Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2022

Mathematics: applications and interpretation

Higher level

Paper 3

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1****,* ***A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) (i) mean $= 4.23$ $(4.23333\ldots)$ ***A1***

variance $= 4.27$ $(4.26777...)$ ***A1***

[2 marks]

(ii) mean is close to the variance ***A1***

[1 mark]

(b) *One of the following:*
the number of bags sold each day is independent of any other day
the sale of one bag is independent of any other bag sold
the sales of bags of rice (each day) occur at a *constant mean* rate ***A1***

Note: Award ***A1*** for a correct answer in context. Any statement referring to independence must refer to either the independence of each bag sold or the independence of the number of bags sold each day. If the third option is seen, the statement must refer to a "constant mean" or "constant average". Do not accept "the number of bags sold each day is constant".

[1 mark]

(c) attempt to find Poisson probabilities and multiply by 90 ***(M1)***
$a = 7.018$ ***A1***
$b = 17.498$ ***A1***

EITHER
$90 \times \mathrm{P}(X \geq 8) = 90 \times \left(1 - \mathrm{P}(X \leq 7)\right)$ ***(M1)***
$c = 5.755$ ***A1***

OR
$90 - 7.018 - 11.903 - 16.665 - 17.498 - 14.698 - 10.289 - 6.173$ ***(M1)***
$c = 5.756$ ***A1***

Note: Do not penalize the omission of clear a, b and c labelling as this will be penalized later if correct values are interchanged.

[5 marks]

(d) (i) 7 ***A1***

[1 mark]

(ii) H_0 : The number of bags of rice sold each day follows a Poisson distribution with mean 4.2. ***A1***

H_1 : The number of bags of rice sold each day does not follow a Poisson distribution with mean 4.2. ***A1***

Note: Award ***A1A1*** for **both** hypotheses correctly stated and in correct order. Award ***A1A0*** if reference to the data and/or "mean 4.2" is not included in the hypotheses, but otherwise correct.

evidence of attempting to group data to obtain the observed frequencies for ≤ 1 and ≥ 8 ***(M1)***

p-value $= 0.728$ $(0.728100...)$ ***A2***

0.728 $(0.728100...) > 0.05$ ***R1***

the result is not significant so there is no reason to reject H_0
(the number of bags sold each day follows a Poisson distribution) ***A1***

Note: Do not award ***R0A1***. The conclusion MUST follow through from their hypotheses. If no hypotheses are stated, the final ***A1*** can still be awarded for a correct conclusion as long as it is in context (e.g. therefore the data follows a Poisson distribution).

[7 marks]

(e) (i) **METHOD 1**

evidence of multiplying 4.2×60 (seen anywhere) ***M1***

$H_0 : \mu = 252$

$H_1 : \mu > 252$ ***A1***

Note: Accept $H_0 : \mu = 4.2$ and $H_1 : \mu > 4.2$ for the ***A1***.

evidence of finding probabilities around critical region ***(M1)***

Note: Award ***(M1)*** for any of these values seen:
$P(X \geq 277) = 0.0630518\ldots$ **OR** $P(X \leq 276) = 0.936948\ldots$
$P(X \geq 278) = 0.0558415\ldots$ **OR** $P(X \leq 277) = 0.944158\ldots$
$P(X \geq 279) = 0.0493055\ldots$ **OR** $P(X \leq 278) = 0.950694\ldots$.

critical value $= 279$ ***A1***

$282 \geq 279$, ***R1***

the null hypothesis is rejected ***A1***

(the advertising increased the number of bags sold during the 60 days)

Note: Do not award ***R0A1***. Accept statements referring to the advertising being effective for ***A1*** as long as the ***R*** mark is satisfied. For the ***R1A1***, follow through within the part from their critical value.

METHOD 2

evidence of dividing 282 by 60 (or 4.7 seen anywhere) ***M1***

$H_0 : \mu = 4.2$

$H_1 : \mu > 4.2$ ***A1***

attempt to find critical value using central limit theorem ***(M1)***

(e.g. sample standard deviation $= \sqrt{\frac{4.2}{60}}$, $\bar{X} \sim N\left(4.2, \sqrt{\frac{4.2}{60}}\right)$, etc.)

