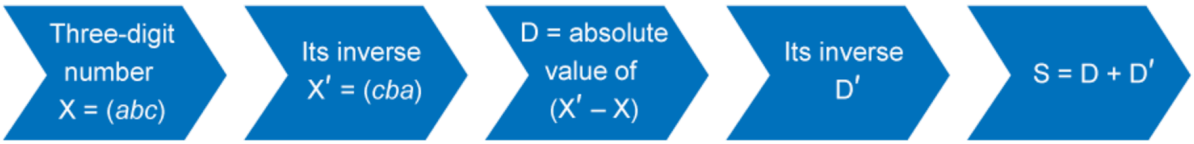
Question 1 (7 marks)

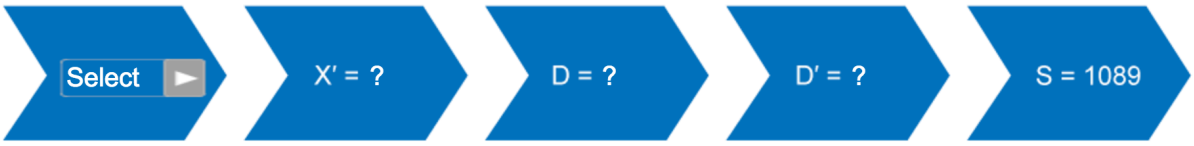
Question 1a (3 marks)

In this question, we will discover an interesting and magical property of three-digit numbers using a special algorithm, illustrated in the algorithm flow diagram below.

Algorithm flow diagram



Here is a simulator for the algorithm flow diagram which provides some examples of how this algorithm affects three-digit numbers. Select a number and see what happens.



Reset

Apply the same algorithm to 437 to complete the missing values.

Question 1b (2 marks)

A three-digit number can be written in terms of sum of multiples of its digits. For example, 437 can be written in the format shown below.

X is a three-digit number *abc*. **Write down** X and X′ as a sum of multiples of their digits.

X =

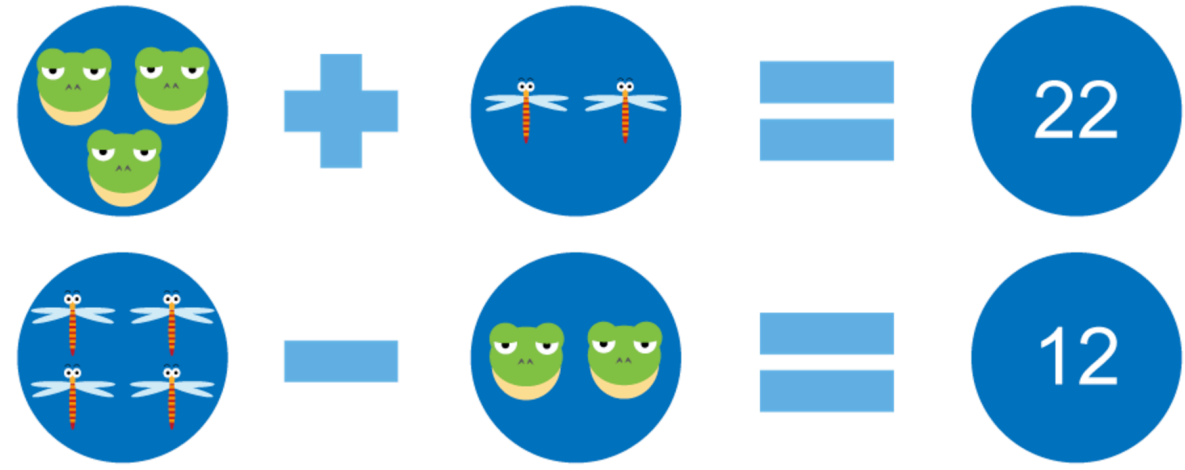
X′ =

Question 1c (2 marks)

Using your answer from part (b), **determine** the difference D in terms of *a* and *c*.

B I U x_2 x^2 Ω Σ Styles

Question 2 (6 marks)



©

Find the value of *X* for the following expression.

B I U x_2 x^2 Ω Σ Styles

Question 3 (8 marks)

Question 3a (4 marks)

To raise money for their graduation party, senior students organize a cookie and muffin sale.

x represents the number of cookies and y represents the number of muffins. The amount of cookies and muffins sold are represented by the shaded region in the diagram below.

