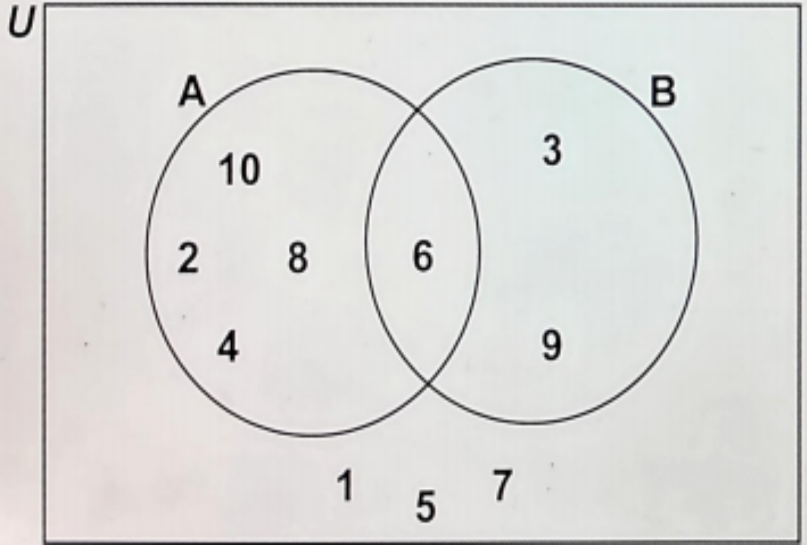
Question 1 (4 marks)

The elements of the universal set U are {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

Consider two subsets of U

Set A contains the multiples of 2.

Set B contains the multiples of 3.



Question 1a (1 mark)

Determine the probability $P(A \mid B')$.

B I U x_2 x^2 Ω Σ

Styles

Question 1b (3 marks)

Two numbers are selected at random from U. **Calculate** the probability that only one is an element of $A \cap B'$.

B I U x_2 x^2 Ω Σ

Styles

Question 2a (3 marks)

The table below shows an example of a completed addition grid.

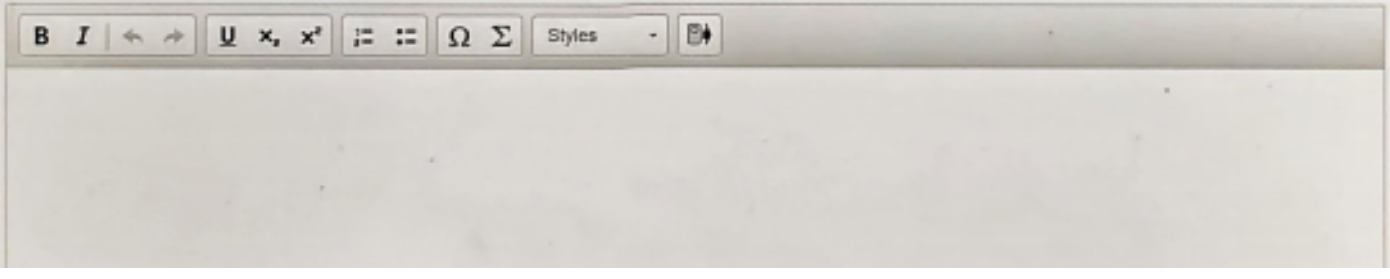
Addition grid	+	4	5
	3	7	8
	10	14	15

Write down the values of a, b and c, in a simplified exact form.

Addition grid	+	log(2)	a
	log(6)	b	log(3)
	log(16)	c	3 log(2)

b:

c:



Question 2b (3 marks)

The table below shows an example of a completed multiplication grid.

Multiplication grid

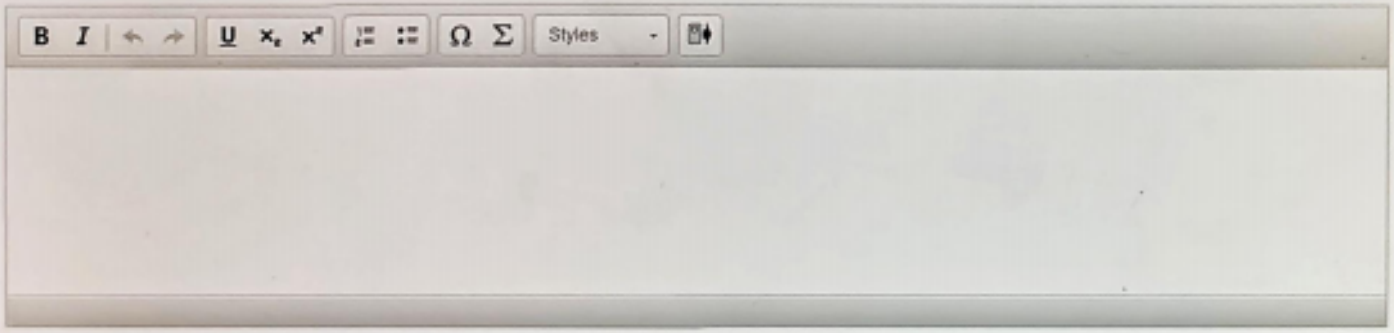
×	4	5
3	12	15
10	40	50

Write down the missing values of a, b and c, in a simplified index form.

