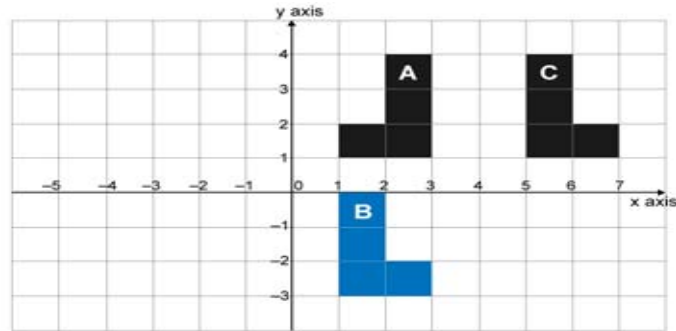


Question 1 (6 marks)



Question 1a (1 mark)

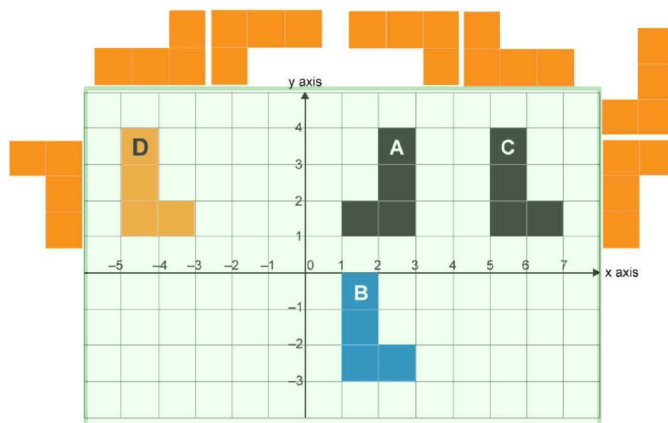
Write down the equation of the line that reflects shape A to shape C.

Question 1b (1 mark)

The vector \mathbf{c} translates the shape B to shape C. **Write down** the vector \mathbf{c} in the form $\begin{pmatrix} p \\ q \end{pmatrix}$.

Question 1c (1 mark)

Translate shape B by $\mathbf{v} = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$, **draw** and label the image D.



Question 1d (2 marks)

Find the scalar product $\mathbf{c} \cdot \mathbf{d}$.

Question 1e (1 mark)

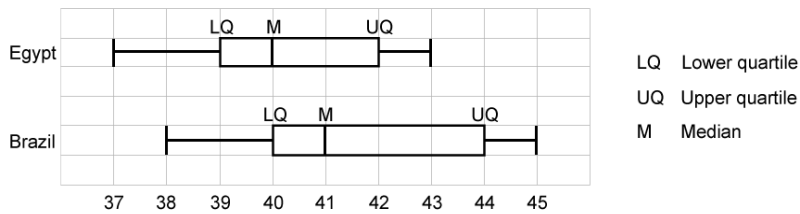
Explain the significance of your result in part (d).

Question 2 (8 marks)



A manufacturer makes shoes to sell internationally. The manufacturer is contracted to make shoes for Egypt and Brazil.

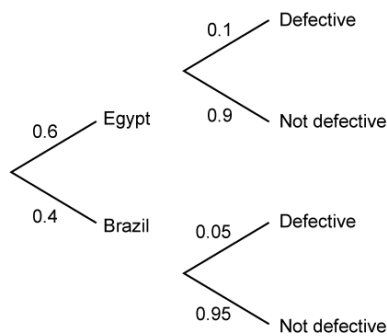
The box plots show the sizes of shoes sold in Egypt and Brazil. The distributions shown in the box plots are based on the medians and quartiles of the shoe sizes sold.



Question 2a (2 marks)

Compare the sizes of shoes sold in the two countries.

The manufacturer makes shoes in two factories; one in Egypt and one in Brazil. The factory in Egypt makes 60 % of the shoes and 10 % are defective. The factory in Brazil makes the rest of the shoes and 5 % are defective.