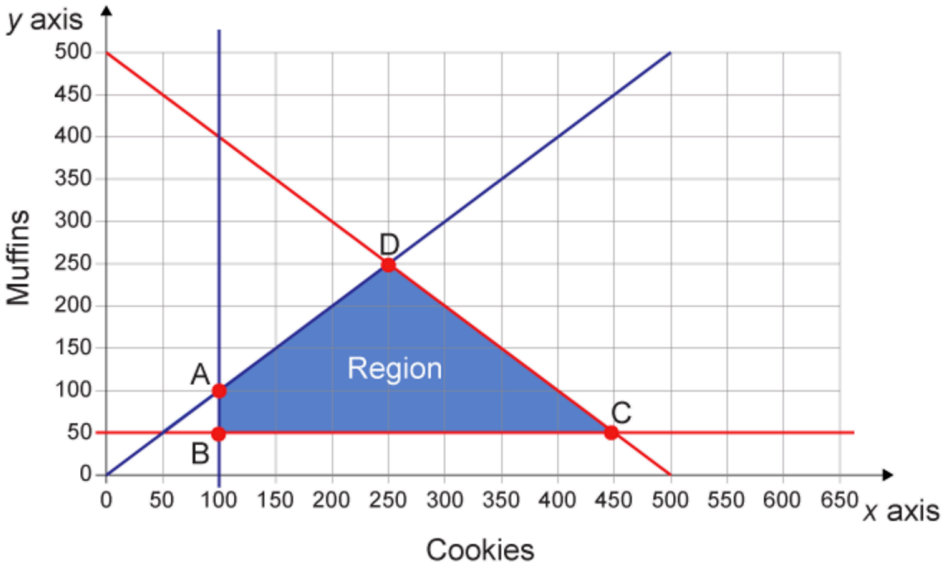
Using the information provided in the diagram below:

- **Identify** the shaded region by completing the inequalities below.
- The first constraint is that they must sell 100 cookies or more. **State** the other three constraints in the spaces provided.

Draggable inequalities: ≥ ≤

Inequalities	Constraints
$x \geq 100$	The number of cookies is at least 100
y ☐ 50	
y ☐ x	
$x + y$ ☐ 500	

Diagram



Question 3b (1 mark)

The profit P, in Canadian dollars (CAD), for the cookies and muffins sale is calculated using the formula $P = x + 1.5y$.

Interpret the meaning of the coefficients of x and y in the profit formula above.

Question 3c (3 marks)

The table below shows the profit at vertex A and vertex B.

Vertices (x, y)	$P = x + 1.5y$ (CAD)
A (100, 100)	250
B (100, 50)	175
C	
D	

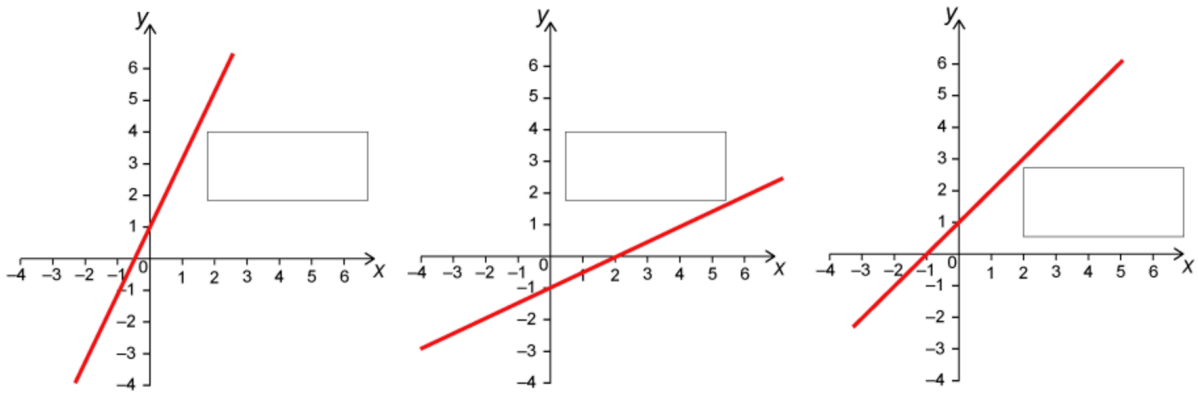
Find the number of cookies and the number of muffins that the students must sell in order to maximize their profit.

Question 4 (6 marks)

Question 4a (2 marks)

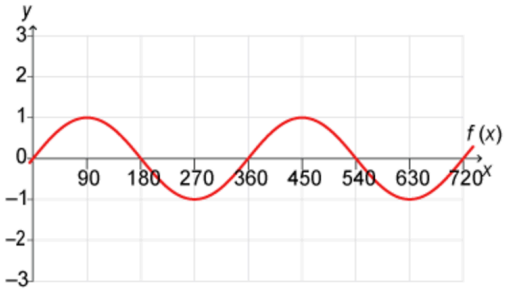
Select the line equations and place them with the corresponding graphs.