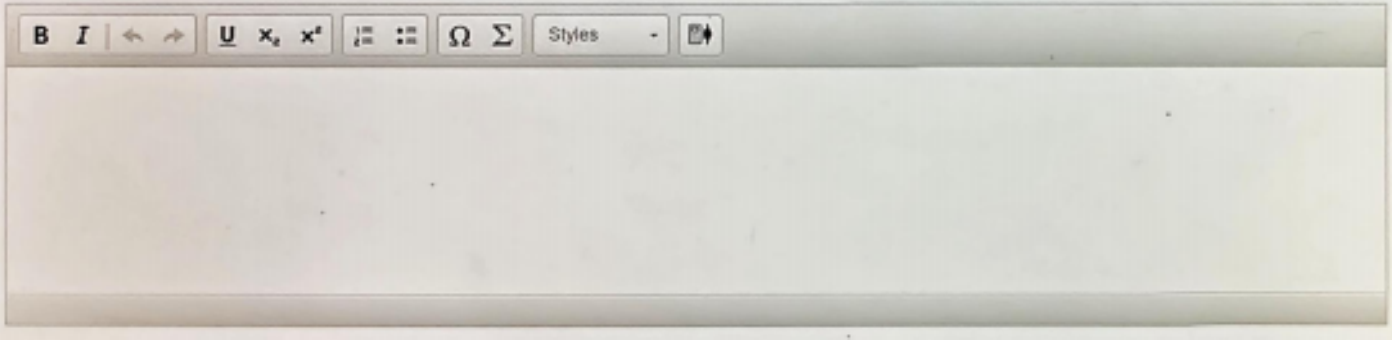
Multiplication grid

×	$2x^{\frac{1}{2}}$	a
$x^{\frac{1}{2}}y$	b	$2y^3$
c	$x^{\frac{1}{2}}$	$x^{-\frac{1}{2}}y^2$

a:



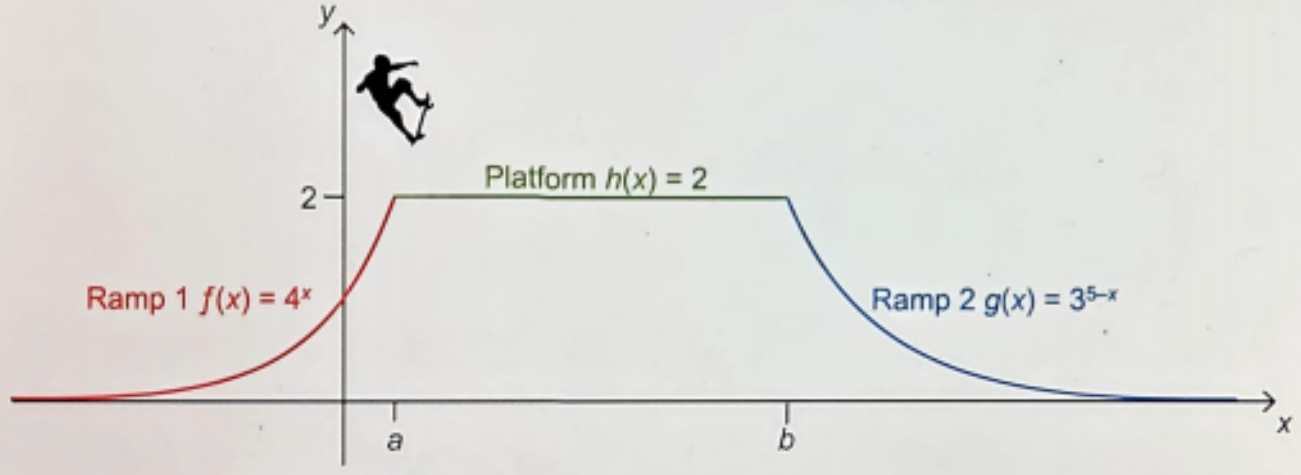
b:



Question 3 (7 marks)

A skateboard ramp set is built using functions provided in the diagram below:

The diagram is interactive. Hover over the intersection of each line.



Question 3a (2 marks)

Determine the value of *a*.

B I U x_2 x^2 Ω Σ

Styles

Question 3b (4 marks)

Calculate the value of *b*.

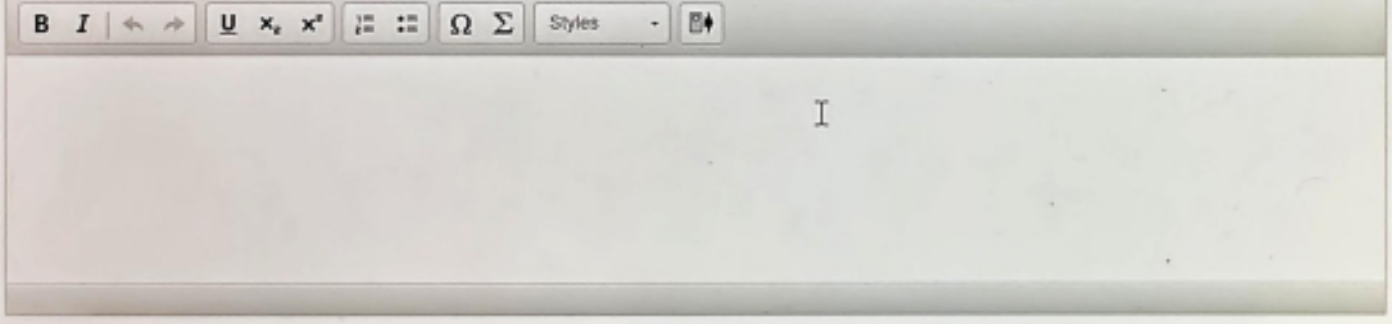
B I U x_2 x^2 Ω Σ

Styles

Formatting Styles

Question 3c (1 mark)

Hence, **determine** the length of the platform.