Question 2b (3 marks)

Find the probability that a shoe, chosen at random from either factory, is defective.

Question 2c (3 marks)

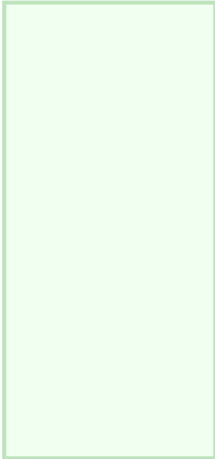
Given that a shoe is defective, **find** the probability that it was made in Brazil.

Question 3 (8 marks)

Consider the functions $f(x) = \frac{x-2}{3x-11}$, $x \neq \frac{11}{3}$ and $g(x) = x+3$, $x \in \mathbb{R}$.

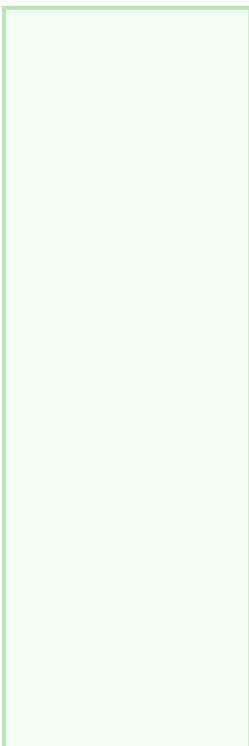
Question 3a (2 marks)

Show that $f \circ g(x) = \frac{x+1}{3x-2}$, $x \neq \frac{2}{3}$



Question 3b (6 marks)

Find $(f \circ g)^{-1}(x)$, the inverse function of $f \circ g(x)$, and **write down** its domain.



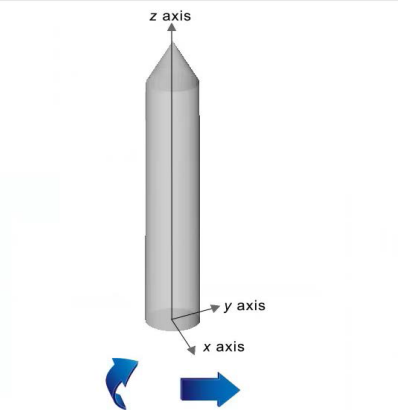
Question 4 (9 marks)

Here is information on Glendalough round tower in Ireland.