Draggable: $y = x + 1$ $y = 2x + 1$ $y = \frac{1}{2}x - 1$



Question 4b (4 marks)

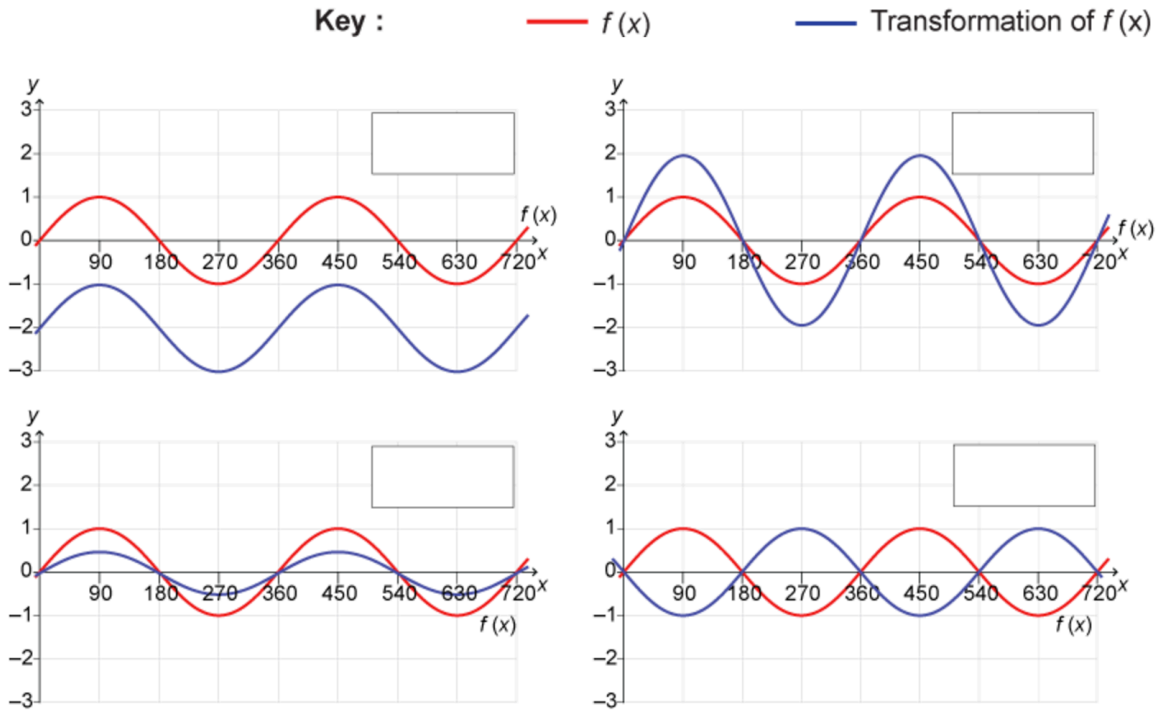
The graph below represents the function $f(x)$. Transformations of $f(x)$ are shown in the following graphs.



Select the equations and place them with the corresponding graphs.

Draggable:

- $f(x) + 2$
- $f(x) - 2$
- $2f(x)$
- $-f(x)$
- $\frac{1}{2}f(x)$
- $f(-x)$
- $f(2x)$
- $f\left(\frac{1}{2}x\right)$



Question 5 (8 marks)

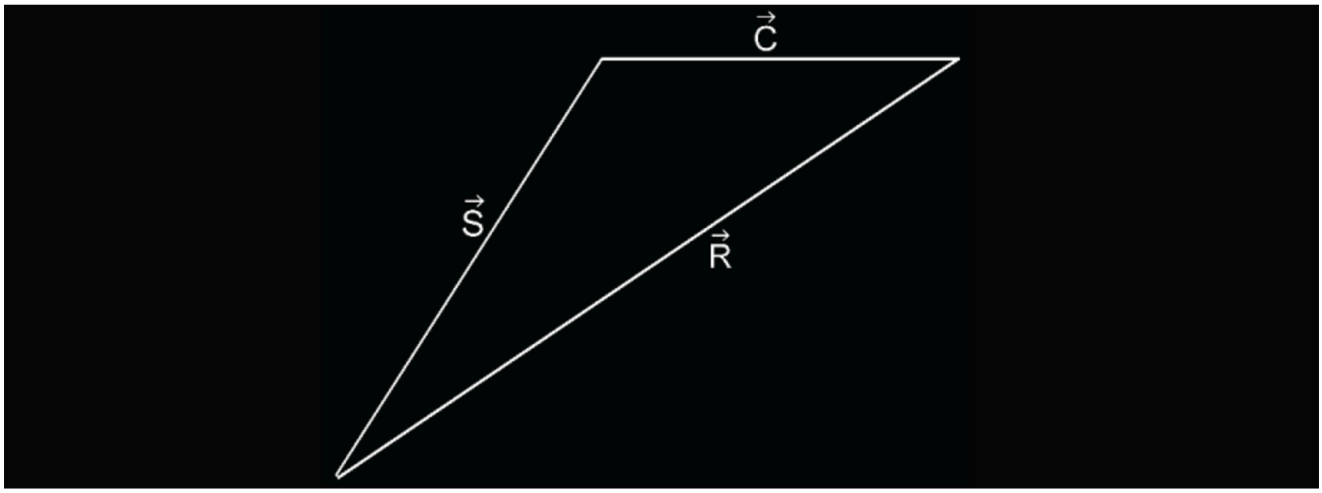
The animation illustrates the effect of the river current ($\vec{C}$) on the resultant direction ($\vec{R}$) of the boat.