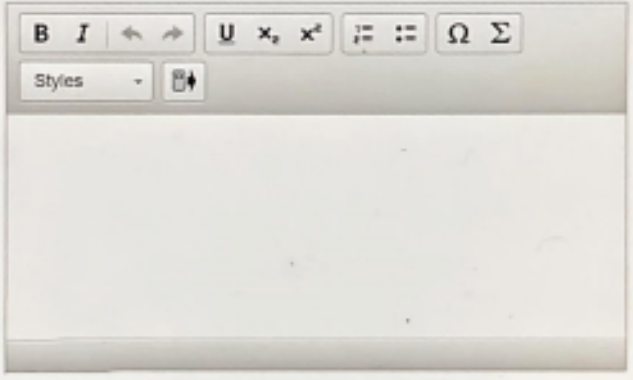
Question 4 (10 marks)

Below is a geometric sequence with first term $U_1 = 4$ and common ratio r

$$4, 2\sqrt{2}, 2, \ldots$$

Question 4a (1 mark)

Write down the value of r,



Below is a geometric sequence with first term $U_1 = 4$ and common ratio r

$$4, 2\sqrt{2}, 2, \ldots$$

Question 4b (2 marks)

By continuing the pattern, **determine** the value of n when $U_n = r$.

B I U x_2 x^2 Ω Σ

Styles

Below is a geometric sequence with first term $U_1 = 4$ and common ratio r

$$4, 2\sqrt{2}, 2, \ldots$$

Question 4c (3 marks)

Given that $U_{21} = 2^k$; $k \in Z$, **find** the value of k.

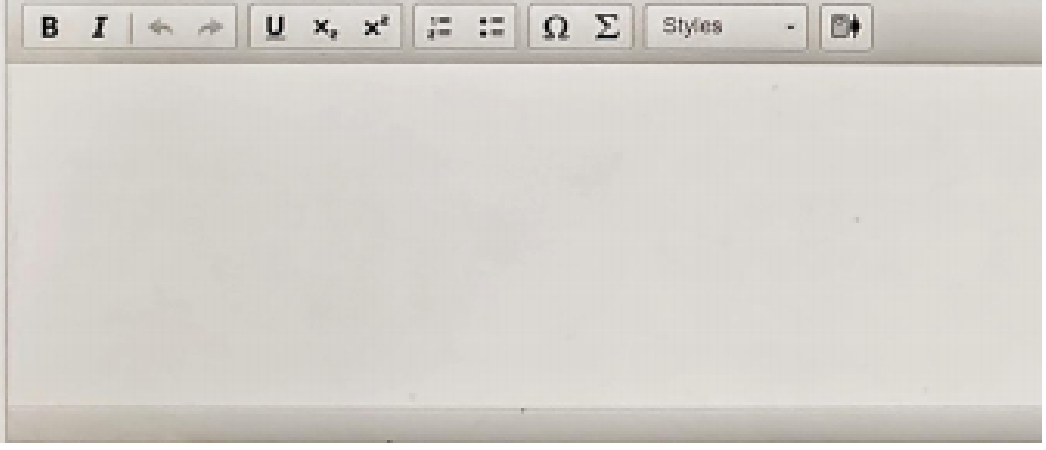
B I U x_2 x^2 Ω Σ

Styles

Question 4d (4 marks)

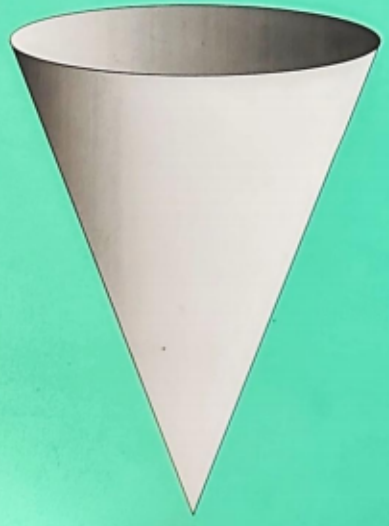
Find the sum to infinity of $4 + 2\sqrt{2} + 2 + \ldots$

Write your answer in the form $a + b\sqrt{2}$; $a, b \in N$.

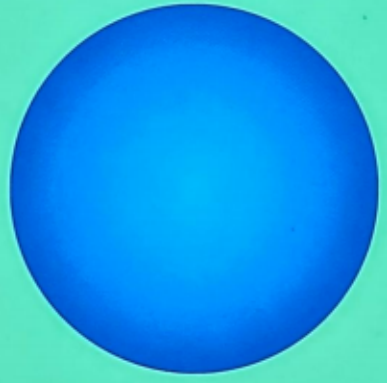


Question 5 (9 marks)

Cone



Sphere of ice



Question 5 (9 marks)

