

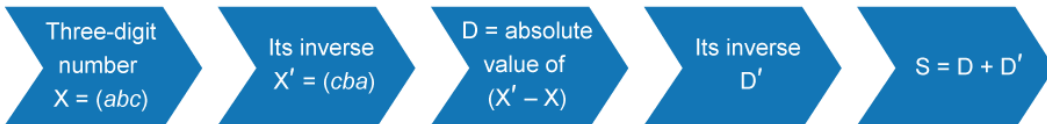
2018 May eAssessment Mathematics



Question 1a (3 marks)

In this question, we will discover an interesting and magical property of three-digit numbers using a special algorithm, illustrated in the algorithm flow diagram below.

Algorithm flow diagram



Here is a simulator for the algorithm flow diagram which provides some examples of how this algorithm affects three-digit numbers. Select a number and see what happens.



Reset

Apply the same algorithm to 437 to complete the missing values.



Question 1b (2 marks)

A three-digit number can be written in terms of sum of multiples of its digits. For example, 437 can be written in the format shown below.



X is a three-digit number *abc*. **Write down** X and X' as a sum of multiples of their digits.

X =

X' =



Question 1c (2 marks)

Using your answer from part (b), **determine** the difference D in terms of a and c .

B *I* ← → U x_2 x^2 $\frac{1}{2}$ $\frac{3}{4}$ Ω Σ Styles -



Question 2 (6 marks)



$$\begin{array}{ccccccc} \text{Frog} & + & \text{Dragonfly} & = & 22 \\ \text{Frog} & - & \text{Dragonfly} & = & 12 \end{array}$$



Find the value of X for the following expression.

$$2 \text{ Frogs} + 4 \text{ Dragonflies} = X$$

B *I* ← → U x_2 x^2 $\frac{1}{2}$ $\frac{3}{4}$ Ω Σ Styles -



Question 3a (4 marks)

To raise money for their graduation party, senior students organize a cookie and muffin sale. x represents the number of cookies and y represents the number of muffins. The amount of cookies and muffins sold are represented by the shaded region in the diagram below.

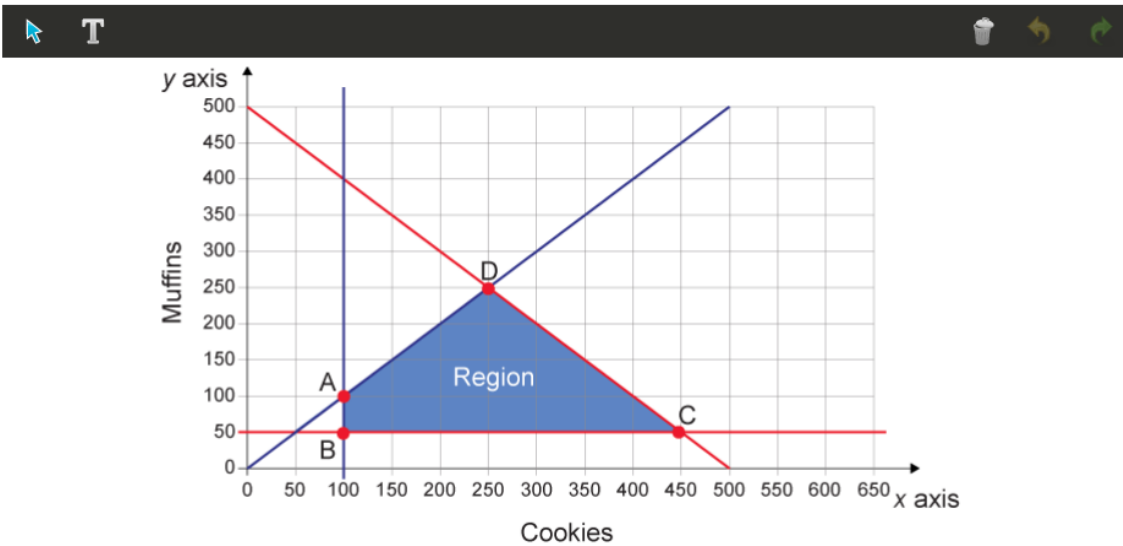
Using the information provided in the diagram below:

- **Identify** the shaded region by completing the inequalities below.
- The first constraint is that they cannot sell more than a total of 500 cookies and muffins. **State** the other three constraints in the spaces provided.

| Draggable inequalities | Inequalities | Constraints |
|------------------------|------------------|---|
| ≥ | $x + y \leq 500$ | Total number of cookies and muffins cannot exceed 500 |
| ≤ | $x \leq 100$ | |
| ≤ | $y \leq 50$ | |
| ≤ | $y \leq x$ | |

Diagram

Diagram



**Question 3b** (1 mark)

The profit P , in Canadian dollars (CAD), for the cookies and muffins sale is calculated using the formula $P = x + 1.5y$.

