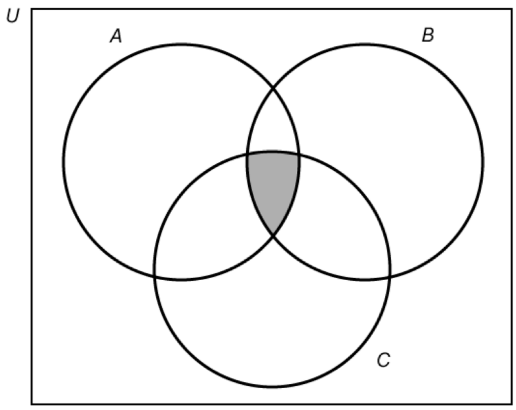
Question 1

Question 1 (7 marks)

Question 1a (1 mark)

Diagram 1



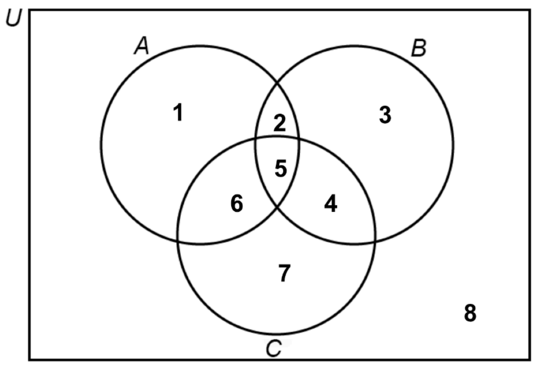
Using Diagram 1, **write down** an expression for the shaded area in set notation using the symbol ∩.

Styles

The elements of the universal set (*U*) are {1,2,3,4,5,6,7,8}. Click inside the sections of the Venn diagram below to interact with the simulation.

Venn diagram

This media is interactive



©

Question 1b (1 mark)

List the elements of A ∩ C.

Styles

Question 1c (2 marks)

List the elements of A ∩ (B ∪ C) and (A ∩ B) ∪ (A ∩ C).

Question 1d (1 mark)

Compare A ∩ (B ∪ C) with (A ∩ B) ∪ (A ∩ C).

Question 1e (2 marks)

Write down (A ∩ B) ∪ (B′ ∩ A) in its simplest equivalent set form.

Question 2

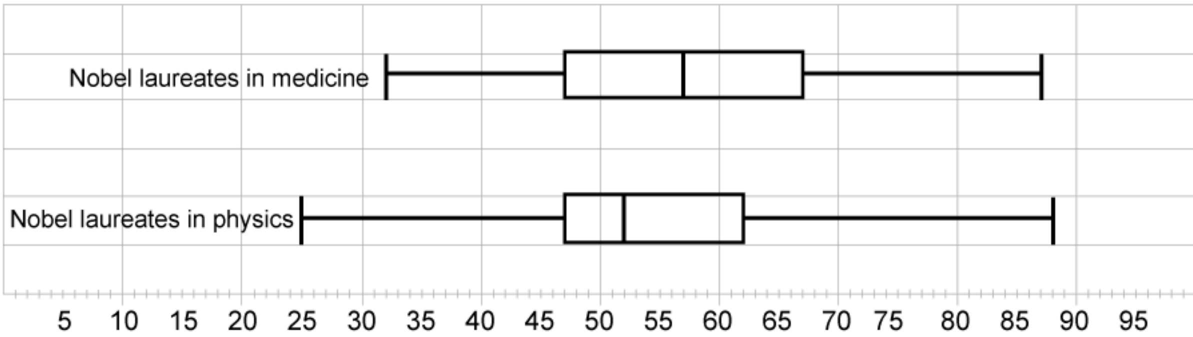
Question 2 (10 marks)

The Nobel Prize is a set of awards presented annually in recognition of academic, cultural and scientific advances. Every year, the Nobel Prize is awarded in recognition of discoveries made. Winners of a Nobel Prize are called Nobel laureates.

The box-and-whisker plots below show the distribution of the ages of the medicine and physics laureates the year they received their Nobel Prize. The data is taken from 2016.

Hover over each box-and-whisker plot to reveal the values.

This media is interactive



Question 2a (4 marks)

Interpret the distribution of the ages of the Nobel Prize laureates in physics and medicine by considering the medians and interquartile ranges in the context provided.

