 + 1.19)$ $\cdot^2 8 \text{ subtractions}$ $\cdot^3 \text{ Accept correct to 1 decimal place}$	3
<p>For candidates that do not apply the scale 1:100 treat as a misread MR on the first occasion, ECF can be awarded so candidates can gain full marks for subsequent working. The final answer for 7a) 7b) 7c) should be in the answer box. If there is not a value in the answer box, award marks for final answers in the response box.</p>				

7	e	Aspect	1 mark	2 marks	6
		Identify the relevant information required	Identify one of the following: Total measurements of the model Volume of the model Elements of the pyramid (temple and platform) the scale 1:100 Practicality of the size of the model	Identify more than one of the following: Total measurements of the model Volume of the model Elements of the pyramid (temple and platform) the scale 1:100 Practicality of the size of the model	
		Consider the degree of accuracy	Consideration the implication of the degree of accuracy on the given values. Examples: Recognizing that the lengths given to the nearest cm Recognizing that the angles given to one decimal place	Consideration the implication of the degree of accuracy on the calculated values. Examples: 2.7x9+6 =30.3 and not 30cm (2.5 instead of 2.7 we get 2.5x9+6=28.5 which is far from 30 or 3cm instead of 2.7 we get 3x9+6=33 which is far from 30) The width of first platform 55.3cm while it could be 55 and in this case 9 th platform 55-2x2x8=23 cm The width of 9 th platform could be 23 cm instead of 23.3 and in this case the 1 st platform 55cm Recognizing the difficulty of using a model with dimensions more accurate than nearest mm because of measuring tools available	
Comment on the validity	Comment not supported with evidence. Examples: The model will be close enough to look like the real pyramid The model was calculated with correct mathematical steps so it is valid	Comment supported with evidences. Examples: The model is practical and referring to overall size (height being about 30, width being about 50 cm) and can be carried/placed on a table easily The model is not very valid because the total height needs to be to the nearest mm like the height of the platform (27 mm). this will lead to inaccurate angles compared to real pyramid The erosion had an effect on the real pyramid and this is not taken into account in the model			

Task 3

Answers			Notes	Total
8	a	• states the limit as 8	Correct limit	1
	b	• ¹ two correct from p=5 q=6 r=7 • ² all three correct	Award 1 mark for two correct values	2
	c	• ¹ any correct patterns • ² in words give a rule e.g. divide column a by 3 and then add 2	For example Column a increases by 3 or multiples of 3 Column L increase by 1 Do not accept column b are the same / all 3	2
	d	• ¹ $L = \frac{1}{3}a + b$ or $L = xc + 2$ • ² $L = \frac{1}{3}a + 2$	Any correct rearrangement Award 1 mark for each correct term	2

e	Mark holistically			12
	Strand	Holistic markscheme	Mark band	
	Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	Nothing from below	0	
	Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	One prediction made Attempt to describe a pattern in words	1 - 3	
	Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	More than one prediction made A pattern correctly described in words Attempt to test their described pattern	4 - 6	
	Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	More than one prediction made A pattern described as suggested general rule consistent with some of the findings Their general rule is tested correctly	7 - 9	
	Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	More than one prediction made A pattern described as correct general rule consistent with findings A general rule is fully proved or verified and justified	10 - 12	
	<p>SC More than one prediction made with constant value for column A without any pattern description award 3 marks Patterns need to refer to a relation involving a b and L Predictions could be seen in the table or in the answer box</p>			

Exemplification

The candidates can populate the table using the simulator.

Candidates who restrict themselves to the pattern shown can only discover simple patterns e.g.

For L in terms of row number [$L = n+3$] and a in terms of row number [$a = 3(n+1)$]. Simple patterns can only gain credit up to Mark band 7–9.

Candidates who use the simulator to vary a and b will be able to described more complex patterns and find $\frac{1}{3}a + \frac{2}{3}b$.

Using random values of a and b will probably not prove useful. Some systematic approach will be more helpful. Here are some examples.

Candidates keep the same a values and change b – they will soon see that multiples of 3 are better to use.

	a	b	L
	3	6	5
	6	6	6
	9	6	7
	12	6	8

They might write down a rule and test it again using the simulator.

They might try $b = 0$

	a	b	L
	3	0	1
	6	0	2
	9	0	3
	12	0	4

Candidates need to use other a values. They might keep a constant and vary b . And again put $a = 0$

	a	b	L
	3	3	3
	3	6	5
	3	9	7
	3	12	9

	a	b	L
	0	3	2
	0	6	4
	0	9	6
	0	12	8

9	a	$\bullet^1 \left(b + \frac{1}{2}a + \frac{1}{2}b \right) \div 2$ $\bullet^2 \left(\frac{1}{2}a + 1\frac{1}{2}b \right) \div 2 = \frac{1}{4}a + \frac{3}{4}b \quad \mathbf{AG}$		2
	b	$\bullet^1 \text{ student generates a sequence}$ $\bullet^2 \frac{3}{4}$ $\bullet^3 \frac{5}{8}$ $\bullet^4 \text{ this sequence represents the coefficients of } \mathbf{b}$		4

c	Mark holistically using a best fit approach			14
	Strand	Holistic markscheme	Mark band	
	Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	Nothing from below	0	
	Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	Attempt at patterns or rules	1 - 2	
	Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	One or more patterns in words Attempt to test one pattern	3 - 5	
	Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	One or more patterns in words, evidence of working to find a rule, any general rule Evidence of testing the rule	6 - 9	
	Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	Two or more patterns or rules found, a general rule found that links to 8e Attempt to prove or verify or justify a rule linking to 8e	10 - 12	
	Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	Patterns or rules found Prove the rule from 8e or verify and justify the rule from 8e	13 - 14	

Exemplification

Candidates may look at **their** developing table and find simple and more complex patterns.