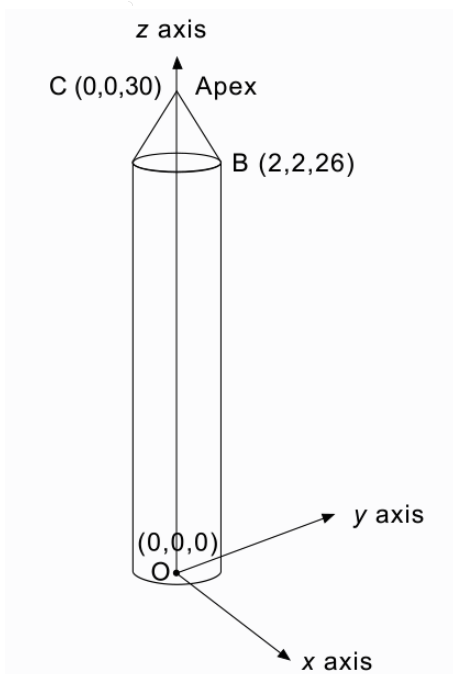
Video



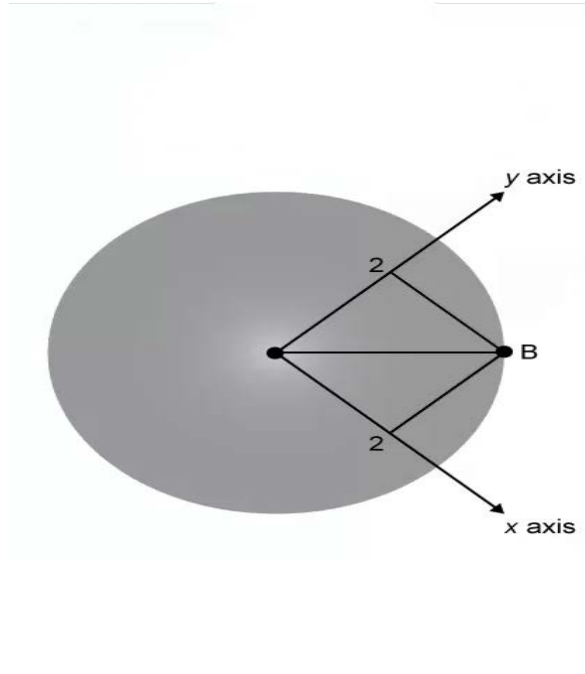
Simulator



Tower dimensions



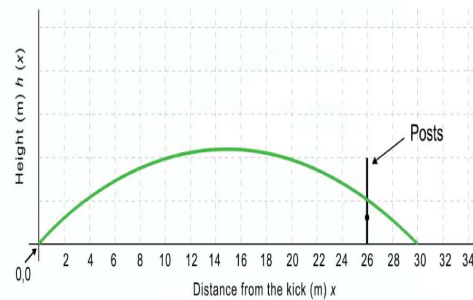
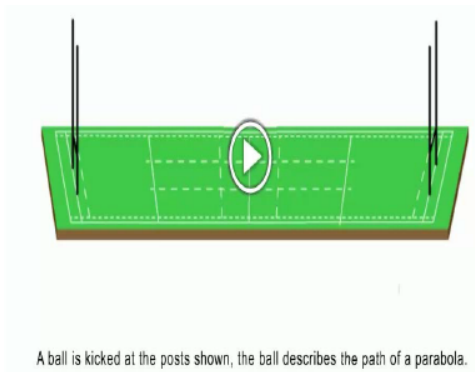
Tower cross section



The round tower is modelled as a cylinder and a cone. **Find** the total volume of the round tower. Express your answer as an exact value.

Question 5 (12 marks)

A ball is kicked from (0,0) towards the posts and describes the path of a parabola as shown in Tab 2.



The height in metres (m) of the ball above the ground is given by the function $h(x) = \frac{x(a-x)}{25}$.

Question 5a (2 marks)

Show that $a = 30$.

Question 5b (2 marks)

Find the maximum height achieved by the ball.

To score points the ball is kicked from the point (0,0) and must go over a bar on the posts that is 3 m above the ground and 26 m away from the point the ball was kicked. This is illustrated in the diagram in Tab 2 above.

Remember the function for the height of the ball is $h(x) = \frac{x(a-x)}{25}$

Question 5c (3 marks)

Did the team score points? **Show** your working and reasoning.

Question 5d (5 marks)

Find the total horizontal distance for which the ball is above 3 m.

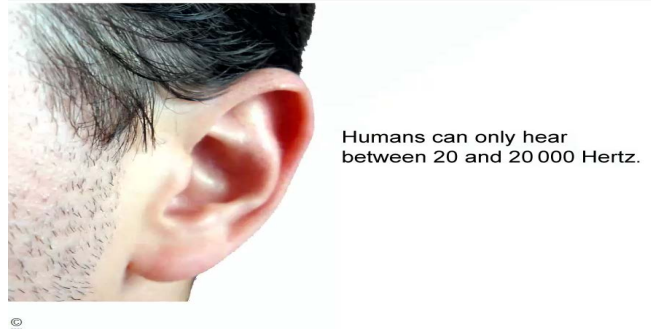
Question 6 (20 marks)

Music is a widely accepted form of artistic expression and creativity. In this question you will be presented with the instrument design of a piano.