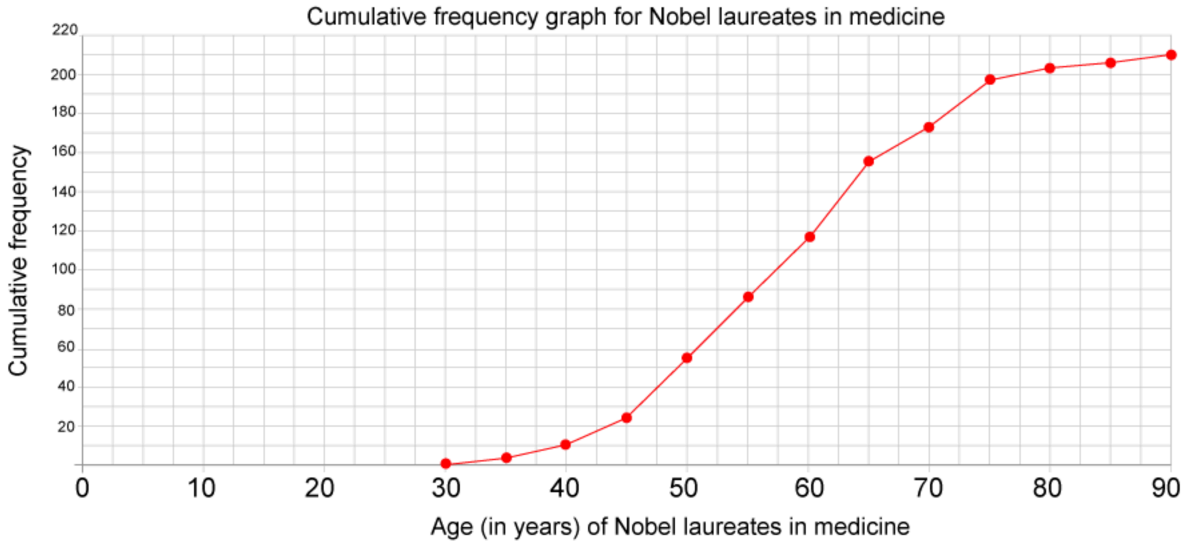
B I U Styles

Cumulative frequency graph | Cumulative frequency table

The cumulative frequency graph below shows the age (in years) of 210 Nobel laureates in medicine.

Hover over the points to reveal the coordinates.

This media is interactive



Cumulative frequency graph | Cumulative frequency table

Age (in years) of Nobel laureates in medicine	Cumulative frequency
Under 30	0
Under 35	3
Under 40	12
Under 45	22
Under 50	55
Under 55	88
Under 60	118
Under 65	156
Under 70	172
Under 75	189
Under 80	201
Under 85	206
Under 90	210

Question 2b (2 marks)

Using the cumulative frequency graph or table, **find** an estimate for the probability that the next Nobel laureate in medicine will be over 50 years old.

Twelve of the Nobel Prize laureates in medicine are women and one of them was below 50 years old when she was awarded the Nobel Prize.

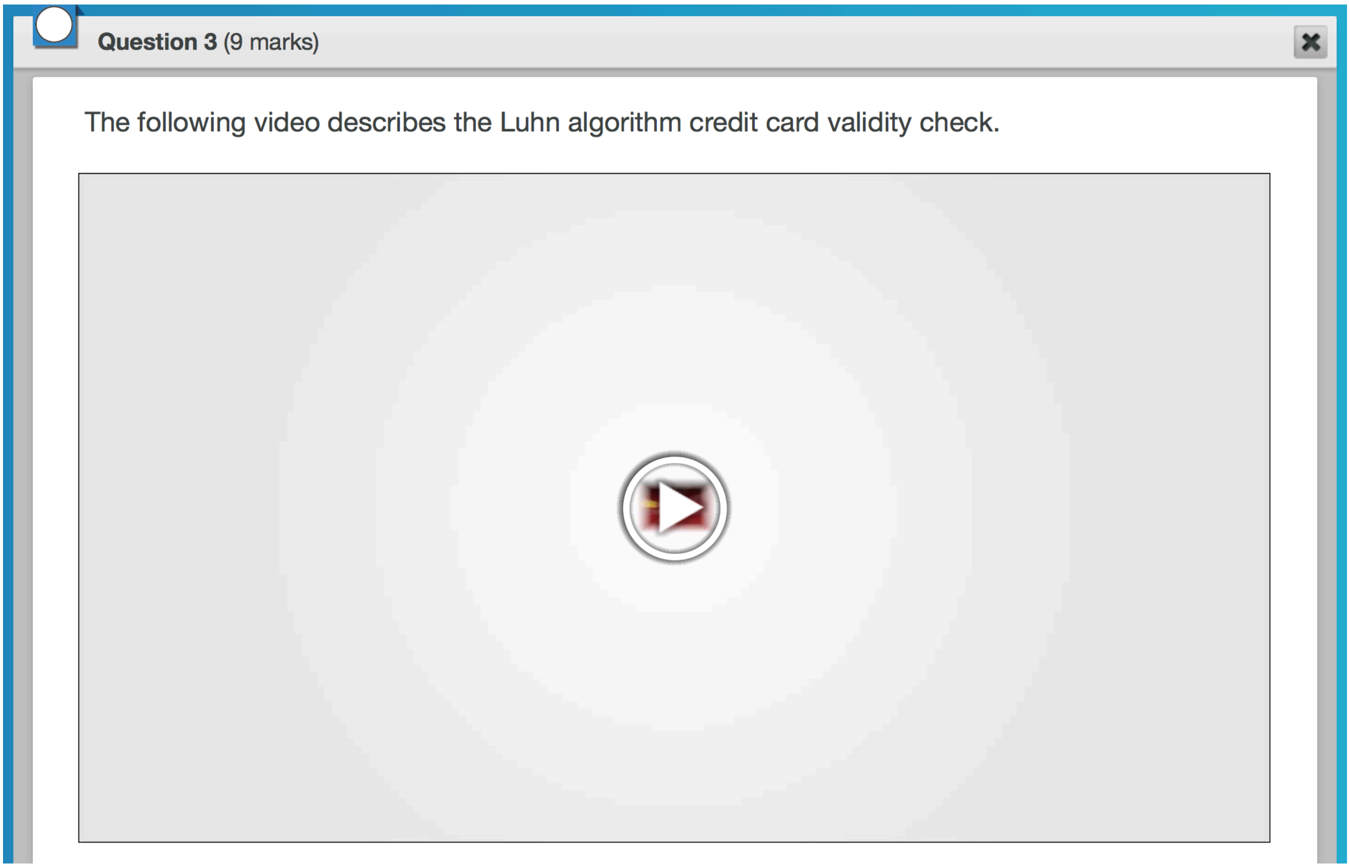
Question 2c (2 marks)