The boat must adjust its sailing direction ($\vec{S}$) in order to arrive at its destination.

This media contains no audio

A boat sails with velocity vector $\vec{S} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$, which is given in component form.

Diagram not to scale



Question 5a (2 marks)

The speed is the magnitude of the velocity vector. **Determine** the speed of the boat.

Question 5b (1 mark)

A current flows with velocity vector $\vec{C} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$, **Show that** the resultant velocity vector $\vec{R}$ is $\begin{pmatrix} 8 \\ 12 \end{pmatrix}$ in component form.

Question 5c (5 marks)

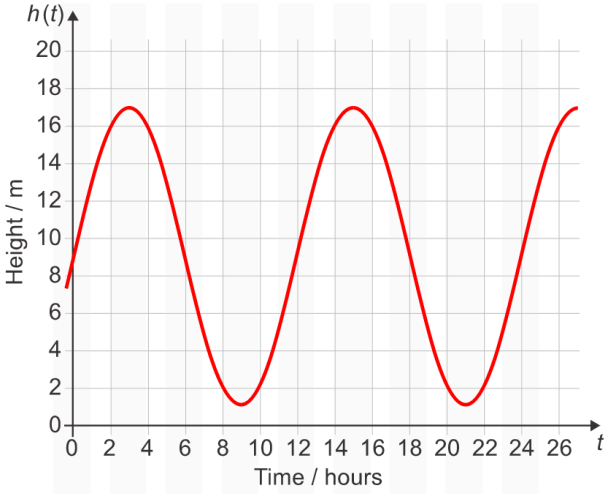
Find the angle between the resultant vector $\vec{R}$ and the vector $\vec{S}$.

Question 6 (10 marks)

The following video illustrates how tidal range can be modelled over time by a sine function.

Below is the sine curve modelling the tide in Saint-Malo on a day in November 2017.

$h(t)$ is the height in metres (m) of water in the harbour and t is the number of hours after midnight.



Question 6a (2 marks)

Determine the tidal range which is the difference between the height of the low and high tides in the harbour.

B I U x_2 x^2 Ω Σ Styles

Question 6b (2 marks)

The period can be measured as the time difference between two consecutive high tides. **Determine** the period of this tide.

Question 6c (6 marks)

The behaviour of this tide can be modelled by the function,

$$h(t) = 8\sin\left(\frac{\pi}{6}t\right)+9$$

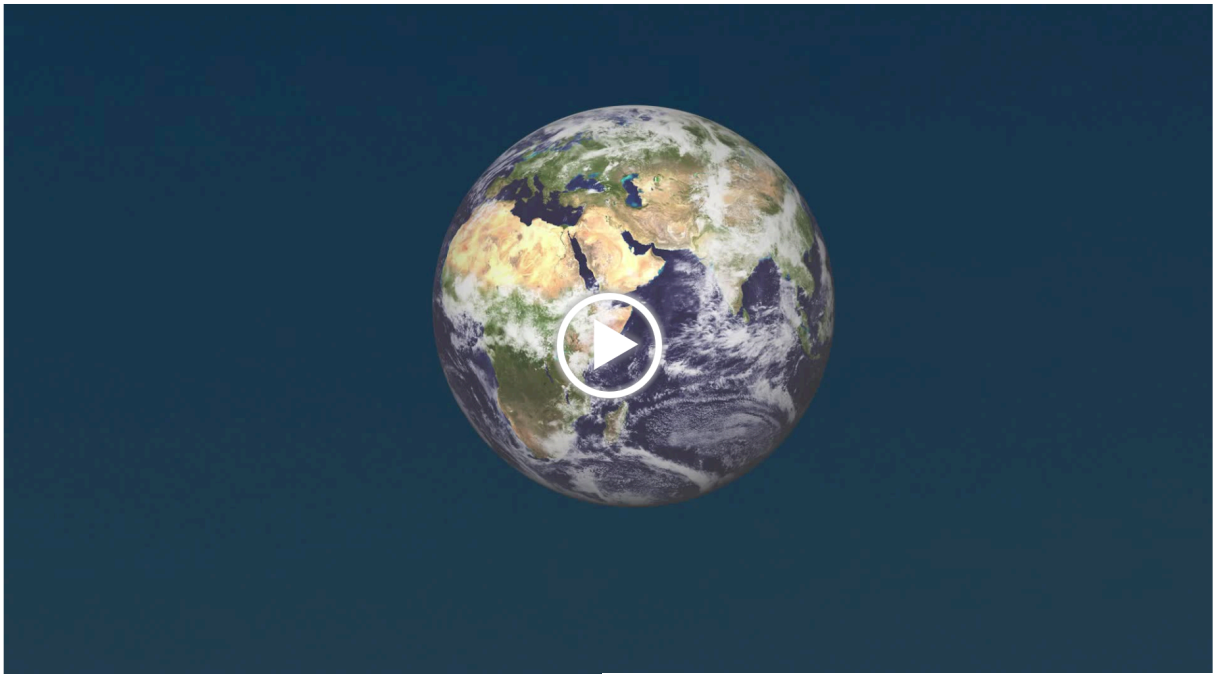
By substituting into the equation,

$$h(t) = 8\sin\left(\frac{\pi}{6}t\right)+9,$$

calculate at what times, during the first 12 hours after midnight, the height of water in the harbour is 13 m.