Interpret the meaning of the coefficients of x and y in the profit formula above.

B *I* ← → x_e x² ☰ ☷ Ω Σ Styles ↕

**Question 3c** (2 marks)

The table below shows the profit at the vertices A, B and D.

| Vertices (x, y) | $P = x + 1.5y$ (CAD) |
|---------------------|----------------------|
| A (100, 100) | 250 |
| B (100, 50) | 175 |
| C | |
| D (250, 250) | 625 |

Determine the missing profit at vertex C.

B *I* ← → x_e x² ☰ ☷ Ω Σ Styles ↕

**Question 3d** (1 mark)

Write down the number of cookies and the number of muffins that the students must sell in order to maximize their profit.

Number of cookies:

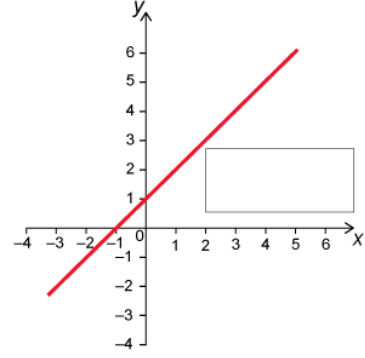
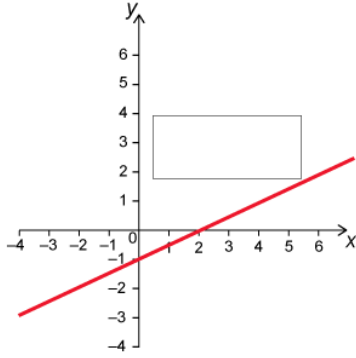
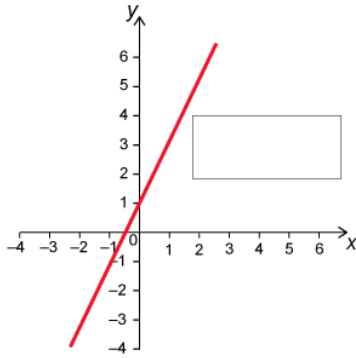
Number of muffins:



Question 4a (2 marks)

Select the line equations and place them with the corresponding graphs.

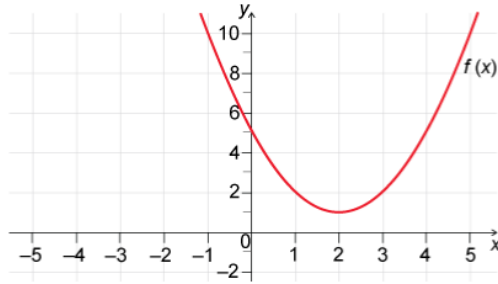
Draggable: $y = x + 1$ $y = 2x + 1$ $y = \frac{1}{2}x - 1$





Question 4b (4 marks)

The graph below represents the function $f(x)$. Transformations of $f(x)$ are shown in the following graphs.

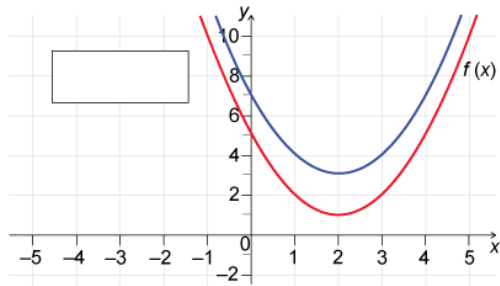
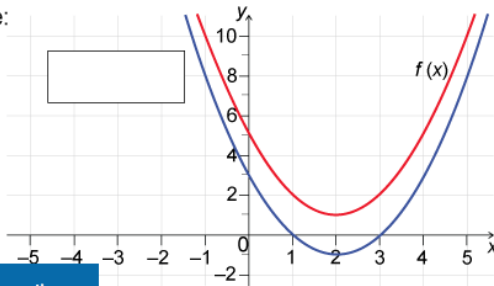


Select the equations and place them with the corresponding graphs.