Video



Hearing range



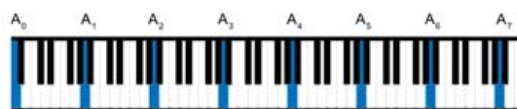
Here is a table with some information on the octaves and frequencies of the note A on the piano. The frequencies increase by a geometric progression.

| | | | | | | | | |
|-----------------------------|-------|-------|-------|-------|-------|-------|-------|-------|
| Octave (n) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Note (A) | A_0 | A_1 | A_2 | A_3 | A_4 | A_5 | A_6 | A_7 |
| Frequency (F) Hertz (Hz) | 27.5 | 55 | 110 | 220 | 440 | 880 | ? | ? |

Question 6a (1 mark)

Write down what happens to the frequency F, as the octaves increase.

Question 6b (2 marks)



Deduce that all the A notes on the piano are within human hearing.

Question 6c (2 marks)

Find a formula for F in terms of n.

Question 6d (3 marks)

Hence, rearrange your formula for part (c) to give n in terms of F .

Question 6e (2 marks)

A note has frequency 28 160 Hz. Is it an A note? **Justify** your answer.

Question 6f (10 marks)

A new design for a piano will contain all of the A notes within human hearing.

Use the information provided in the tabs to **discuss** the practicality of the new design for the piano.

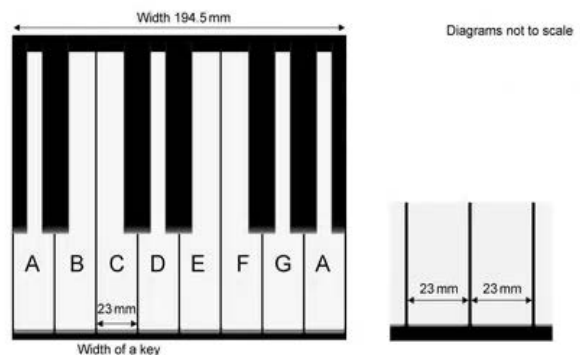
In your discussion you should:

- identify the relevant information for the new design
- calculate relevant measurements for the new design
- justify your degree of accuracy
- justify the practicality of the new design.

Piano



Keys

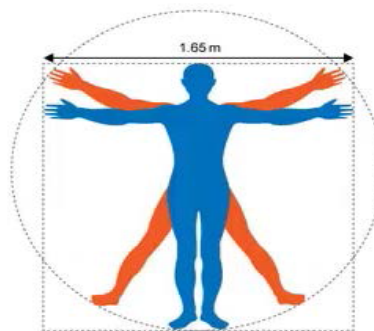


Hearing range



Humans can only hear between 20 and 20 000 Hertz.

Arm span



Question 7 (18 marks)

Ancient civilizations built pyramids to express cultural identity. In this question you will make calculations for a model of Chichén Iztá pyramid.

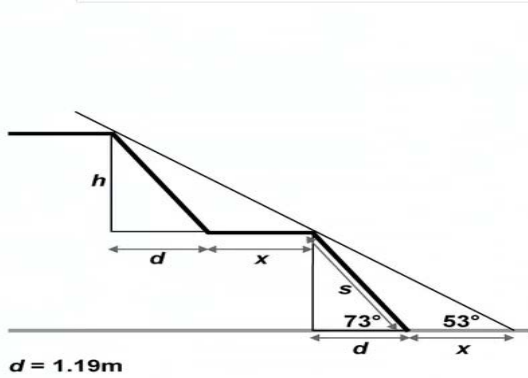


Chichén Iztá pyramid is composed of nine platforms. The width of the first platform is 55.3 m and on the ninth platform there is a temple that is 6 m high.

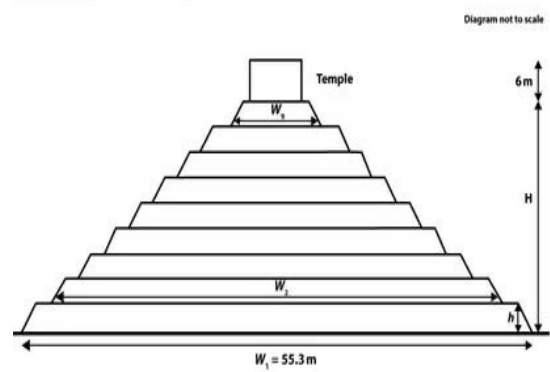
All lengths provided are given to the nearest centimetre (cm) and angles are to the nearest whole degree.