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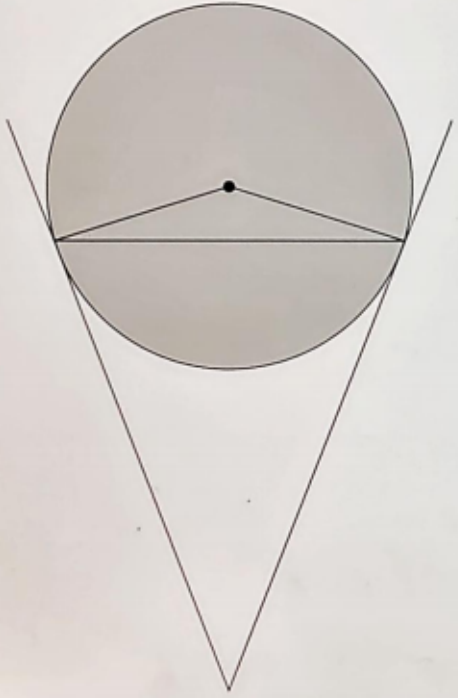
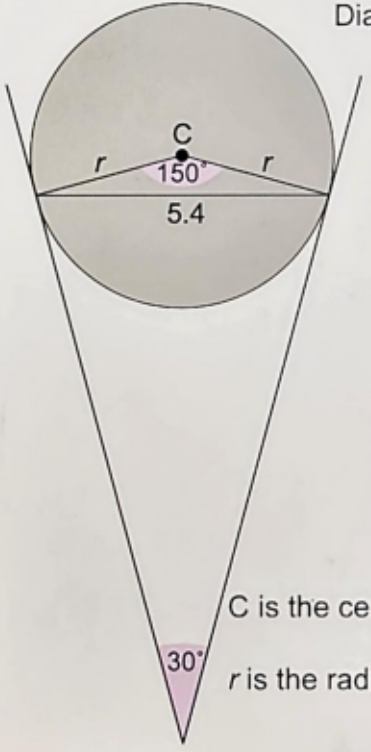


This is a cross-section of the cone

Question 5a (3 marks)

Diagram not to scale



C is the centre of the sphere

r is the radius of the sphere

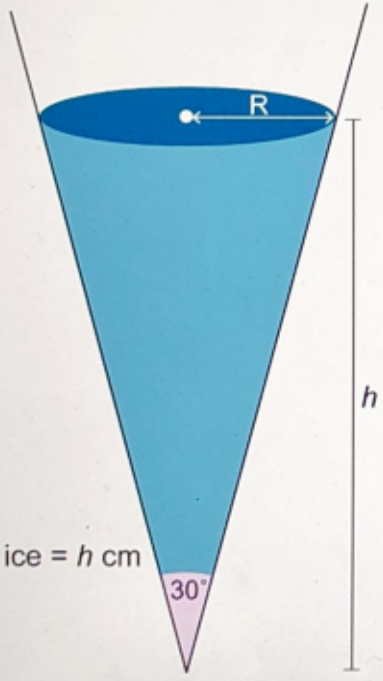
Show that $r = 2.80$ cm, correct to **three** significant figures.

B I U x_2 x^2 Ω Σ

Styles

Question 5b (6 marks)

Diagram not to scale



Radius = R cm

Height of melted ice = h cm

Find the value of h.

B I U x_2 x^2 Ω Σ

Styles

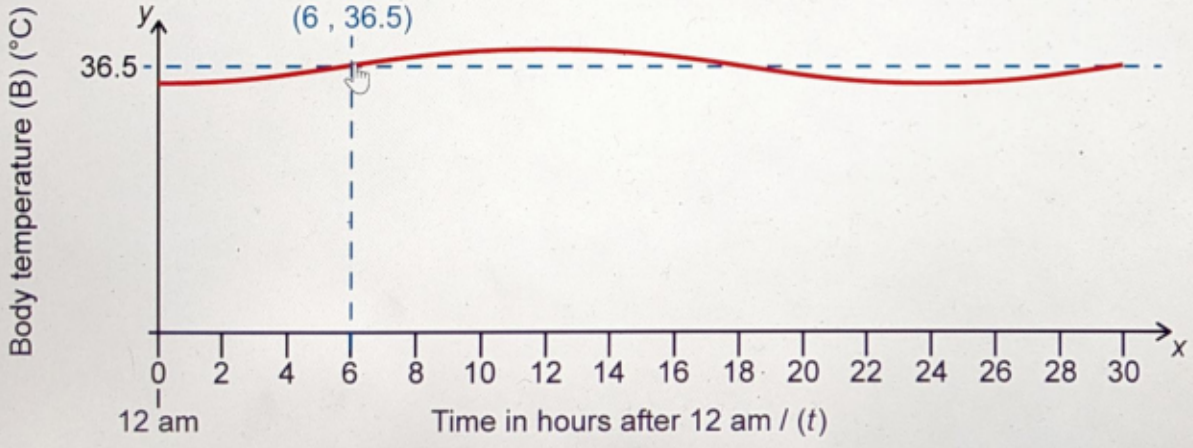
Question 6 (15 marks)

Body temperature changes during the day. The graph below shows a cosine curve modelling the body temperature for Ingrid.

B is the temperature in degrees Celsius (°C)

t is the time in hours after midnight.

This media is interactive



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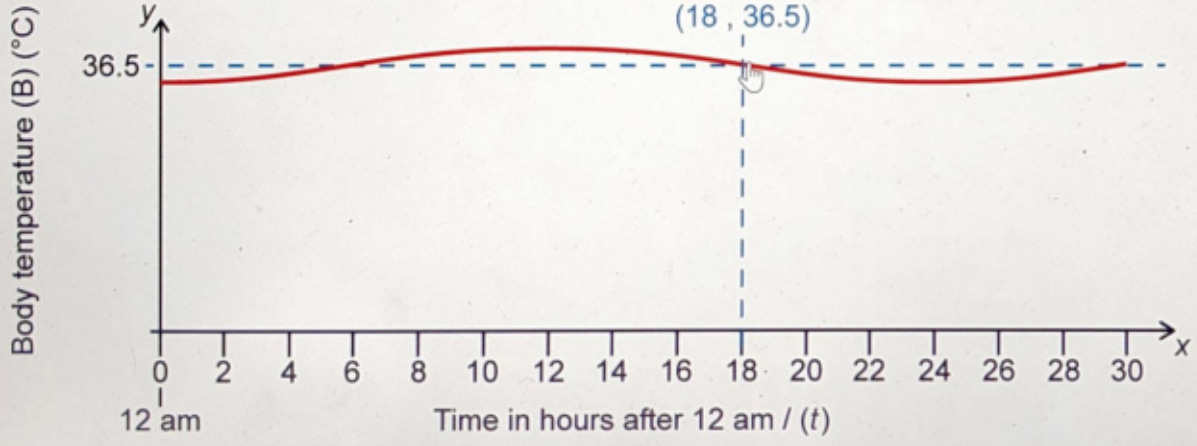