Key : — $f(x)$ — Transformation of $f(x)$

Draggable:

- $f(x) + 2$
- $-f(x)$
- $\frac{1}{2}f(x)$



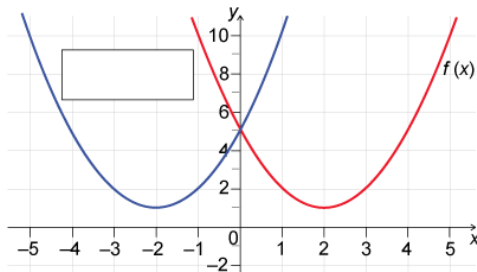
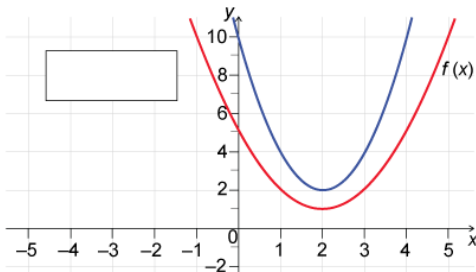
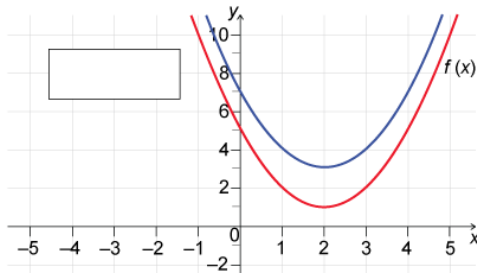
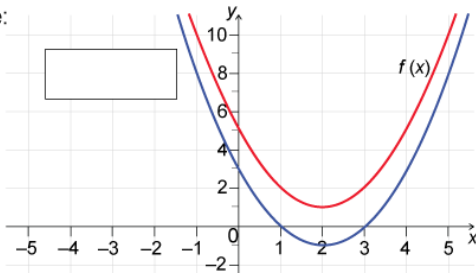
Scroll down to continue

Select the equations and place them with the corresponding graphs.

Key : — $f(x)$ — Transformation of $f(x)$

Draggable:

- $f(x) + 2$
- $-f(x)$
- $\frac{1}{2}f(x)$
- $f(-x)$
- $f(x - 2)$
- $f(x + 2)$
- $2f(x)$
- $f(x) - 2$



Question 5 (8 marks)

Students are planning to construct a kite which will be a scaled model of the wings of a hang glider.

This media contains no audio

Diagram not to scale

Image 1: Hang glider

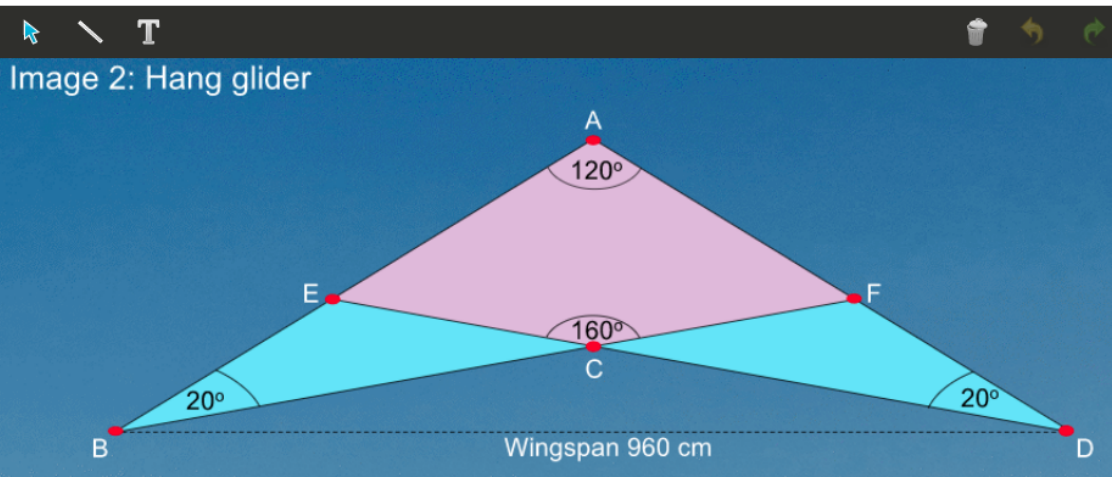


Question 5a (2 marks)

The hang glider is a quadrilateral with the following properties:

| Sides | Angles |
|-----------------------|------------------------|
| $AB = AD$ | $ABC = ADC = 20^\circ$ |
| $BC = DC$ | $ECF = 160^\circ$ |
| $BD = 960 \text{ cm}$ | $BAD = 120^\circ$ |

Diagram not to scale



Question 5a (2 marks)

Show that the triangles BCE and DCF are congruent.