Example

The denominators go 1, 2, 4, 8, 16, 32, ...

The denominators are 2^n

The numerators always add to give the denominator e.g. $5 + 11 = 16$

The coefficients always add to give 1

The coefficient of **a** is always smaller than that of **b**

Recognizes geometric sequence with $r = -\frac{1}{2}$

The coefficient of **a** is always nearly half that of **b**

The numerators go 1, 3, 5, 11, 21, 43, 87, starting at term 3

The rule for the numerator of **b** is double the numerator of **a** and then add **or** subtract 1

The numerators always add to the denominator

The numerator of **b** is the numerator of **a** plus twice the previous numerator of **a**

Recognizes it is a summation

$$1 - \frac{1}{2} + \frac{1}{4} - \dots$$

Example for minimum justification for $\frac{1}{3}a + \frac{2}{3}b$ the further term coefficients tends to be that the

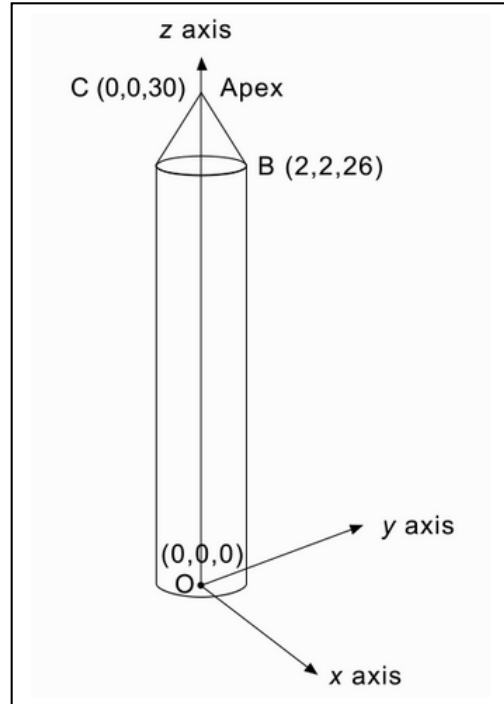
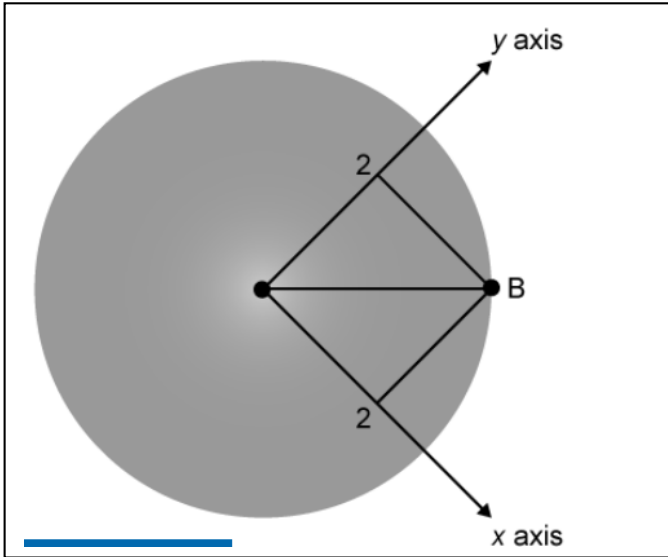
coefficient **b** is double the coefficient of **a** **or** the limit of the sequence is $0.33333\dots a + 0.66666\dots b$
(for 9 marks)

Sums geometric sequence for coefficient of **b** $= \frac{2}{3}$, deduces coefficient of **a** must be $\frac{1}{3}$.