Question 7 (23 marks)

The following video shows how mathematics can be used to make predictions for population growth over time and space.



Tab 1 Tab 2

The table below shows the age distribution of the population of Australia (percentage to nearest 1 %, data correct as at 2015).

Age (A)	Percentage
$0 < A \leq 20$	23
$20 < A \leq 40$	32
$40 < A \leq 60$	27
$60 < A \leq 80$	17
$80 < A \leq 100$	1

Tab 1 Tab 2

The table below shows the country of birth of immigrants to Australia (data correct as at 2015, top ten countries only).

Country of birth	Number of immigrants
United Kingdom	1 207 000
New Zealand	611 400
China	481 800
India	432 700
Philippines	236 400
Vietnam	230 200
Italy	198 200
South Africa	178 700
Malaysia	156 500
Germany	125 900

Question 7a (1 mark)

A person was selected at random from the population of Australia in 2015. Using the data in Tab 1, **show that** the probability of this person being over 60 years old is 0.18.

Question 7b (2 marks)

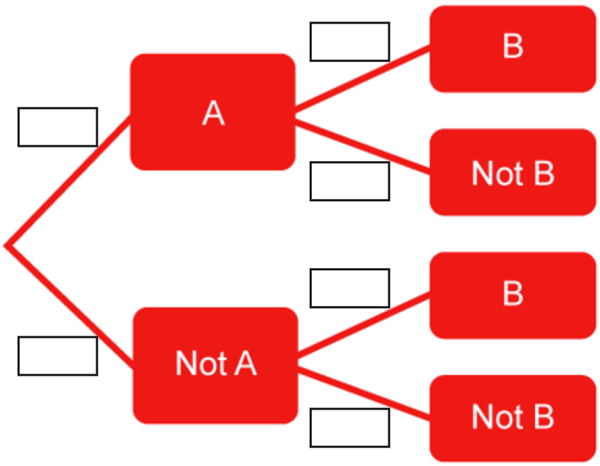
The total population of Australia in 2015 was estimated to be 23 858 000 to the nearest 1000. Using the data in Tab 2, **show that** 5 % of the Australian population were immigrants from the United Kingdom (UK), to the nearest percent.

Question 7c (2 marks)

A person was selected at random from the Australian population in 2015.

Event A represents: The person is an immigrant from the UK
Event B represents: The person is over 60 years old.

Write down the missing probabilities in the tree diagram below to the nearest 2 decimal places.



Question 7d (3 marks)

A person was selected at random from the population of Australia in 2015.

Using your tree diagram, **find** the probability that this person was over 60 years old.

Question 7e (2 marks)

Given that this person was over 60 years old, **determine** the probability that the person was an immigrant from the UK.

Question 7f (1 mark)