B *I* ← → U \times_2 \times^2 \equiv \equiv Ω Σ
Styles -

Question 5b (3 marks)

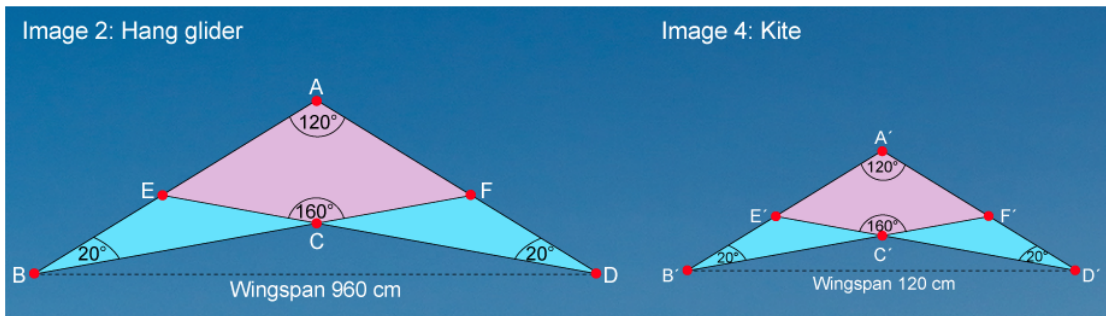
Calculate the length of AB for the hang glider.

B *I* ← → U \times_2 \times^2 \equiv \equiv Ω Σ
Styles -

Question 5c (1 mark)

The hang glider and kite are similar shapes, where B'D' is 120 cm. **Determine** the scale factor of enlargement.

Diagram not to scale



Question 5d (2 marks)

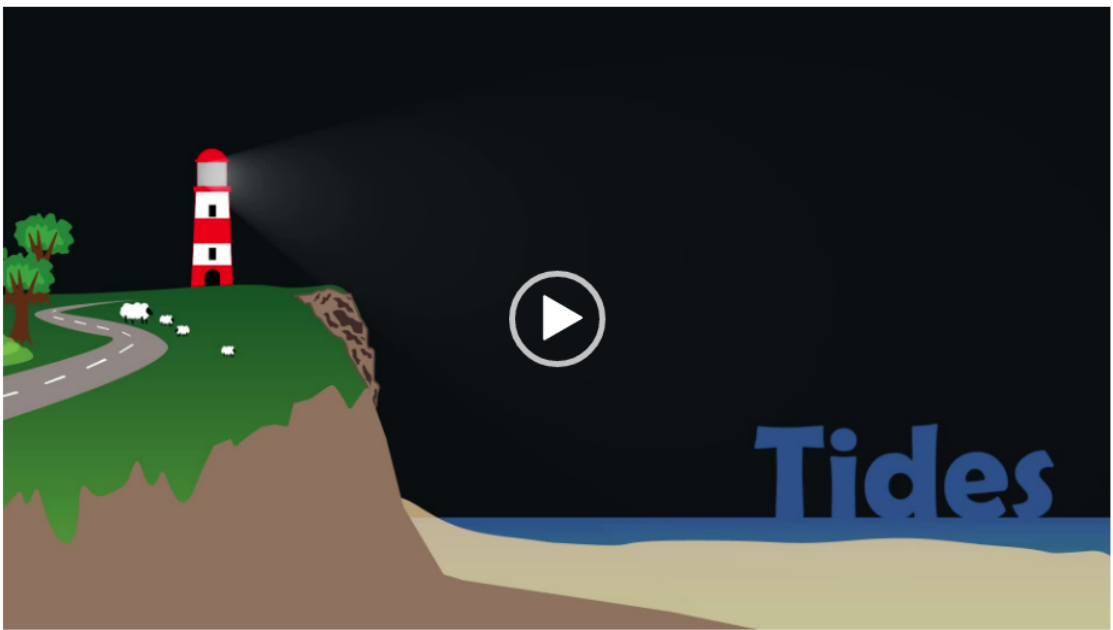
The area of the hang glider is 92 395 cm².

Determine the area of the kite.

B *I* ← → U \times_2 \times^2 \equiv \equiv Ω Σ Styles -

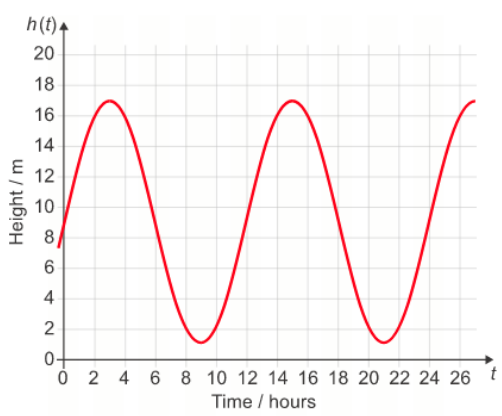
Question 6 (10 marks)

The following video illustrates how tidal range can be modelled over time by a sine function.