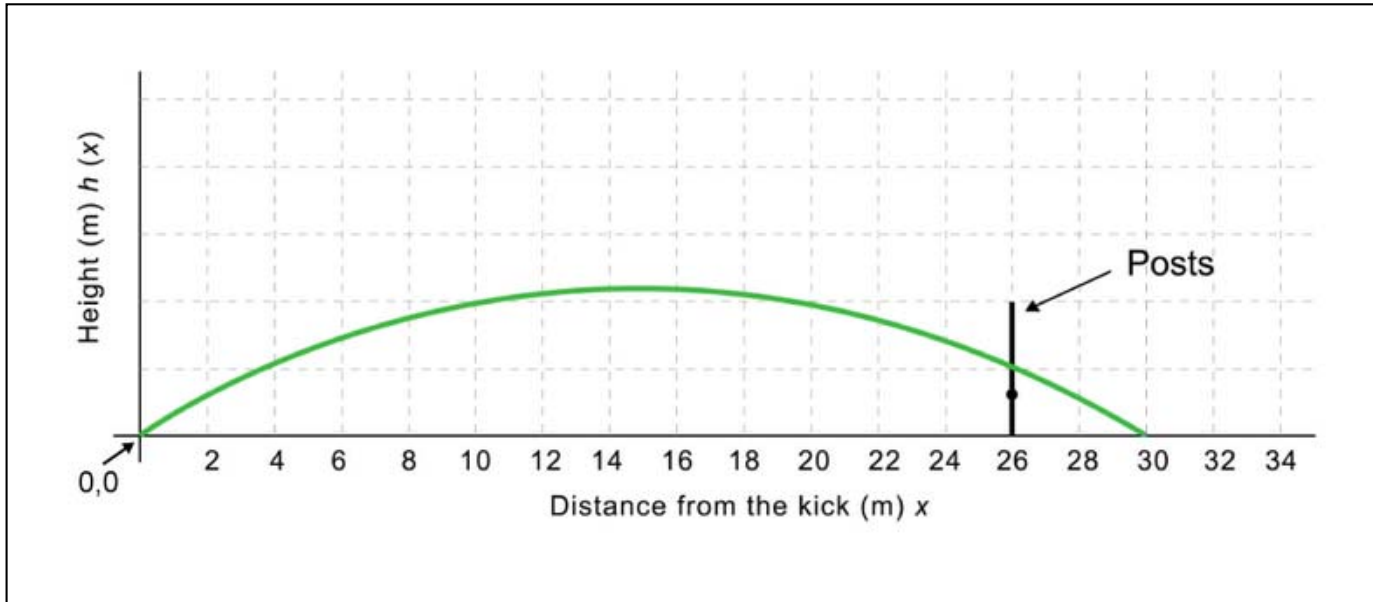
Concludes proof

Appendices

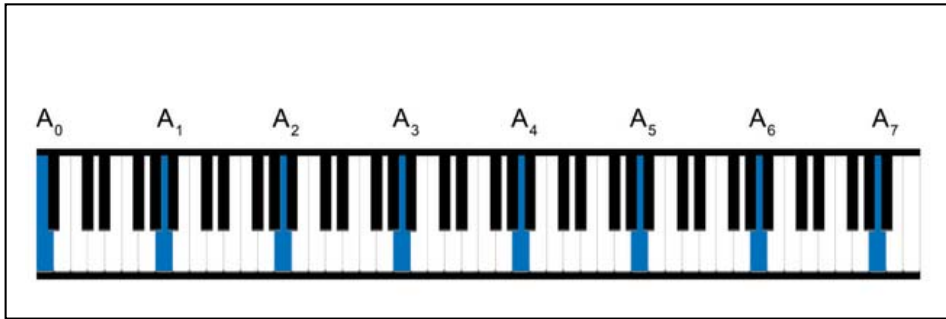
Question 4



Question 5



Question 6



Humans can only hear between 20 and 20 000 Hertz.

Piano Keys Hearing range Arm span

Grotrian-Steinweg Friedrich
Width A: 148 cm
Depth B: 52 cm
Height C: 111 cm

The diagram shows a black upright piano with three dimensions labeled: A (width), B (depth), and C (height). The piano is shown from a three-quarter perspective.

Piano Keys Hearing range Arm span

Width 194.5 mm

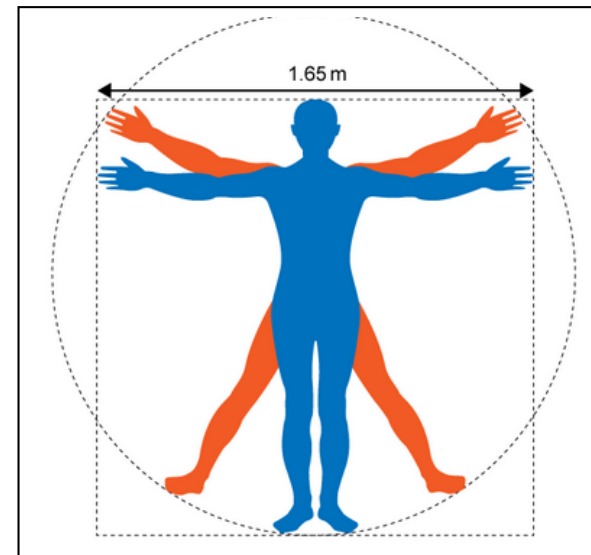
Diagrams not to scale

A B C D E F G A

23 mm

Width of a key

The diagram shows a close-up of piano keys. The white keys are labeled with the letters A, B, C, D, E, F, G, and A. A dimension line indicates a width of 23 mm between two adjacent white keys. A larger dimension line above shows a total width of 194.5 mm for a group of keys. A note says 'Diagrams not to scale'.



Question 7