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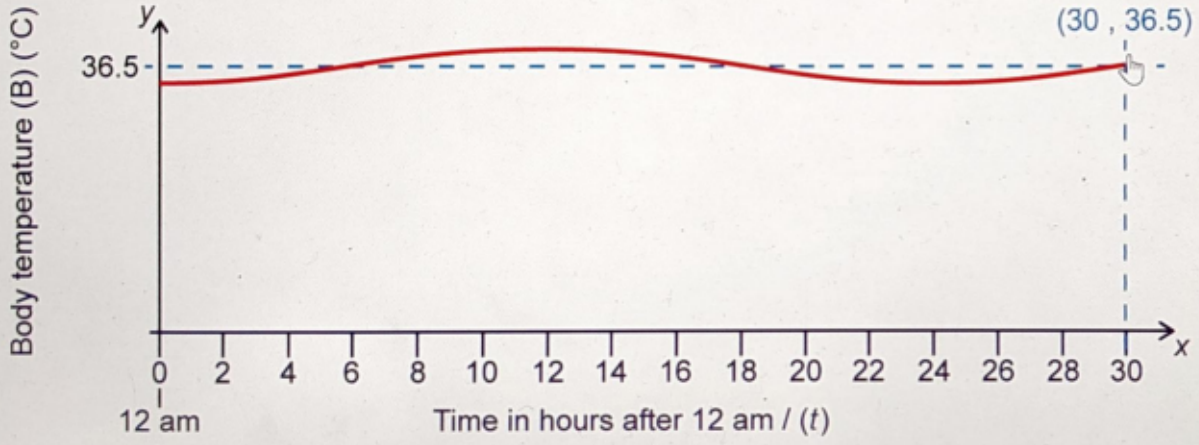


Ingrid knows it is best to sleep for 8 to 10 hours when her body temperature is 36.5 °C or below.

Question 6a (1 mark)

Suggest a sleeping schedule for Ingrid.

Sleep time: : Select

Wake up time: :

Select
am
pm

Question 6b (2 marks)

Write down the time when Ingrid's body temperature is at a maximum and a minimum.

Maximum: : Select

Minimum: : Select

During the day, Ingrid's body temperature (B) can be modelled using the equation

$$B = -0.5\cos\frac{\pi}{12}t + 36.5$$

where t is the time in hours after 12 am.
Angles are in radians

Question 6c (2 marks)

Write down the amplitude and period.

Amplitude:

Period:

During the day, Ingrid's body temperature (B) can be modelled using the equation

$$B = -0.5\cos\frac{\pi}{12}t + 36.5$$

where t is the time in hours after 12 am.
Angles are in radians

Question 6d (2 marks)

Determine the values of the maximum and minimum temperatures.

Maximum: ____ °C

Minimum: ____ °C

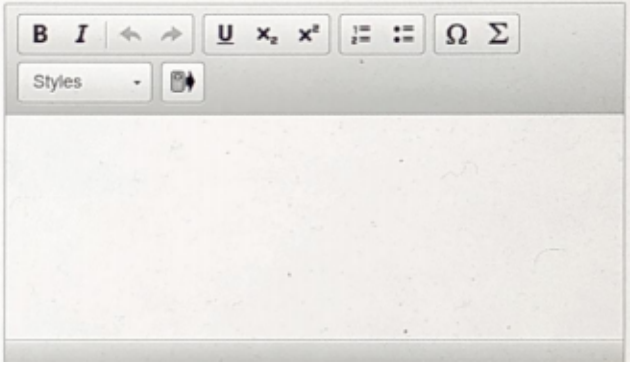
During the day, Ingrid's body temperature (B) can be modelled using the equation