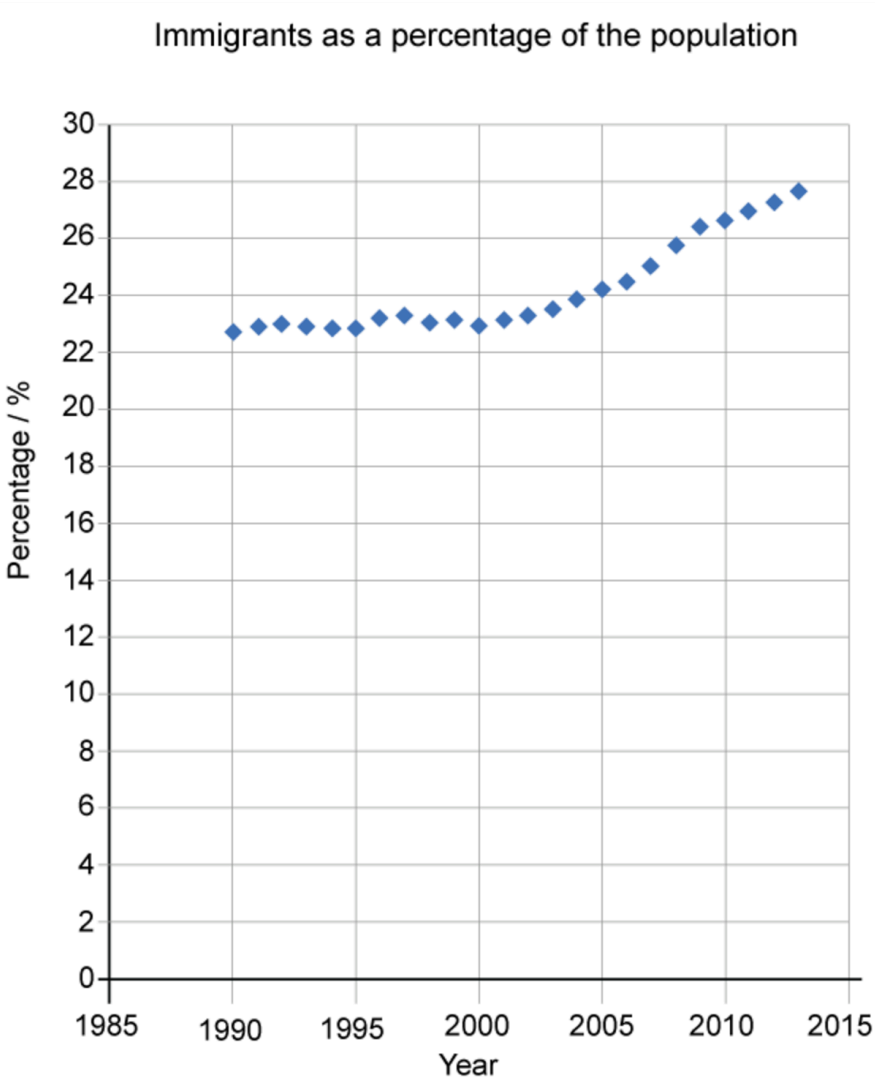
Using parts (b) and (e), or otherwise, **state** the type of relationship between events A and B.

Question 7g (2 marks)

The graph below shows immigrants as a percentage of the population of Australia from 1990 to 2013.

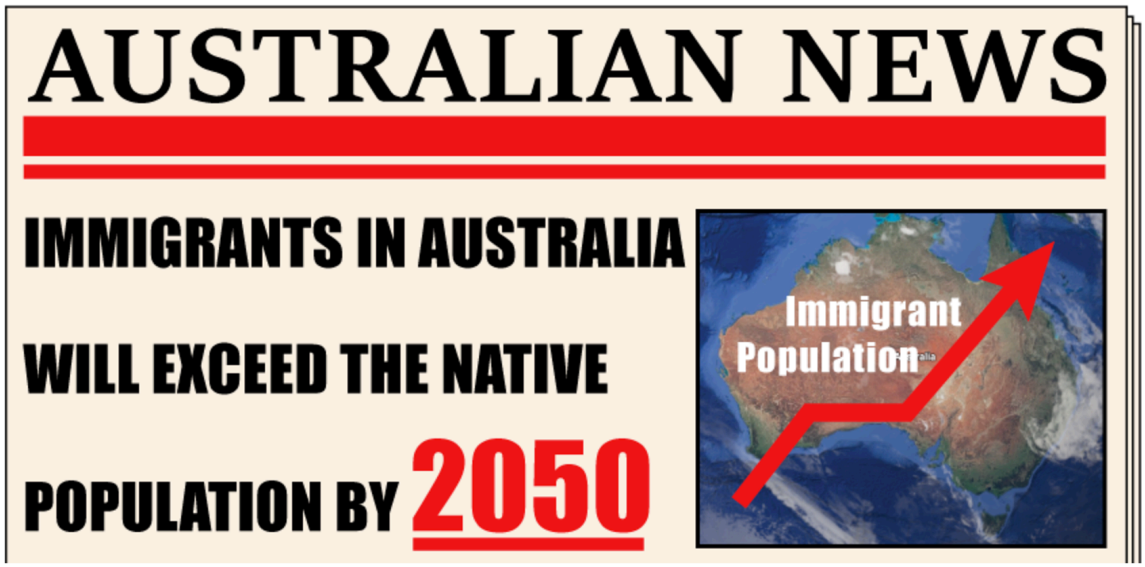
Draw a line of best fit on the graph.

Draggable:



Scroll down to continue

Question 7h (10 marks)



Analyse the information provided to comment on this news headline.

In your answer, you should:

- identify the factors to be considered when making your prediction
- estimate in what year the percentage of immigrant population in Australia will exceed 50 %
- use a suitable degree of accuracy for your results
- justify whether the year you estimated makes sense in the context of the problem
- comment on the news headline.