©

Below is the sine curve modelling the tide in Saint-Malo on a day in November 2017. $h(t)$ is the height in metres (m) of water in the harbour and t is the number of hours after midnight.



Question 6a (2 marks)

Determine the tidal range which is the difference between the height of the low and high tides in the harbour.

Rich text editor interface with a toolbar containing icons for bold (B), italic (I), left arrow, right arrow, underline (U), subscript (x₂), superscript (x²), bulleted list, numbered list, link (Ω), and insert link (Σ). Below the toolbar is a text input area.

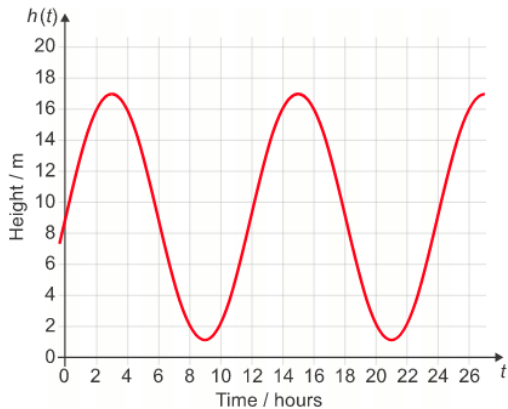


Question 6b (2 marks)

The period can be measured as the time difference between two consecutive high tides. **Determine** the period of this tide.

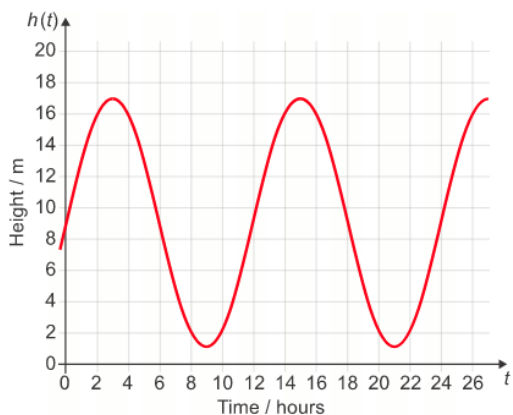
Below is the sine curve modelling the tide in Saint-Malo on a day in November 2017.

$h(t)$ is the height in metres (m) of water in the harbour and t is the number of hours after midnight.



Below is the sine curve modelling the tide in Saint-Malo on a day in November 2017.

$h(t)$ is the height in metres (m) of water in the harbour and t is the number of hours after midnight.



Question 6c (2 marks)

Using the graph, **estimate** at what times, during the first 12 hours after midnight, the height of the water is 2 m.

Rich text editor interface with formatting options (B, I, U, x₂, x^e, list, link, Ω, Σ) and a text input area.



Question 6d (2 marks)

The behaviour of this tide can be modelled by the function $h(t) = a \sin(30t) + c$.

Using the graph characteristics, **justify** that $a = 8$ and $c = 9$.

Empty text input area for the answer.



Question 6e (2 marks)

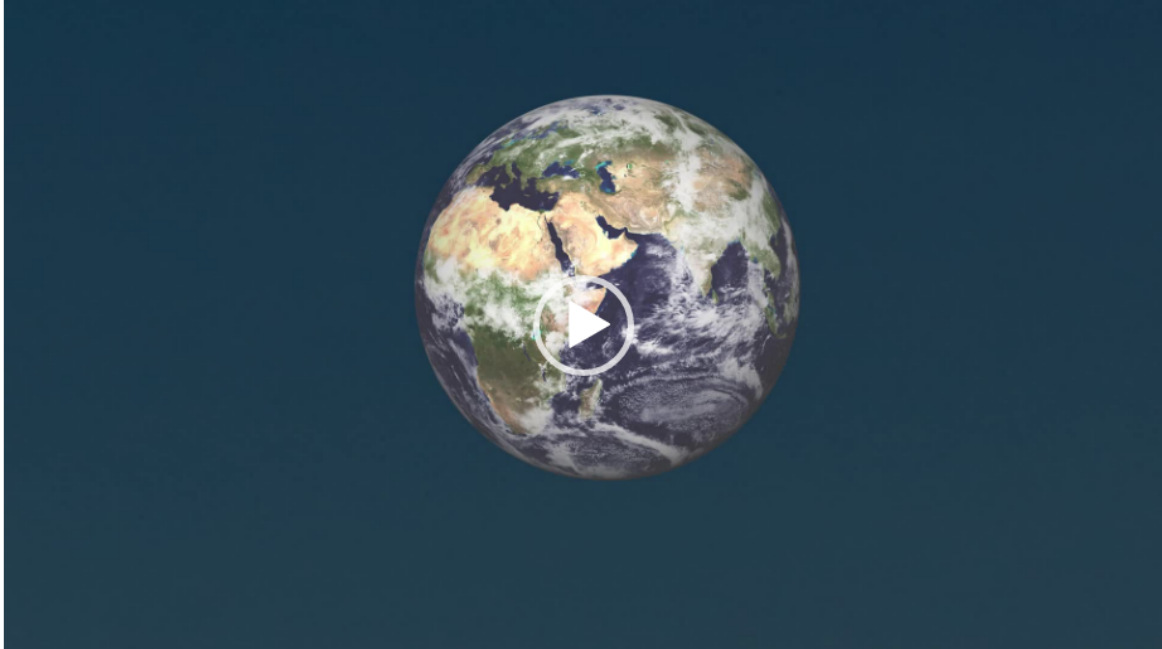
By substituting into the equation $h(t) = 8 \sin(30t) + 9$, **calculate** the height in metres of water in the harbour at 17:00.

