Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2022

Mathematics: applications and interpretation

Higher level

Paper 3

15 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1****,* ***A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) (i) $Q(t) = 3090t - 54000$ $(3094.27...t - 54042.3...)$ ***A1A1***

Note: Award at most ***A1A0*** if answer is not an equation. Award ***A1A0*** for an answer including either x or y.

[2 marks]

(ii) 0.755 $(0.754741\ldots)$ ***A1***

[1 mark]

(iii) t is not a random variable **OR** it is not a (bivariate) normal distribution
OR data is not a sample from a population
OR data appears nonlinear
OR r only measures linear correlation ***R1***

Note: Do not accept "r is not large enough".

[1 mark]

(b) (i) attempt to separate variables ***(M1)***

$$\int \frac{1}{Q}\,\mathrm{d}Q = \int \beta N\,\mathrm{d}t$$

$$\ln|Q| = \beta Nt + c$$ ***A1A1A1***

Note: Award ***A1*** for LHS, ***A1*** for βNt, and ***A1*** for $+c$.
Award full marks for $Q = \mathrm{e}^{\beta Nt + c}$ **OR** $Q = A\mathrm{e}^{\beta Nt}$.
Award ***M1A1A1A0*** for $Q = \mathrm{e}^{\beta Nt}$

[4 marks]

(ii) attempt at exponential regression ***(M1)***

$Q = 1.15\mathrm{e}^{0.292t}$ $(Q = 1.14864...\mathrm{e}^{0.292055...t})$ ***A1***

OR

attempt at exponential regression ***(M1)***

$Q = 1.15 \times 1.34^{t}$ $(1.14864... \times 1.33917...^{t})$ ***A1***

Note: Condone answers involving y or x. Condone absence of "Q ="
Award ***M1A0*** for an incorrect answer in correct format.

[2 marks]

(iii) 0.999 $(0.999431...)$ ***A1***

[1 mark]

(iv) comparing something to do with R^2 and something to do with r ***M1***

Note: Examples of where the ***M1*** should be awarded:
$R^2 > r$
$R > r$
$0.999 > 0.755$
$0.999 > 0.755^2 \quad (= 0.563)$
The "correlation coefficient" in the exponential model is larger.
Model B has a larger R^2
Examples of where the ***M1*** should **not** be awarded:
The exponential model shows better correlation (since not clear how it is being measured)
Model 2 has a better fit
Model 2 is more correlated

an unambiguous comparison between R^2 and r^2 or R and r leading to the conclusion that the model in part (b) is more suitable / better ***A1***

Note: Condone candidates claiming that R is the "correlation coefficient" for the non-linear model.

[2 marks]

(v) it suggests that there will be more infected computers than the entire population ***R1***

Note: Accept any response that recognizes unlimited growth.

[1 mark]

(c) $1.15\,e^{0.292t} = 2.3$ **OR** $1.15 \times 1.34^t = 2.3$ **OR** $t = \dfrac{\ln 2}{0.292}$ **OR** using the model to find two specific times with values of $Q(t)$ which double ***M1***

$t = 2.37$ (days) ***A1***

Note: Do not ***FT*** from a model which is not exponential. Award ***M0A0*** for an answer of 2.13 which comes from using $(10, 20)$ from the data or any other answer which finds a doubling time from figures given in the table.

[2 marks]

(d) an attempt to calculate β for city X ***(M1)***

$\beta = \dfrac{0.292055...}{2.6 \times 10^6}$ **OR** $\beta = \dfrac{\ln 1.33917...}{2.6 \times 10^6}$

$= 1.12328... \times 10^{-7}$ ***A1***

this is larger than 9.64×10^{-8} so the virus spreads more easily in city X ***R1***

Note: It is possible to award ***M1A0R1***.
Condone "so the virus spreads faster in city X" for the final ***R1***.

[3 marks]

(e) $a = 38.3,\ b = 3086.1$ ***A1A1***

Note: Award ***A1A0*** if values are correct but not to 1 dp.

[2 marks]

(f) (i) $\dfrac{Q'}{Q} = 0.42228 - 2.5561\times10^{-6}Q$ ***(A1)(A1)***

Note: Award ***A1*** for each coefficient seen – not necessarily in the equation. Do not penalize seeing in the context of y and x.

identifying that the constant is k **OR** that the gradient is $-\dfrac{k}{L}$ ***(M1)***

therefore $k = 0.422 \quad (0.422228...)$ ***A1***

$\dfrac{k}{L} = 2.5561\times10^{-6}$

$L = 165000 \quad (165205)$ ***A1***

Note: Accept a value of L of 164843 from use of 3 sf value of k, or any other value from plausible pre-rounding.
Allow follow-through **within** the question part, from the equation of their line to the final two ***A1*** marks.