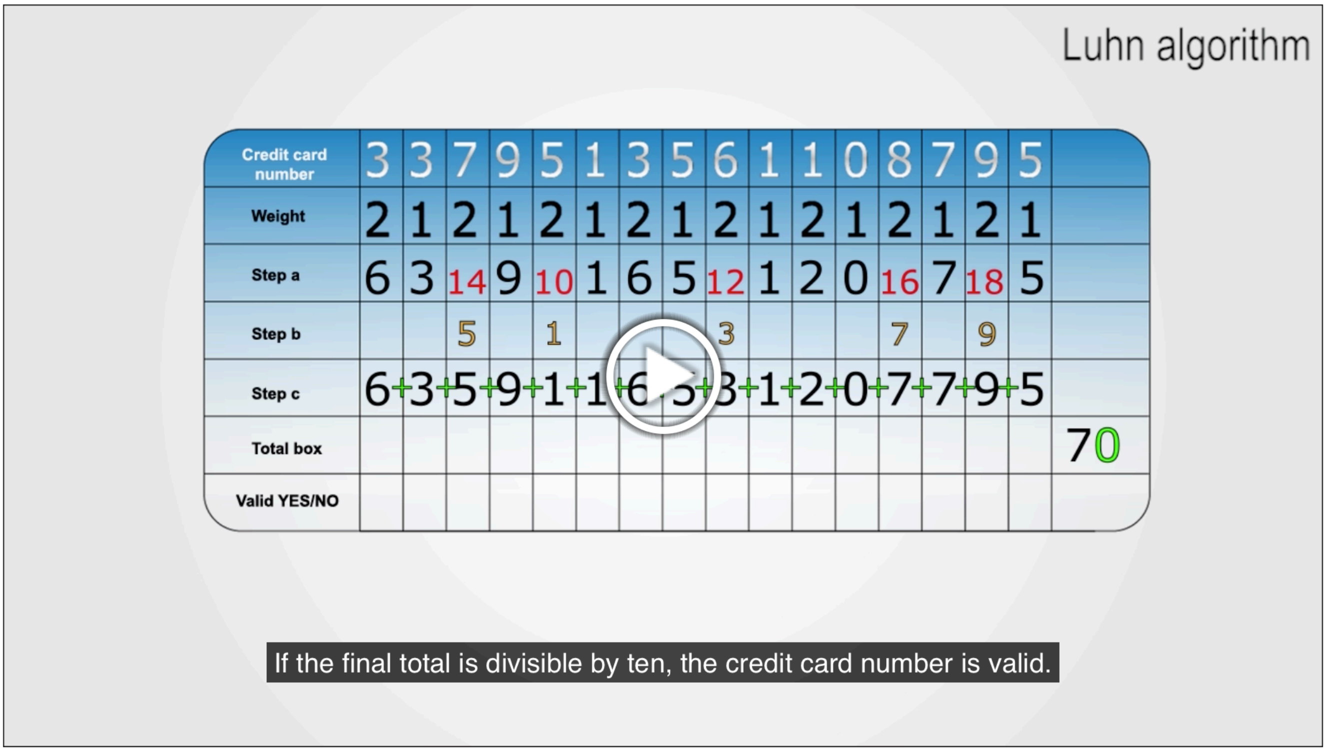
Find an estimate for the probability that the next Nobel laureate in medicine is a woman, given that the laureate is over 50 years old.

Question 2d (2 marks)

Given that 20 % of the Nobel Prize laureates in medicine are over x years old, **find** an estimate for the age x.

Question 3





Credit card number	3	3	7	9	5	1	3	5	6	1	1	0	8	7	9	5	
Weight	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	
Step a	6	3	14	9	10	1	6	5	12	1	2	0	16	7	18	5	
Step b			5		1				3				7		9		
Step c	6+	3+	5+	9+	1+	1+	6+	5+	3+	1+	2+	0+	7+	7+	9+	5	
Total box																	70
Valid YES/NO																	

Question 3a (2 marks)

Determine the missing digits represented by the letters A, B, C, D and E to complete the table below.