Rich text editor interface with formatting options (B, I, U, x₂, x^e, list, link, Ω, Σ) and a text input area.



Question 7 (23 marks)

The following video shows how mathematics can be used to make predictions for population growth over time and space.



©

Use the table in Tab 1 below to answer questions (a), (b) and (c).

Tab 1

Tab 2

The table below shows the age distribution of the population of Australia (percentage to nearest 1 %, data correct as at 2015).

| Age (A) | Percentage |
|-------------------|------------|
| $0 < A \leq 20$ | 23 |
| $20 < A \leq 40$ | 32 |
| $40 < A \leq 60$ | 27 |
| $60 < A \leq 80$ | 17 |
| $80 < A \leq 100$ | 1 |

The table below shows the country of birth of immigrants to Australia (data correct as at 2015, top ten countries only).

| Country of birth | Number of immigrants |
|------------------|----------------------|
| United Kingdom | 1 207 000 |
| New Zealand | 611 400 |
| China | 481 800 |
| India | 432 700 |
| Philippines | 236 400 |
| Vietnam | 230 200 |
| Italy | 198 200 |
| South Africa | 178 700 |
| Malaysia | 156 500 |
| Germany | 125 900 |



Question 7a (1 mark)

Write down the modal class for the age distribution in Australia.

B *I* | ← → | U x_2 x^2 | $\frac{1}{2}$ $\frac{3}{2}$ | Ω Σ

Styles ▾ |



Question 7b (4 marks)

Show that the estimated mean age is 38.2



Question 7c (2 marks)

A person was selected at random from the population of Australia in 2015. **Determine** the probability that this person was over 60 years old.

Rich text editor toolbar with buttons for Bold (B), Italic (I), Undo, Redo, Underline (U), Subscript (x_2), Superscript (x^2), Bulleted List, Numbered List, Omega (Ω), and Sum (Σ). Below the toolbar is a text input area and a mobile device icon.



Question 7d (2 marks)

The total population of Australia in 2015 was estimated to be 23 858 000 to the nearest 1000. Using the data in Tab 2, **show that** 5 % of the Australian population were immigrants from the United Kingdom (UK), to the nearest percent.

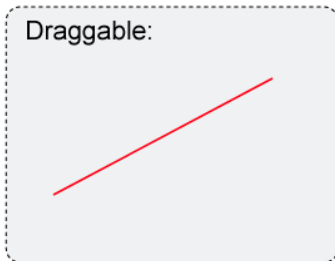


Question 7e (2 marks)