You are planning to construct a scale model of the Chichén Iztá Pyramid and will use 1:100 as scale for your model. In this question you will work out values of lengths. The validity of the accuracy of your model will be required in part (e).

Tab 1



Tab 2



Question 7a (2 marks)

Using the diagram in Tab 1, **show that** the sloping length $s = 4.07$ m to the nearest cm.

Question 7b (2 marks)

Using the diagram in Tab 1, **find** the height h of each platform in your model.

Model height h cm

Question 7c (5 marks)

Using the diagram in Tab 2, **find** the width W_2 for the second platform in your model.

Model width W_2 cm

Question 7d (3 marks)

Using the diagram in Tab 2, **find** the width W_9 of the top platform in your model.

Model width W_9 cm

Question 7e (6 marks)

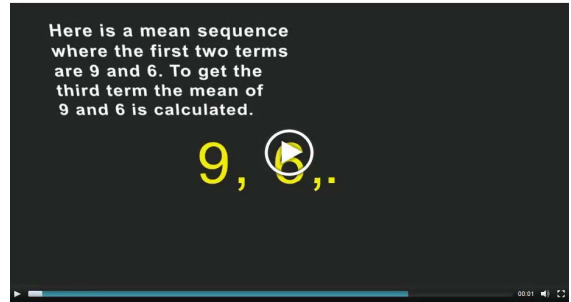
Discuss the validity of your model for Chichén Iztá pyramid. In your discussion you should:

- identify relevant information required to discuss the validity
- consider the implications of your chosen degree of accuracy
- comment on the validity of the model to Chichén Iztá.

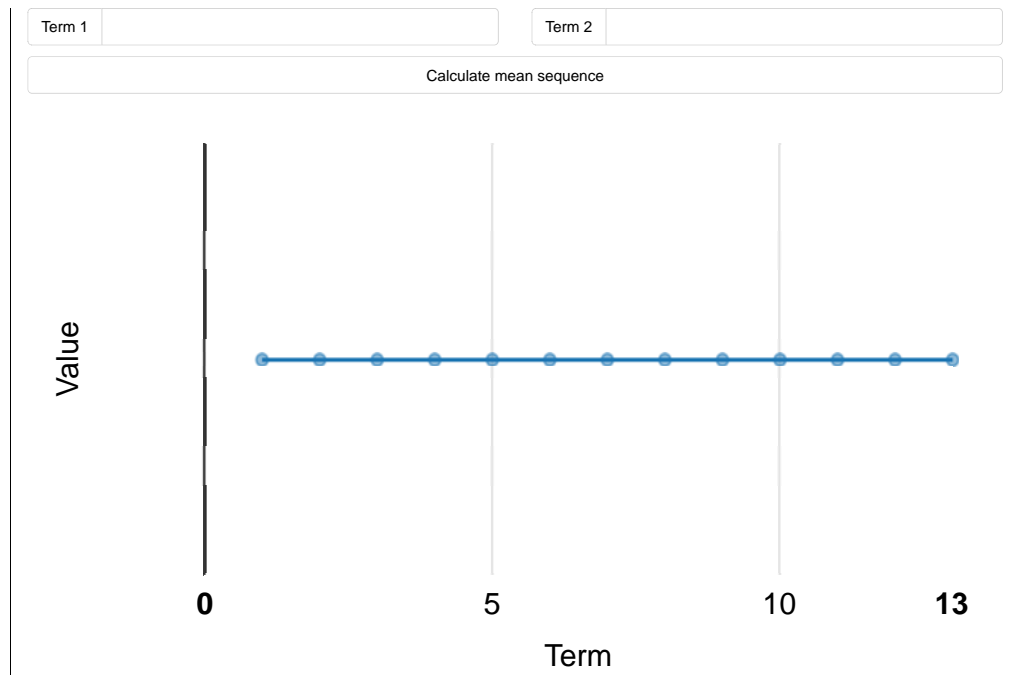
Question 8 (19 marks)

A recursive sequence is one where the next term is a specific combination of the previous terms.