To insert your answers in the table, click inside the box and replace the letters with your answers in the "Add Label" box.

Credit card number	1	5	4	3	4	1	3	8	7	2	3	5	8	3	4	6
Weight	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1
Step a	2	5	8	3	8	1	6	8	A	2	6	5	16	B	8	6
Step b																
Step c	2	5	8	3	8	1	6	8	C	2	6	5	D	E	8	6

Question 3b (2 marks)

Justify why the credit card number is invalid.

Question 3c (2 marks)

The credit card 4150 0811 1727 967X is a valid credit card number.

After the Luhn algorithm is applied, the final total is 61 + X. **Determine** the value of X.

Question 3d (3 marks)

Label the table above with further instructions F, G and H to clarify the workings of the algorithm. Some instructions have already been given.

Credit card number	**3379 5135 6110 8795**
Weight row	**Identify the digits that are doubled**
Product	**Double the appropriate digits**
Instruction F	
Result	**Write down the new digit after instruction F**
Instruction G	
Instruction H	

Reset

Please enter your data into the table (including any headings)

Question 4

Question 4 (8 marks)

Question 4a (4 marks)

The function f(x) has the following points within the domain $0 \le x \le 6$.

x	0	2	4	6
f(x)	−4	0	2	3

Find x for $2f(x-2) - 1 = 5$.

B I U x₂ x² Ω Σ
Styles

Question 4b (2 marks)

The functions f(x) and g(x) have the following points within the domain $0 \le x \le 6$.

Determine the missing values for f(g(x)) and complete the table below.

To insert your answer in the table, click inside the box and write your answer in the "Add Label" box.

x	0	2	4	6
f(x)	−4	0	2	3
g(x)	0	2	0	4
f(g(x))	−4		−4	

Question 4c (2 marks)

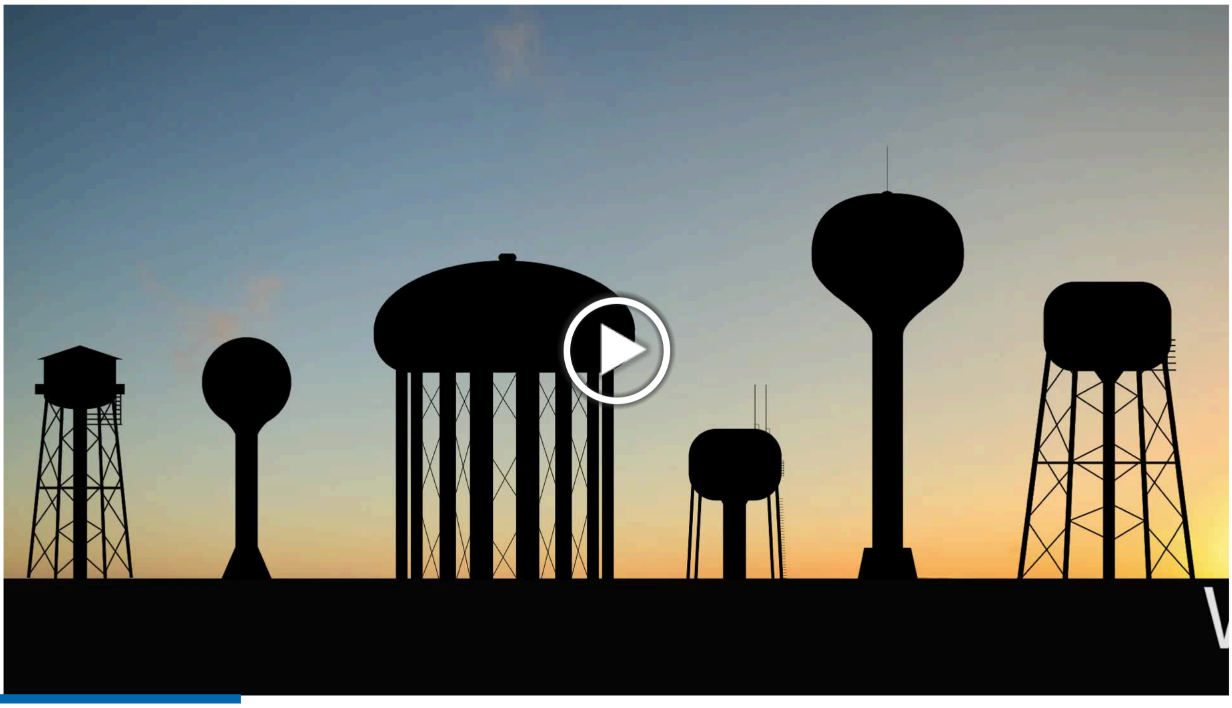
Hence or otherwise, **justify** why the inverse of f(g(x)) is not a function in the given domain.