$$B = -0.5\cos\frac{\pi}{12}t + 36.5$$

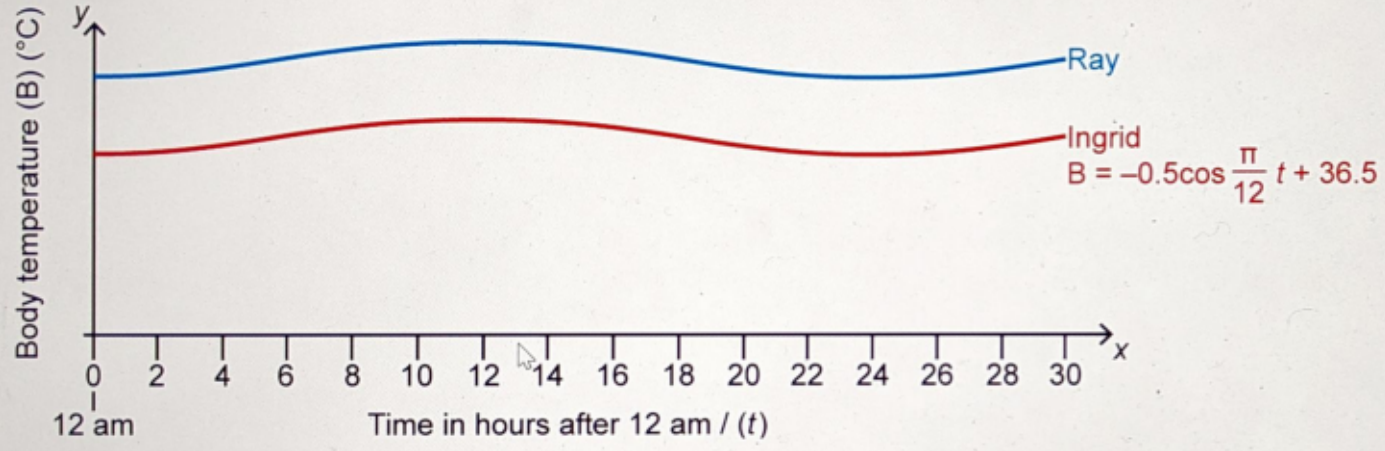
where t is the time in hours after 12 am.
Angles are in radians

Question 6e (3 marks)

Calculate Ingrid's body temperature at 7:15 am, to the nearest **one** decimal place.



Ray's body temperature is 0.25 °C higher than Ingrid's. The graph below shows two cosine curves modelling the body temperatures for Ingrid and Ray.



Question 6f (1 mark)

Write down the equation modelling Ray's body temperature.

B I U x_2 x^2 Ω Σ

Styles

Question 6g (4 marks)

Hence, **calculate** the first time when Ray's body temperature will reach 36.5 °C

B I U x_2 x^2 Ω Σ

Styles

Question 7 (19 marks)

In this question you will predict the reaction times of sprinters based on previously acquired data and their sleep pattern.

Video | Script

Sprinters competing in a 100 metre race should ensure they are well rested before competitions.

Studies have shown that a good sleeping habit can improve reaction time.

The start of the race is one of the most important factors for an overall fast time. Sprinters need to react as quickly as possible to the start signal.

In this question you will explore the reaction times of sprinters with different sleeping habits and how this affects their probability of winning a race.

These sprinters take a test that records their reaction time. The table below shows the results.

©

Reaction time in seconds (s)	0.75	0.76	0.77	0.78	0.79	0.80
Number of sprinters	4	3	5	6	1	1

Question 7a (2 marks)

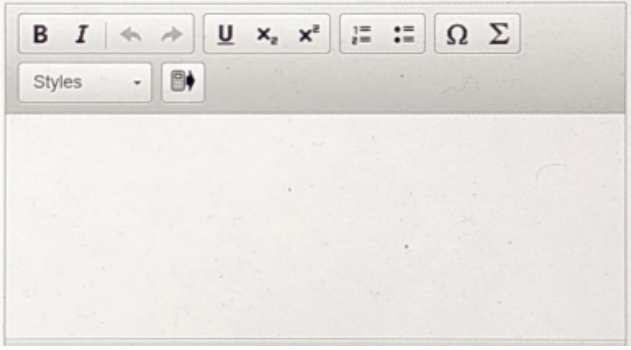
Write down the mode and median reaction times.

Mode: _____ seconds

Median: _____ seconds

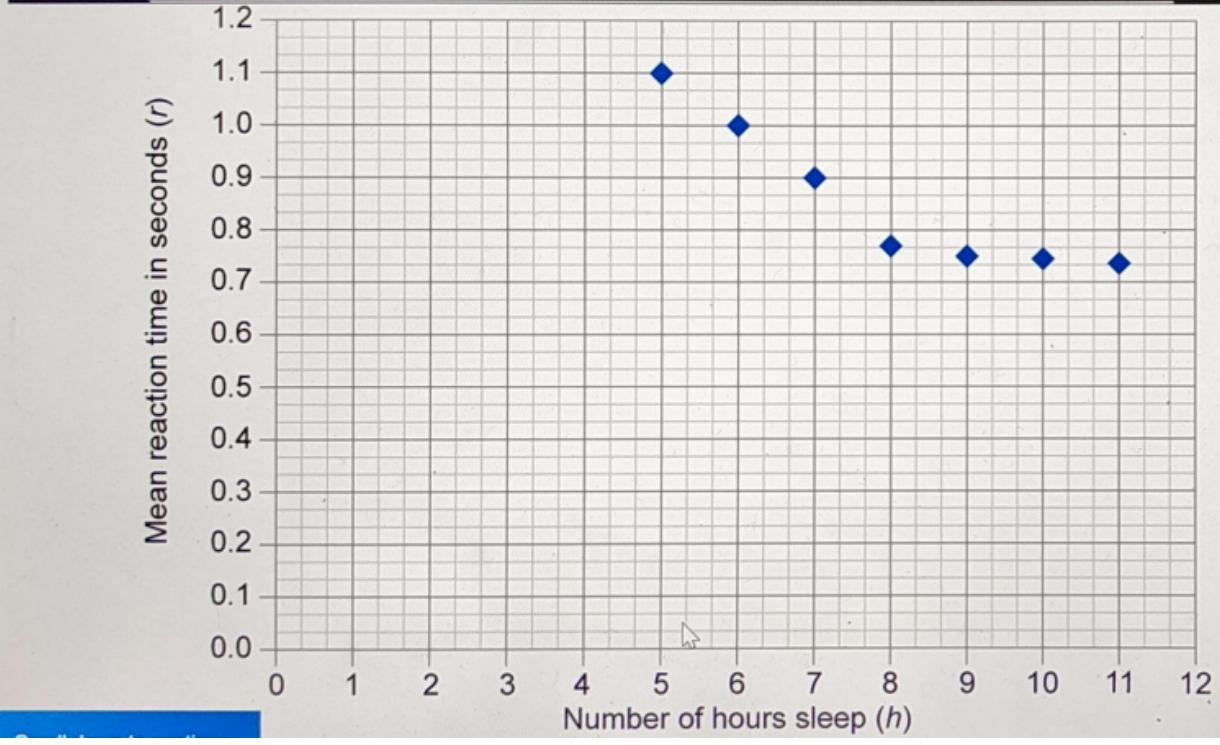
Question 7b (2 marks)