In this question the recursive sequence is created where the next term is the mean of the previous two terms. This is called a mean sequence.



You can use the simulator to find the limit of a mean sequence. Input the first two terms of the sequence into the simulator and click 'Calculate mean sequence'.



Question 8a (1 mark)

Here is a mean sequence with starting terms 6 and 9.

6, 9, 7.5, 8.25, 7.875, ...

Write down the limit of the sequence.

All mean sequences have limits and there is a connection between the first two terms a and b and the limit L of each sequence.

Question 8b (2 marks)

The table below shows the first two terms and the limit L , for some mean sequences. The different starting terms a are provided and b is fixed at 3.

Write down the limit values L , for row 2, 3 and 4 in the table for the mean sequences provided. Use the simulator to help you.

Question 8c (2 marks)

Describe in words **two** patterns you have found from the table.

Question 8d (2 marks)

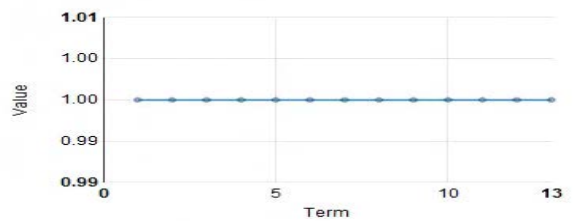
Find a general rule connecting a and L .

Question 8e (12 marks)

Using your previous results, **investigate** and **find** a general rule connecting a , b and L . The simulator and a blank table is provided below to support your investigation. In your answer you should:

- describe patterns
- find a general rule
- test your general rule
- use correct mathematical notation
- notation prove or verify and justify your general rule.

Term 1 Term 2
Calculate mean sequence



Question 9 (20 marks)

We can also use an algebraic approach to investigate mean sequences.

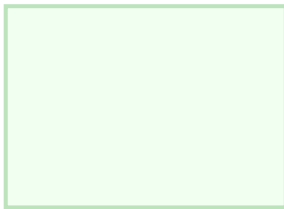
If a and b are the first two terms, the mean sequence will start with the terms below.

$$a, b, \frac{1}{2}a + \frac{1}{2}b, \dots$$

Question 9a (2 marks)

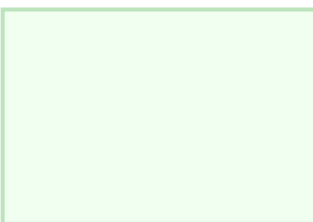
Show that the fourth term is $\frac{1}{4}a + \frac{3}{4}b$.

| Term (T) | Sequence | Change in coefficient of b |
|----------|----------------------------------|------------------------------|
| 1 | a | |
| 2 | b | + 1 |
| 3 | $\frac{1}{2}a + \frac{1}{2}b$ | $-\frac{1}{2}$ |
| 4 | $\frac{1}{4}a + \frac{3}{4}b$ | $+\frac{1}{4}$ |
| 5 | $\frac{3}{8}a + \frac{5}{8}b$ | $-\frac{1}{8}$ |
| 6 | $\frac{5}{16}a + \frac{11}{16}b$ | $+\frac{1}{16}$ |



Question 9b (4 marks)

Write down the values of x and y and explain how this series relates to the sequence in the table above.



Series

$$1 = 1$$

$$1 - \frac{1}{2} = \frac{1}{2}$$

$$1 - \frac{1}{2} + \frac{1}{4} = x$$

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} = y$$

Question 9c (14 marks)

Investigate the algebraic approach further to **prove** or **verify** and **justify** the formula in question 8e.

In your answer you should:

- describe patterns
- find one or more general rules
- test your general rule(s)
- use correct mathematical notation
- prove or verify and justify your general rule(s).