B I U x₂ x² Ω Σ Styles

Question 5

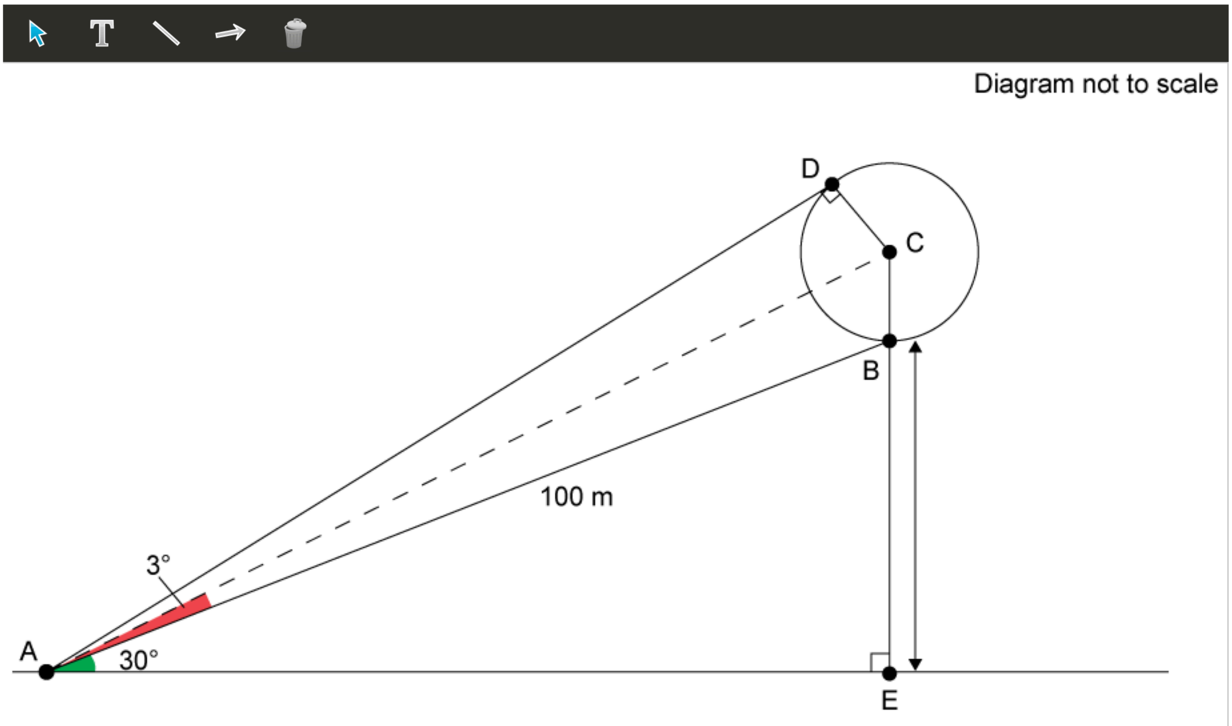
Question 5 (20 marks)

The following video describes how a water tower can be used to maintain residential water pressure for a community.



Question 5a (1 mark)

Diagram 1



The water is stored in a spherical water tank that is modelled by the cross section in Diagram 1, with centre C which is vertically above E. The distances are recorded to the nearest metre (m) and angles are recorded to the nearest degree. AB is 100 m, angle BAC is 3° and angle BAE 30°. Angles AEB and ADC are 90°.

Write down the size of angle BCA.

Question 5b (4 marks)

Show that BC, the length of the radius of the spherical water tank is 6.24 m, to the nearest cm.

Question 5c (3 marks)

Hence, **calculate** the volume of the spherical water tank to the nearest cubic metre (m^3).

Question 5d (2 marks)

Given that the maximum volume of the water held in the spherical water tank is 950 m^3, **identify two** possible reasons for the difference between the volume you calculated and the maximum volume.

Reason 1

B I U x_2 x^2 Ω Σ
Styles

Reason 2

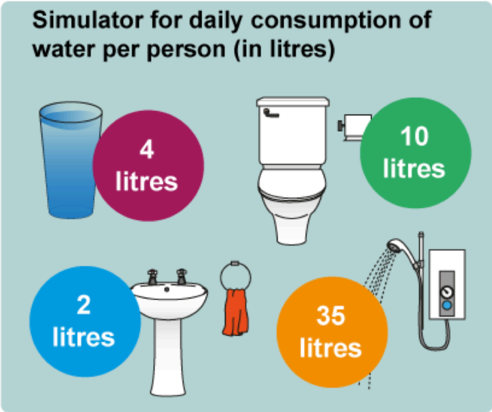
B I U x_2 x^2 Ω Σ
Styles

The MYP water tower serves the MYP county area with a population of approximately 300 000 people. On average there are four people per household and the typical water consumption distribution is shown in the table below (1000 litres = 1 m^3).

The water tower provides a supply of water for household use during a power outage. There will be a planned power outage for maintenance reasons on Tuesday 30th May 2017 between 9am and 1pm. You are responsible for gathering information on the possible impact to the water supply for the community during the power outage. An interactive simulator is provided for the different household activities.

Use the water consumption simulators below for the different activities.