Tab 1 | Tab 2

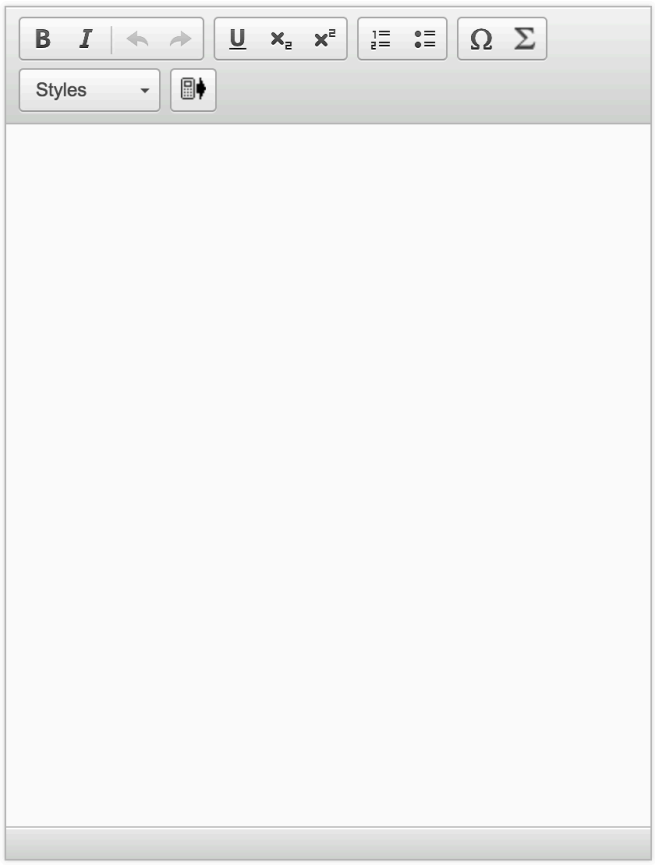
The table below shows the age distribution of the population of Australia (percentage to nearest 1 %, data correct as at 2015).

Age (A)	Percentage
$0 < A \leq 20$	23
$20 < A \leq 40$	32
$40 < A \leq 60$	27
$60 < A \leq 80$	17
$80 < A \leq 100$	1

Tab 1 | Tab 2

The table below shows the country of birth of immigrants to Australia (data correct as at 2015, top ten countries only).

Country of birth	Number of immigrants
United Kingdom	1 207 000
New Zealand	611 400
China	481 800
India	432 700
Philippines	236 400
Vietnam	230 200
Italy	198 200
South Africa	178 700
Malaysia	156 500
Germany	125 900

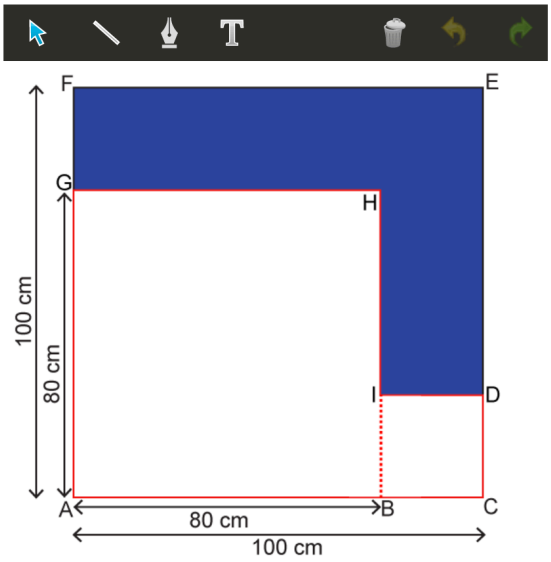

Styles

Question 8 (32 marks)

Question 8a (2 marks)

The diagram below shows squares ACEF, ABHG and BCDI.

Diagram not to scale

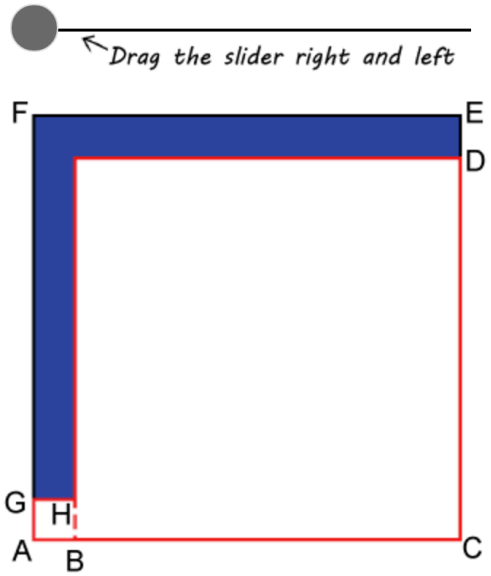


The large square shown has side AC = 100 cm. AB has length 80 cm.

Show that the shaded area is 3200 cm^2.

Styles

On the simulator below, use the slider to change the length of AB and see the corresponding values of the shaded area when AC is 100 cm.



Length of AB	Shaded area / cm^2
10	1800

Question 8b (1 mark)

Write down the missing values in the table.

Length of AC in cm	Length of AB (L) in cm	Length of BC in cm	Shaded area (A) in cm²
100	20	80	3200
100	30	70	4200
100	40		
100	50	50	5000
100		40	
100	70	30	4200
100	80	20	3200

Reset

Question 8c (1 mark)

Write down, in words, a pattern from the table for the shaded area (A).