Note: Award ***(M1)*** for a p-value of $0.0293907\ldots$ seen.

critical value $= 4.63518\ldots$ ***A1***

$4.7 > 4.63518\ldots$ ***R1***

the null hypothesis is rejected ***A1***

(the advertising increased the number of bags sold during the 60 days)

Note: Do not award ***R0A1***. Accept statements referring to the advertising being effective for ***A1*** as long as the ***R*** mark is satisfied. For the ***R1A1***, follow through within the part from their critical value.

[6 marks]

(ii) $\left(P(X \geq 279 \mid \mu = 252) =\right)$ 0.0493 $(0.0493055\ldots)$ ***A1***

Note: If a candidate uses **METHOD 2** in part (e)(i), allow an ***FT*** answer of 0.05 for this part but only if the candidate has attempted to find a p-value.

[1 mark]

(f) attempt to compare profit *difference* with cost of advertising ***(M1)***

Note: Award ***(M1)*** for evidence of candidate mathematically comparing a profit difference with the cost of the advertising.

EITHER
(comparing profit from 30 extra bags of rice with cost of advertising)
$14850 < 18000$ ***A1***

OR
(comparing total profit with and without advertising)
$121590 < 124740$ ***A1***

OR
(comparing increase of average daily profit with daily advertising cost)
$247.50 < 300$ ***A1***

THEN

EITHER
Even though the number of bags of rice increased, the advertising is not worth it as the overall profit did not increase. ***R1***

OR
The advertising is worth it even though the cost is less than the increased profit, since the number of customers increased (possibly buying other products and/or returning in the future after advertising stops) ***R1***

Note: Follow through within the part for correct reasoning consistent with their comparison.

[3 marks]

[Total 27 marks]

2. (a) $AF^2 = 89.2^2 + 104.9^2 - 2(89.2)(104.9)\cos 83$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into the cosine rule and ***(A1)*** for correct substitution.

$AF = 129 \text{ m} \quad (129.150...)$ ***A1***

[3 marks]

(b) $21310 \div 129.150\ldots$ ***(M1)***

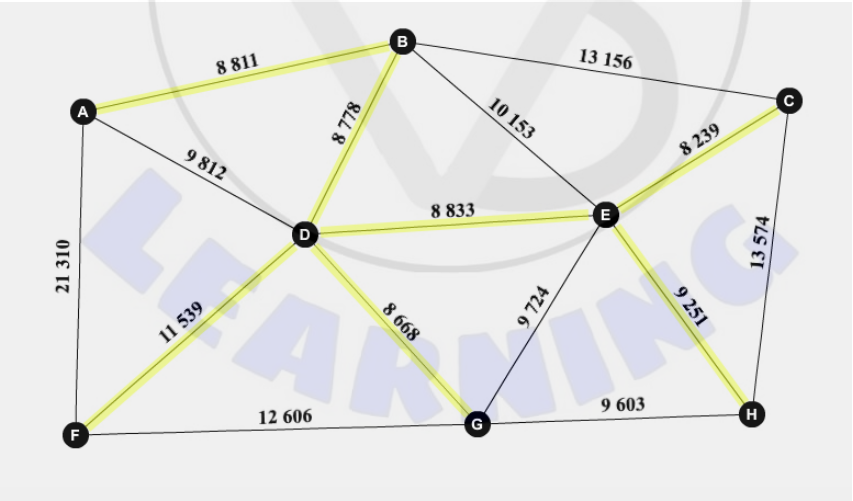
$\$ 165$ ***A1***

[2 marks]

(c) any reasonable statement referring to the lake ***R1***
(eg. there is a lake between A and F, the cables would need to be installed under/over/around the lake, special waterproof cables are needed for lake, etc.)