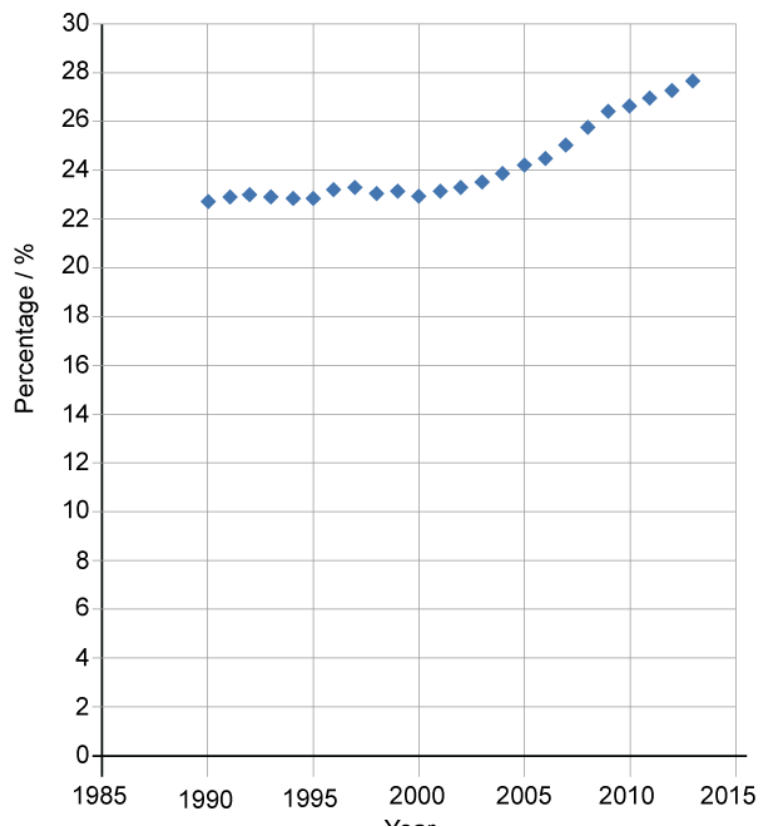
A person was selected at random from the population of Australia in 2015. **Determine** the probability that this person was an immigrant from the UK over 60 years old.

The graph below shows immigrants as a percentage of the population of Australia from 1990 to 2013.

Draw a line of best fit on the graph.




Immigrants as a percentage of the population



Scroll down to continue

AUSTRALIAN NEWS

IMMIGRANTS IN AUSTRALIA WILL EXCEED THE NATIVE POPULATION BY **2050**



©

Analyse the information provided to comment on this news headline.

In your answer, you should:

- identify the factors to be considered when making your prediction
- estimate in what year the percentage of immigrant population in Australia will exceed 50 %
- use a suitable degree of accuracy for your results
- justify whether the year you estimated makes sense in the context of the problem
- comment on the news headline.

Tab 1

Tab 2

The table below shows the age distribution of the population of Australia (percentage to nearest 1 %, data correct as at 2015).

| Age (A) | Percentage |
|-------------------|------------|
| $0 < A \leq 20$ | 23 |
| $20 < A \leq 40$ | 32 |
| $40 < A \leq 60$ | 27 |
| $60 < A \leq 80$ | 17 |
| $80 < A \leq 100$ | 1 |

Tab 1

Tab 2

The table below shows the country of birth of immigrants to Australia (data correct as at 2015, top ten countries only).

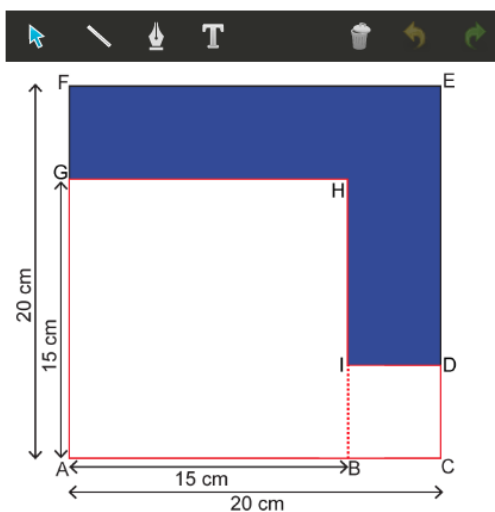
| Country of birth | Number of immigrants |
|------------------|----------------------|
| United Kingdom | 1 207 000 |
| New Zealand | 611 400 |
| China | 481 800 |
| India | 432 700 |
| Philippines | 236 400 |
| Vietnam | 230 200 |
| Italy | 198 200 |
| South Africa | 178 700 |
| Malaysia | 156 500 |
| Germany | 125 900 |



Question 8a (2 marks)

The diagram below shows squares ACEF, ABHG and BCDI.

Diagram not to scale



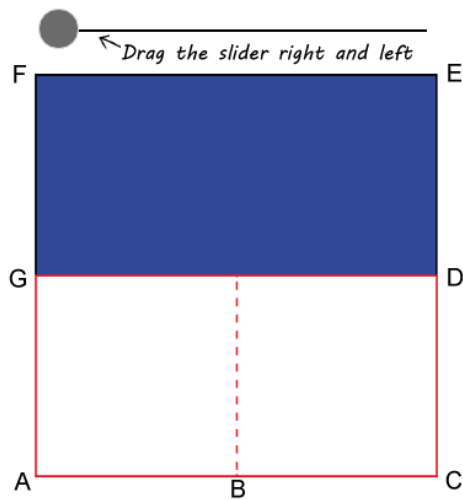
The large square shown has side $AC = 20$ cm. AB has length 15 cm.