Show that the mean reaction time is 0.77 s, for this group of sprinters.



Groups of sprinters with different sleeping habits take the same test. The graph below shows the mean reaction time of each group.

Draw a line of best fit.



Question 7d (2 marks)

Using your line of best fit from (c), **write down** the value of r for $h = 4$ hours and $h = 7.5$ hours.

$h = 4$ hours $r =$ _____ seconds

$h = 7.5$ hours $r =$ _____ seconds

Question 7e (3 marks)

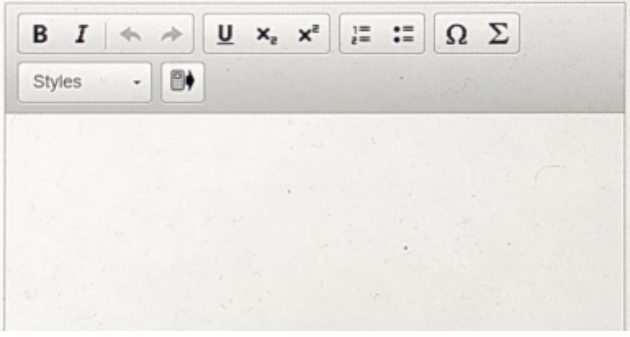
$w = 24\,(100)^{-r}$

Where:

w is the probability of winning a race.

r is the mean reaction time in seconds.

Calculate the value of w when $r = 0.77$ s. Give your answer correct to **two** significant figures.



Question 7f (8 marks)

Explore the probability of winning a race for sprinters with different sleeping habits. In your answer you must:

- identify the **two** relevant factors affecting the probability of winning
- calculate the probability of winning for sprinters with different sleeping habits
- comment on the relationship between the probability of winning and sleeping habits
- justify the accuracy of your findings.

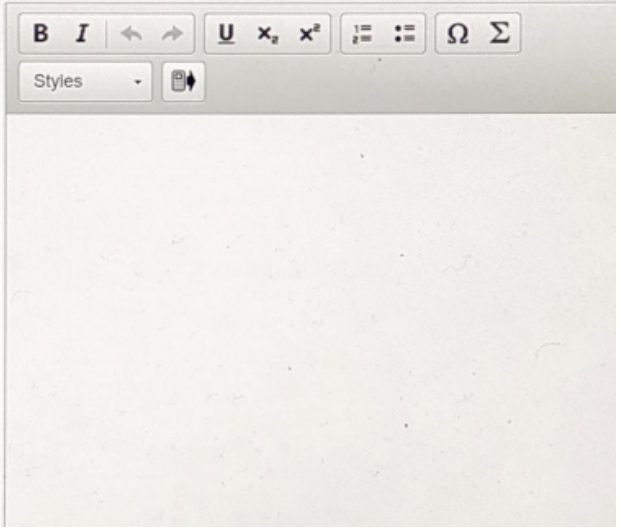
$w = 24\,(100)^{-r}$

Where:

w is the probability of winning a race.

r is the mean reaction time in seconds.

h	4	7.5	8	
r			0.77	
w				



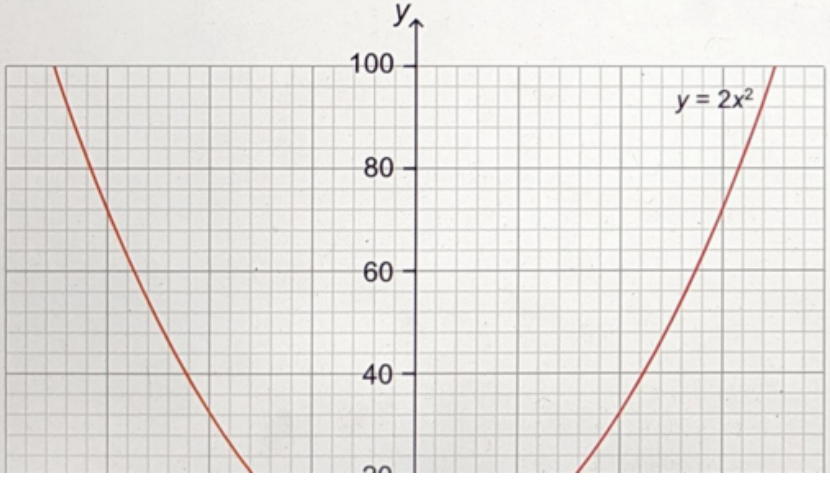
Question 8 (30 marks)

In this question, you will investigate areas of rhombuses.

The parabola $y = 2x^2$ is shown in the graph below. Different sized rhombuses are drawn inside the parabola.