[1 mark]

(d) (i) edges (or weights) are chosen in the order
CE (8239)
DG (8668)
BD (8778)
AB (8811)
DE (8833)
EH (9251)
DF (11 539) ***A1A1A1***



Note: Award ***A1*** for the first two edges chosen in the correct order. Award ***A1A1*** for the first six edges chosen in the correct order. Award ***A1A1A1*** for all seven edges chosen in the correct order. Accept a diagram as an answer, provided the order of edges is communicated**.**

[3 marks]

(ii) Finding the sum of the weights of their edges ***(M1)***
$8239 + 8668 + 8778 + 8811 + 8833 + 9251 + 11539$

total cost = $\$64\,119$ ***A1***

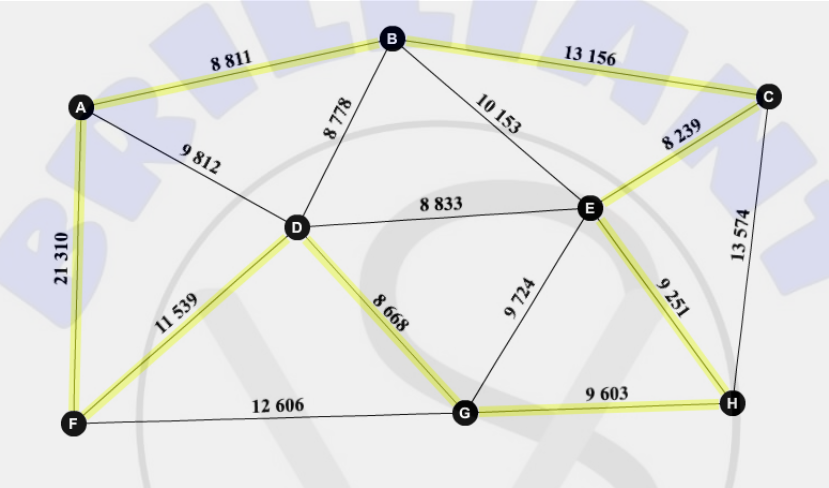
[2 marks]

(e) a Hamiltonian cycle is not always an Eulerian circuit as it does not have to include all edges of the graph (only all vertices) ***R1***

[1 mark]

(f) edges (or weights) are chosen in the order

DG (8668)
GH (9603)
HE (9251)
EC (8239)
CB (13 156)
BA (8811)
AF (21 310)
FD (11 539) ***A1A1A1***



Note: Award ***A1*** for the first two edges chosen in the correct order. Award ***A1A1*** for the first five edges chosen in the correct order. Award ***A1A1A1*** for all eight edges chosen in the correct order. Accept a diagram as an answer, provided the order of edges is communicated.

finding the sum of the weights of their edges ***(M1)***

$8668+9603+9251+8239+13156+8811+21310+11539$

upper bound = \$90 577 ***A1***

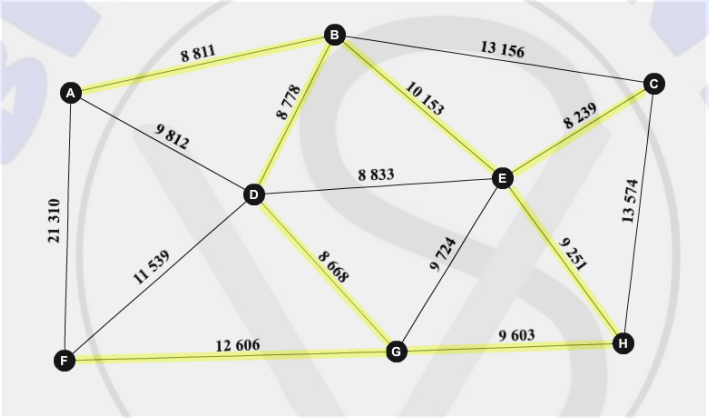
[5 marks]

(g) attempt to find MST after deleting vertex D ***(M1)***
these edges (or weights) (in any order)
CE (8239)
AB (8811)
EH (9251)
GH (9603)
BE (10 153)
FG (12 606) ***A1***

Note: Prim's or Kruskal's algorithm could be used at this stage.