Show that the perimeter of the shaded shape is 70 cm.

Rich text editor toolbar with options for Bold (B), Italic (I), text color, background color, bulleted list, numbered list, link, unlink, and insert link. Below the toolbar is a text input area.



On the simulator below, use the slider to change the length of AB and see the corresponding length of the perimeter of the shaded shape when AC is 20 cm.



| Length of AB | Perimeter of shaded shape / cm |
|--------------|--------------------------------|
| 10 | 60 |

Question 8b (1 mark)

Write down the missing values in the table when the length of AB is $L = 14$ cm.

| Length of AC, in cm | Length of AB (L) in cm | Length of BC in cm | Perimeter of shaded shape (P) in cm |
|---------------------|------------------------|--------------------|-------------------------------------|
| 20 | 10 | 10 | 60 |
| 20 | 11 | 9 | 62 |
| 20 | 12 | 8 | 64 |
| 20 | 13 | 7 | 66 |
| 20 | 14 | | |
| 20 | 15 | 5 | 70 |
| 20 | | | |

Reset

Question 8c (1 mark)

Write down, in words, a pattern from the table for the perimeter (P).

B I ← → U x₂ x^a ≡ ≡ Ω Σ

Styles ↕

Question 8d (2 marks)

Determine a general rule for (P), the perimeter of the shaded shape, in terms of (L), the length of AB.



Question 8e (3 marks)

Verify your general rule.



Question 8f (3 marks)

Traditional shoemakers used a tool as shown below.

Click on "Start" to illustrate the shapes studied by ancient Greeks based on the shoemaker tool.

Start



Traditional shoemakers tool

Diagram not to scale

The diameter AB is 100 cm. The radius of semi-circle AC is 30 cm and radius of semi-circle CB is 20 cm.

Show that the perimeter of the shaded shape is 100π .

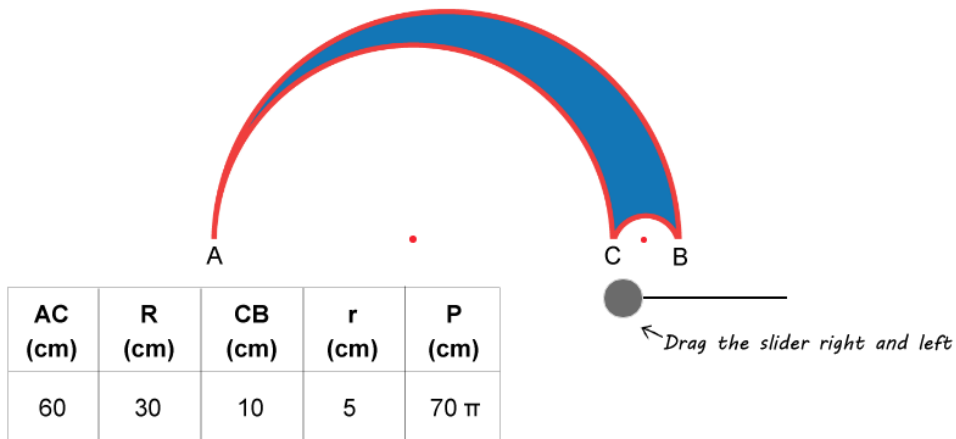


Question 8g (20 marks)

On the simulator below, use the slider to change the diameter CB and see the corresponding length of the perimeter of the shaded shape.

Key:

- D: diameter of the semi-circle AB
- R: radius of the semi-circle AC, which is 30 cm
- r: radius of the semi-circle CB
- P: perimeter of the shaded shape



Investigate the relationship between the radius (r) of semi-circle CB and the perimeter (P) of the shaded shape when radius of semi-circle AC is 30 cm. You may use the simulator above. In your answer, you should:

- predict more values for P
- write down, in words, any patterns you see for P
- find a general rule for the perimeter of the shaded shape (P) in terms of (r)
- test your general rule
- prove or verify and justify your general rule
- ensure that you communicate the above appropriately.

To support your investigation, you may record data in the table below.

| R | r | P |
|----------------------|----------------------|----------------------------|
| 30 | 5 | 70π |
| 30 | 10 | 80π |
| 30 | 15 | 90π |
| 30 | 20 | 100π |
| 30 | 25 | <input type="text"/> π |
| 30 | 30 | <input type="text"/> π |
| 30 | <input type="text"/> | <input type="text"/> π |
| 30 | <input type="text"/> | <input type="text"/> π |
| <input type="text"/> | <input type="text"/> | <input type="text"/> π |
| <input type="text"/> | <input type="text"/> | <input type="text"/> π |