[5 marks]

(ii) recognizing that their L is the eventual number of infected ***(M1)***

$\dfrac{165205...}{2600000} = 6.35\% \quad (6.35403...\%)$ ***A1***

Note: Accept any final answer consistent with their answer to part (f)(i) unless their L is less than 120146 in which case award at most ***M1A0***.

[2 marks]

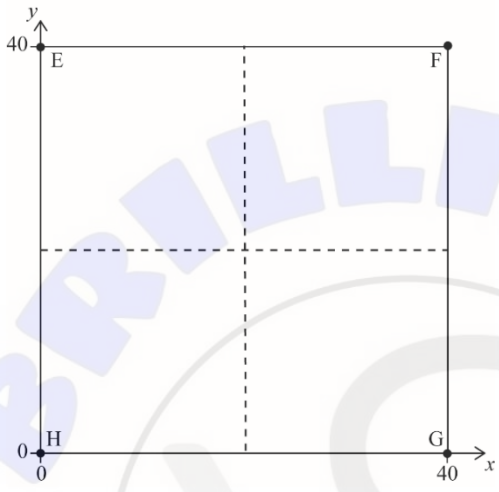
[Total 28 marks]

2. (a) the size of each town is small (in comparison with the distance between the towns)
OR
if towns have an identifiable centre
OR
the centre of the town is at that point ***R1***

Note: Accept a geographical landmark in place of "centre", e.g. "town hall" or "capitol".

[1 mark]

(b)



A1

Note: There is no need for a scale / coordinates here. Condone boundaries extending beyond the metropolitan area.

[1 mark]

(c) (i) the gradient of IF is $\frac{40-20}{40-30}=2$ ***(A1)***

negative reciprocal of any gradient ***(M1)***

gradient of perpendicular bisector $=-\frac{1}{2}$

Note: Seeing $-\frac{2}{3}$ (for example) used clearly as a gradient anywhere is evidence of the "negative reciprocal" method despite being applied to an inappropriate gradient.

midpoint is $\left(\frac{40+30}{2}, \frac{40+20}{2}\right)=(35,\ 30)$ ***(A1)***

equation of perpendicular bisector is $y-30=-\frac{1}{2}(x-35)$ ***A1***

Note: Accept equivalent forms *e.g.* $y=-\frac{1}{2}x+\frac{95}{2}$ or $2y+x-95=0$.
Allow ***FT*** for the final ***A1*** from their midpoint and gradient of perpendicular bisector, as long as the ***M1*** has been awarded.

[4 marks]

(ii) the perpendicular bisector of EH is $y = 20$ ***(A1)***

Note: Award this ***A1*** if seen in the y-coordinate of any final answer or if 20 is used as the y-value in the equation of any other perpendicular bisector.

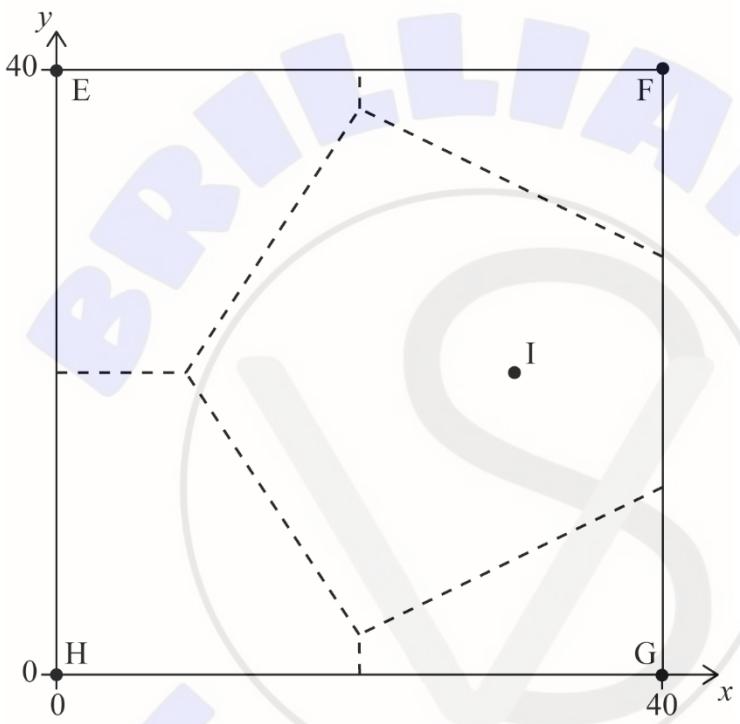
attempt to use symmetry **OR** intersecting two perpendicular bisectors ***(M1)***

$\left(\frac{25}{3}, 20\right)$ ***A1***

$(20, 2.5)$ ***A1***

[4 marks]

(iii)



M1A1

Note: Award ***M1*** for exactly four perpendicular bisectors around I (IE, IF, IG and IH) seen, even if not in exactly the right place.
Award ***A1*** for a completely correct diagram. Scale / coordinates are NOT necessary. Vertices should be in approximately the correct positions but only penalized if clearly wrong (condone northern and southern vertices appearing to be very close to the boundary).