reconnect D to MST with two different edges ***(M1)***
DG (8668)
BD (8778) ***A1***

Note: This ***A1*** is independent of the first ***A*** mark and can be awarded if both DG and BD are chosen to reconnect D to the MST, even if the MST is incorrect.



finding the sum of the weights of their edges ***(M1)***
$8239+8811+9251+9603+10153+12606+8668+8778$

Note: For candidates with an incorrect MST or no MST, the weights of at least seven of the edges being summed (two of which must connect to D) must be shown to award this ***(M1)***.

lower bound = \$76 109 ***A1***

[6 marks]

(h) **METHOD 1**

recognition of a binomial distribution ***(M1)***

$X \sim \mathrm{B}(2,\ 0.014)$

finding the probability that a cable fails (at least one of its connections fails)

$\mathrm{P}(X > 0) = 0.027804$ **OR** $1 - \mathrm{P}(X = 0) = 0.027804$ ***A1***

recognition that **two** cables must fail for the network to go offline ***M1***

recognition of binomial distribution for network, $Y \sim \mathrm{B}(8,\ 0.027804)$ ***(M1)***

$\mathrm{P}(Y \geq 2) = 0.0194\ \ (0.0193602...)$ **OR** $1 - \mathrm{P}(Y < 2) = 0.0194\ \ (0.0193602\ldots)$ ***A1***

therefore, the diagram satisfies the requirement since $1.94\% < 2\%$ ***AG***

Note: Evidence of binomial distribution may be seen as combinations.

METHOD 2

recognition of a binomial distribution ***(M1)***

$X \sim \mathrm{B}(16,\ 0.014)$

finding the probability that at least **two** connections fail

$\mathrm{P}(X \geq 2) = 0.0206473\ldots$ **OR** $1 - \mathrm{P}(X < 2) = 0.0206473\ldots$ ***A1***

recognition that the previous answer is an overestimate ***M1***

finding probability of two ends of the same cable failing, $F \sim \mathrm{B}(2,\ 0.014)$,
and the ends of the other 14 cables not failing, $S \sim \mathrm{B}(14, 0.014)$

$\mathrm{P}(F = 2) \times \mathrm{P}(S = 0) = 0.0000160891\ldots$ ***(A1)***

$0.0000160891\ldots \times 8 = 0.00128713\ldots$

$0.0206473\ldots - 0.00128713\ldots = 0.0194\ \ (0.0193602\ldots)$ ***A1***

therefore, the diagram satisfies the requirement since $1.94\% < 2\%$ ***AG***

METHOD 3

recognition of a binomial distribution ***M1***

$X \sim \mathrm{B}(16,\ 0.014)$

finding the probability that the network remains secure if 0 or 1 connections fail or if 2 connections fail provided that the second failed connection occurs at the other end of the cable with the first failure ***(M1)***

$\mathrm{P}(\text{remains secure}) = \mathrm{P}(X \leq 1) + \frac{1}{15} \times \mathrm{P}(X = 2)$ ***A1***

$= 0.9806397625$ ***A1***

$\mathrm{P}(\text{network fails}) = 1 - 0.9806397625 = 0.0194\ \ (0.0193602...)$ ***A1***

therefore, the diagram satisfies the requirement since $1.94\% < 2\%$ ***AG***

METHOD 4

$\mathrm{P}(\text{network failing})$

$= 1 - \mathrm{P}(0 \text{ connections failing}) - \mathrm{P}(1 \text{ connection failing})$

$\quad - \mathrm{P}(2 \text{ connections on the same cable failing})$ ***M1***

$= 1 - 0.986^{16} - {}^{16}C_1 \times 0.014 \times 0.986^{15} - {}^{8}C_1 \times 0.014^2 \times 0.986^{14}$ ***A1A1A1***

Note: Award ***A1*** for each of 2nd, 3rd and last terms.

$= 0.0194 \ \ (0.0193602\ldots)$ ***A1***

therefore, the diagram satisfies the requirement since $1.94\% < 2\%$ ***AG***

[5 marks]

[Total 28 marks]