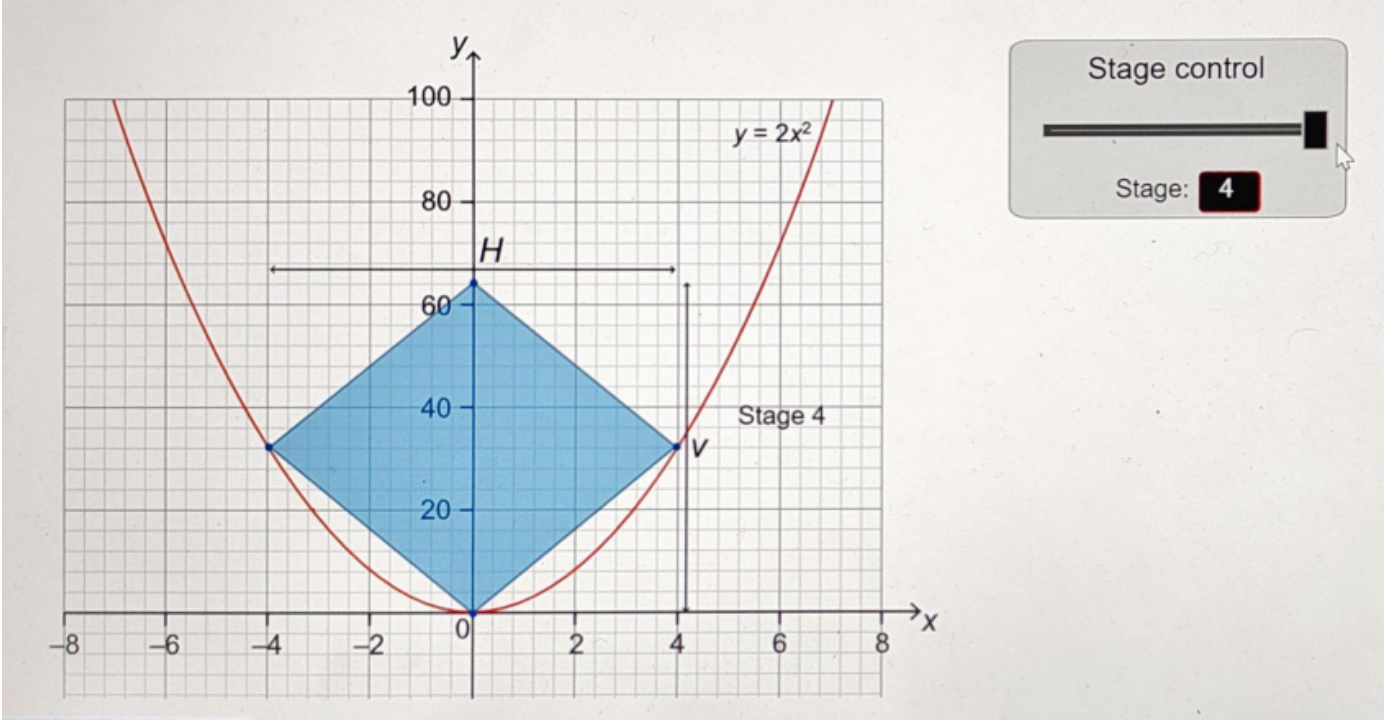
Drag the slider to see how the rhombuses are formed.

This media is interactive



Stage control

Stage: 1


y
100
80
60
40
20
0
H
V
y = 2x²
Stage 4
x
–8
–6
–4
–2
2
4
6
8
Stage control
Stage: 4

Question 8a (1 mark)

Write down the missing values in the table.

Stage (n)	Vertical length of rhombus (V)
1	4
2	16
3	36
4	64
5	
6	

Question 8b (2 marks)

Describe, in words, **two** patterns for V.

B I U x_2 x^2 Ω Σ

Styles

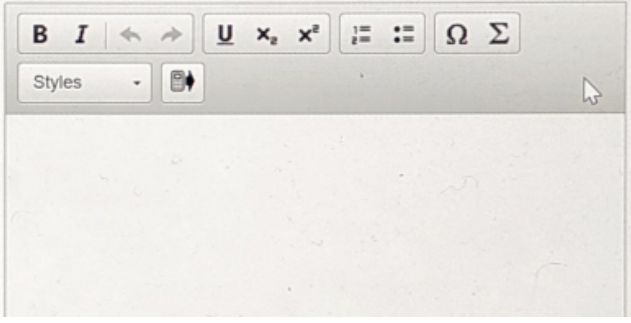
Question 8a (1 mark)

Write down the missing values in the table.

Stage (n)	Vertical length of rhombus (V)
1	4
2	16
3	36
4	64
5	
6	

Question 8c (2 marks)

Write down a general rule for V in terms of n.



Question 8a (1 mark)

Write down the missing values in the table.

Stage (n)	Vertical length of rhombus (V)
1	4
2	16
3	36
4	64
5	
6	

Question 8d (3 marks)

Verify your general rule for V.

B I U x_2 x^2 Ω Σ

Styles

Question 8e (22 marks)

Investigate the values in the table to find a relationship for the area (A) of the rhombus in terms of n. In your answer, you should:

- predict more values and record these in the table
- describe in words **one** pattern for A
- determine a general rule for A in terms of n
- test your general rule for A
- verify and justify your general rule for A
- ensure that you communicate all your working appropriately.

n	Vertical length of rhombus (V)	Horizontal length of rhombus (H)	Area of rhombus (A)	
1	4	2	4	
2	16	4	32	
3	36	6	108	
4	64	8	256	
5				
6				