B I U x_2 x^2 Ω Σ Styles

Question 8d (2 marks)

Determine a general rule for (A), the shaded area, in terms of (L), the length of AB.

B I U x_2 x^2 Ω Σ Styles

Question 8e (3 marks)

Verify your general rule.

B I U x_2 x^2 Ω Σ Styles

Question 8f (3 marks)

Traditional shoemakers used a tool as shown below.

Click on "Start" to illustrate the shapes studied by ancient Greeks based on the shoemaker tool.

Start

Diagram not to scale

Traditional shoemakers tool

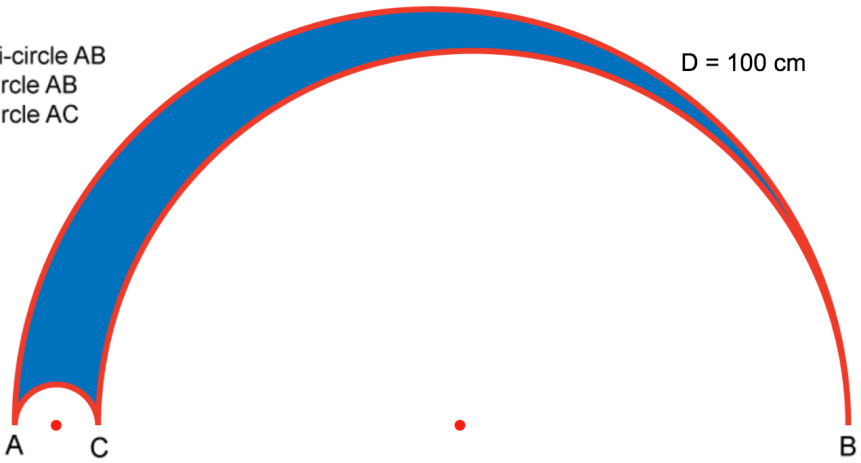
When diameter AB is 100 cm and diameter AC is 40 cm, the area of the semi-circle with diameter AB is 1250π cm^2.

Show that the shaded area is 600π cm^2.

Question 8g (20 marks)

On the simulator below, use the slider to change the diameter AC and see the corresponding shaded area.

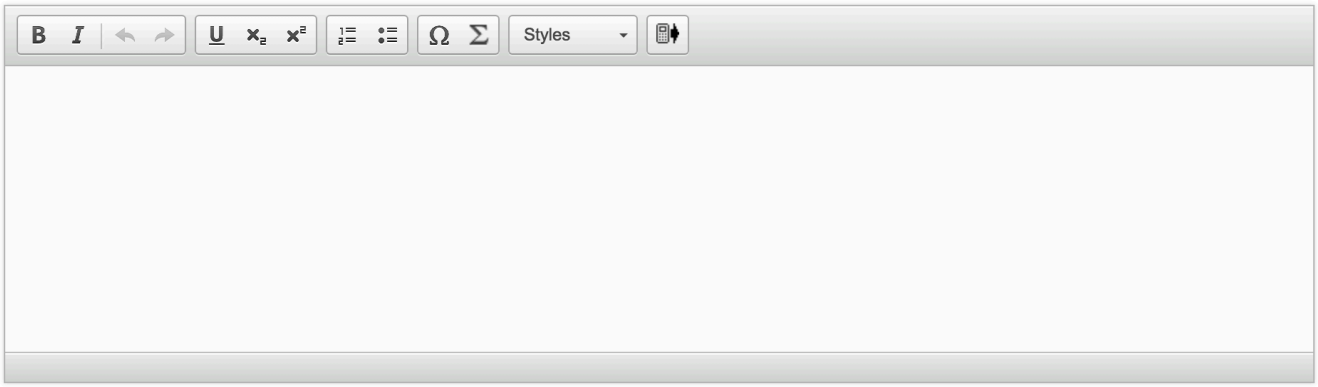
Key:
D: diameter of the semi-circle AB
R: radius of the semi-circle AB
r: radius of the semi-circle AC
A: shaded shape



AB (cm)	R (cm)	AC (cm)	r (cm)	A (cm)
100	50	10	5	225 π

Investigate the relationship between the radius (r) and the shaded area (A) for this semi-circle where D = 100 cm. You may use the simulator above, which is for D = 100 cm. In your answer, you should:

- predict more values for A
- write down, in words, any patterns you see for A
- find a general rule for the shaded area (A) in terms of (r)
- test your general rule
- prove or verify and justify your general rule
- ensure that you communicate the above appropriately.



To support your investigation, you may record data in the table below.

D	R	r		A
100	50	5		225 π
100	50	10		400 π
100	50	15		525 π
100	50	20		600 π
100	50	25		π
100	50	30		π
100	50			π
100	50			π
				π
				π