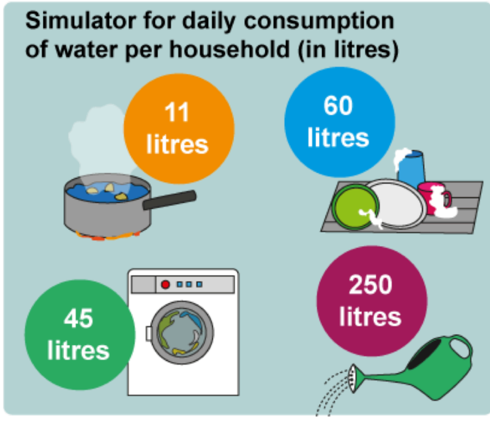
This media is interactive



- [ ] One shower (35 litres)
- [ ] Toilet (10 litres)
- [ ] Washing hands, face and teeth (2 litres)
- [ ] Drinking (4 litres)

Total water used:

This media is interactive



- [ ] Washing dishes (60 litres)
- [ ] Washing clothes (45 litres)
- [ ] Cooking (11 litres)
- [ ] Garden watering (250 litres)

Total water used:

Question 5e (10 marks)

Discuss the implications on water consumption for households in the community during this time.

In your answer you should:

- calculate the consumption in litres per hour (l/h) for the households and individuals in the community
- estimate the amount of time before the water (950 m^3) held in the tank runs out
- advise the community about the activities that should be avoided during the power outage
- justify the degree of accuracy of the time calculation.

Question 6

Question 6 (10 marks)

The following video explains the requirements for a reliable mobile phone signal.
Mobile phones are also known as cellular phones.

Identical transmitters are positioned at the vertices of an equilateral triangle in Diagram 1 and a square in Diagram 2.

In Diagram 1, the equilateral triangle has sides of length 20 km and area 173 km^2. The transmitters are placed at A, B and C, and they emit a frequency in a circular radius up to 10 km.

In Diagram 2, the square has sides of length 20 km and area 400 km^2. The transmitters are placed at A, B, C and D, and they emit a frequency in a circular radius up to 10 km.

Diagram 1

Diagram not to scale

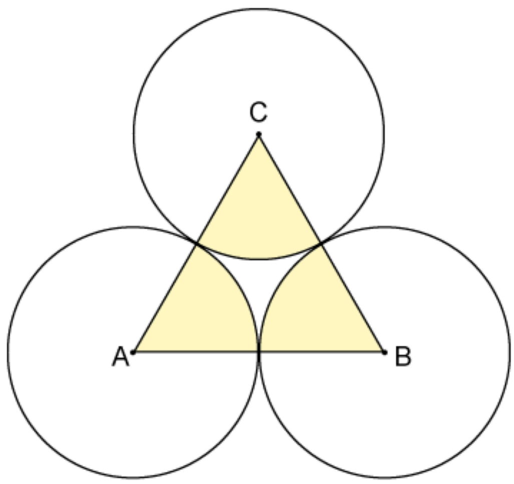
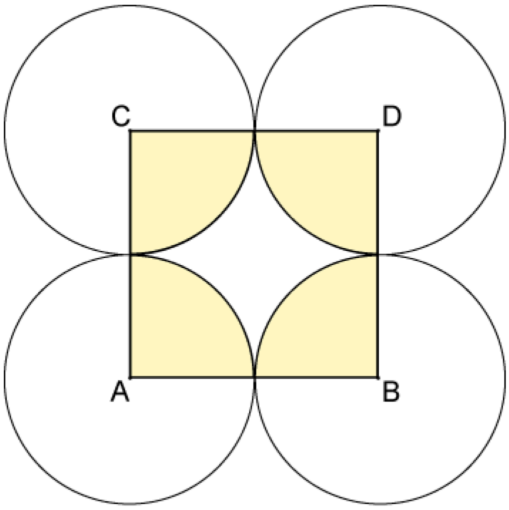


Diagram 2

Diagram not to scale



In Diagrams 1 and 2, you have been presented with two different ways of placing identical transmitters to provide a mobile phone signal. **Suggest** the most suitable layout for the most efficient use of space and the best signal coverage.

In your answer you should:

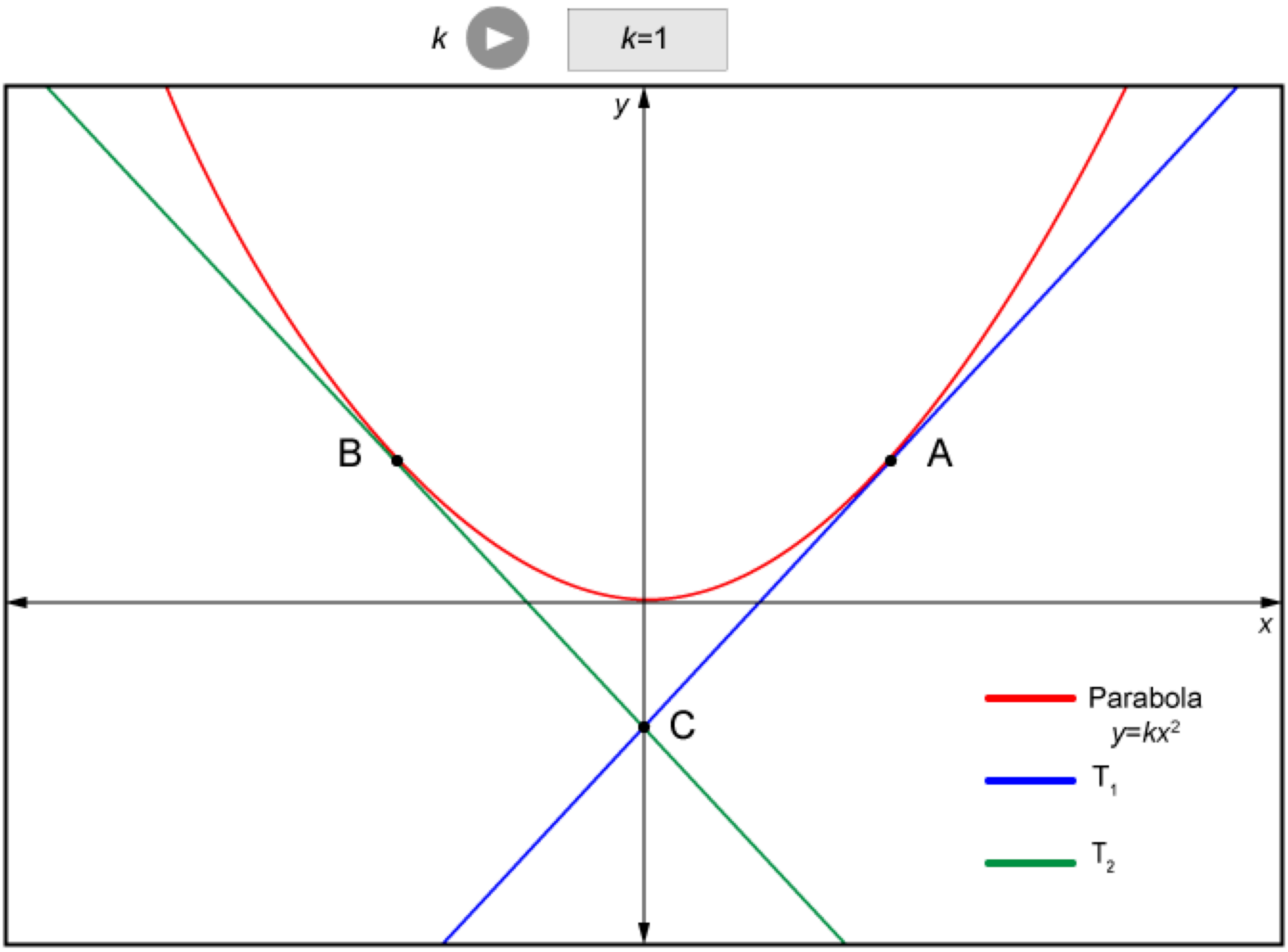
- find the area inside the triangle that does not receive a signal, this is shown as unshaded
- find the area inside the square that does not receive a signal, this is shown as unshaded
- compare the efficiency of the two ways the transmitters have been positioned
- justify the most efficient layout.

Question 7

Question 7 (36 marks)

Question 7a (2 marks)

Using the simulation below for the parabola $y = kx^2$, click on the arrow to see what happens as the value of k, the coefficient of x^2, changes. The lines T_1 and T_2 touch the parabola once as shown in the diagram at points A and B respectively. T_1 and T_2 intersect at the point C on the y-axis. The coordinates of A, B and C change with k. The gradient of T_1 is fixed at 1 and the gradient of T_2 is fixed at –1. The lines T_1 and T_2 are perpendicular. The line T_1 has equation $y = x + c$.



As the value of *k* changes, the following coordinates for A, B, and C have been recorded in the table below.

k	x coordinate of point A (x_A)	y coordinate of point A (y_A)	x coordinate of point B (x_B)	y coordinate of point B (y_B)	y coordinate of point C (y_C)
1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{-1}{2}$	$\frac{1}{4}$	$\frac{-1}{4}$
2	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{-1}{4}$	$\frac{1}{8}$	$\frac{-1}{8}$
3	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{-1}{\square}$	$\frac{1}{\square}$	$\frac{-1}{\square}$
4	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{-1}{\square}$	$\frac{1}{\square}$	$\frac{-1}{\square}$
5	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{-1}{\square}$	$\frac{1}{\square}$	$\frac{-1}{\square}$
$\square$	$\frac{1}{\square}$	$\frac{1}{\square}$	$\frac{-1}{\square}$	$\frac{1}{\square}$	$\frac{-1}{\square}$
$\square$	$\frac{1}{\square}$	$\frac{1}{\square}$	$\frac{-1}{\square}$	$\frac{1}{\square}$	$\frac{-1}{\square}$

Determine the missing values and complete the table above up to $k = 5$.

Question 7b (1 mark)

Write down a general rule for y_c in terms of k.