Condone the Voronoi diagram extending outside of the square.
Do not award follow-though marks in this part.

[2 marks]

(d) 30% of 40 is 12 ***(A1)***

recognizing line intersects bisectors at $y = c$ (or equivalent) but different x-values ***(M1)***

$c = \frac{3}{2}x_1 + \frac{15}{2}$ and $c = -\frac{1}{2}x_2 + \frac{95}{2}$

finding an expression for the distance in Isaacopolis in terms of one variable ***(M1)***

$x_2 - x_1 = (95 - 2c) - \frac{2c - 15}{3} = 100 - \frac{8c}{3}$

equating their expression to 12

$100 - \frac{8c}{3} = 0.3 \times 40 = 12$

$c = 33$

distance $= 33$ (km) ***A1***

[4 marks]

(e) (i) must be a vertex (award if vertex given as a final answer) ***(R1)***
attempt to calculate the distance of at least one town from a vertex ***(M1)***

Note: This must be seen as a calculation or a value.

correct calculation of distances ***A1***

$\frac{65}{3}$ **OR** 21.7 **AND** $\sqrt{406.25}$ **OR** 20.2

$\left(\frac{25}{3}, 20\right)$ ***A1***

Note: Award ***R1M0A0A0*** for a vertex written with no other supporting calculations.
Award ***R1M0A0A1*** for correct vertex with no other supporting calculations.
The final ***A1*** is not dependent on the previous ***A1***. There is no follow-through for the final ***A1***.

Do not accept an answer based on "uniqueness" in the question.

[4 marks]

(ii) *For example, any one of the following:*
decision does not take into account the different population densities
closer to a city will reduce travel time/help employees
it is closer to some cites than others ***R1***

Note: Accept any correct reason that engages with the scenario.
Do not accept any answer to do with ethical issues about whether toxic waste should ever be dumped, or dumped in a metropolitan area.

[1 mark]

(f) (i) **METHOD 1**

attempting $\boldsymbol{M}^3$ ***M1***

attempting $\boldsymbol{M}^4$ ***M1***

e.g.

last row/column of $\boldsymbol{M}^3 = \begin{pmatrix} 3 & 5 & 1 & 6 & 0 & 7 \end{pmatrix}$

last row/column of $\boldsymbol{M}^4 = \begin{pmatrix} 10 & 12 & 4 & 16 & 1 & 18 \end{pmatrix}$

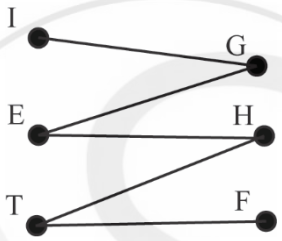
hence Isaacopolis is the last city to be polluted ***A1***

Note: Do not award the ***A1*** unless both $\boldsymbol{M}^3$ and $\boldsymbol{M}^4$ are considered.
Award ***M1M0A0*** for a claim that the shortest distance is from T to I and that it is 4, without any support.

METHOD 2

attempting to translate $\boldsymbol{M}$ to a graph or a list of cities polluted on each day ***(M1)***

correct graph or list ***A1***



hence Isaacopolis is the last city to be polluted ***A1***

Note: Award ***M1A1A1*** for a clear description of the graph in words leading to the correct answer.

[3 marks]

(ii) it takes 4 days ***A1***

[1 mark]

(iii) **EITHER**

the orders of the different vertices are:

E	2
F	1
G	2
H	2
I	1
T	2

(A1)

Note: Accept a list where each order is 2 greater than listed above.

OR

a correct diagram/graph showing the connections between the locations ***(A1)***

Note: Accept a diagram with loops at each vertex.
This mark should be awarded if candidate is clearly using their correct diagram from the previous part.

THEN

"Start at F and end at I" **OR** *"Start at I and end at F"* ***A1***

Note: Award ***A1A0*** for *"it could start at either F or I"*.
Award ***A1A1*** for *"IGEHTF"* **OR** *"FTHEGI"*.
Award ***A1A1*** for "*F and I*" **OR** "*I and F*".

[2 marks]

[Total 27 marks]