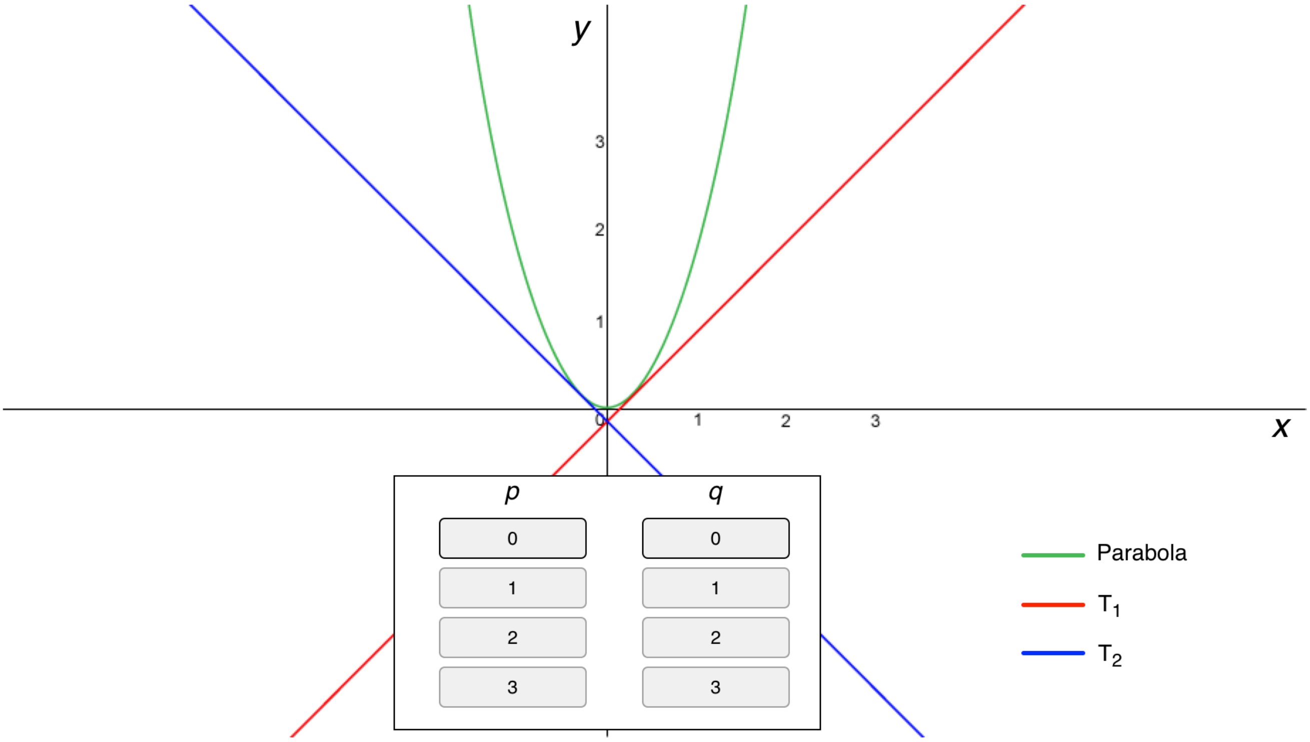
Question 7c (3 marks)

Verify your general rule for y_C.

Question 7d (3 marks)

Given that $x_A = \frac{1}{2k}$ and $x_C = 0$, use the gradient formula to **prove** your general rule for y_C.

The parabola $y = kx^2$ is translated by the vector $\begin{pmatrix} p \\ q \end{pmatrix}$. Using the simulation below you can investigate what happens to the parabola as p and q are changed.



The table below shows the values when k is fixed at 2, and p and q are also controlled.

k	p	q	x coordinate of point C (x_C)	y coordinate of point C (y_C)
2	0	0	0	$\frac{-1}{8}$
2	2	1	2	$\frac{7}{8}$
2	4	2	4	$\frac{15}{8}$
2	6	3	6	$\frac{23}{8}$
2	8	4	8	$\frac{31}{8}$

Question 7e (1 mark)

Write down the relationship between p and x_c.

Question 7f (2 marks)

Describe patterns in the sequence of values of y_c.

Question 7g (2 marks)

Determine a general rule for y_c in terms of q when $k = 2$.

Question 7h (22 marks)

Investigate a general rule for point (x_C , y_C) in terms of k, p, and q, where $p \neq 0$ and $q \neq 0$.
To support your investigation for different values of k, p and q, a simulator is available above to generate graphs and a table is provided for you to record your findings. In your answer you should:

- make predictions for more values of k
- describe the pattern
- find a general rule for x_C and y_C in terms of k, p and q
- test your general rule
- prove or verify and justify your general rule
- ensure you communicate the above appropriately.

B I U Styles

The table below is provided to support your investigation. Other values of k are provided. You can also use the blank table below to record your findings if necessary.

k	p	q	x coordinate of point C (x_C)	y coordinate of point C (y_C)
1	0	0	0	$\frac{-1}{4}$
1	2	1	2	$\frac{3}{4}$
2	0	0	0	$\frac{-1}{8}$
2	2	1	2	$\frac{7}{8}$
3	0	0	0	$\frac{-1}{12}$
3	2	1	2	$\frac{11}{12}$

k	p	q	x coordinate of point C (x_C)	y coordinate of point C (y_C)