Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 2

2 May 2024

Zone A morning | **Zone B** morning | **Zone C** morning

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

15 pages

2224 – 7107

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The functions f and g are both defined for $-1 \le x \le 0$ by

$$f(x) = 1 - x^2$$

$$g(x) = e^{2x}.$$

The graphs of f and g intersect at $x = a$ and $x = b$, where $a < b$.

(a) Find the value of a and the value of b. [3]

(b) Find the area of the region enclosed by the graphs of f and g. [3]

2. [Maximum mark: 5]

Consider the following bivariate data set where $p, q \in \mathbb{Z}^+$.

x	5	6	6	8	10
y	9	13	p	q	21

The regression line of y on x has equation $y = 2.1875x + 0.6875$.

The regression line passes through the mean point $(\bar{x}, \bar{y})$.

(a) Given that $\bar{x} = 7$, verify that $\bar{y} = 16$. [1]

(b) Given that $q - p = 3$, find the value of p and the value of q. [4]

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3. [Maximum mark: 6]

The loudness of a sound, L, measured in decibels, is related to its intensity, I units, by $L = 10\log_{10}(I \times 10^{12})$.

Consider two sounds, S_1 and S_2.

S_1 has an intensity of 10^{-6} units and a loudness of 60 decibels.

S_2 has an intensity that is twice that of S_1.

(a) State the intensity of S_2. [1]

(b) Determine the loudness of S_2. [2]

The maximum loudness of thunder in a thunderstorm was measured to be 115 decibels.

(c) Find the corresponding intensity, I, of the thunder. [3]

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4. [Maximum mark: 6]

A particle moves in a straight line such that its velocity, $v\,\text{m s}^{-1}$, at time t seconds is given by $v(t) = 1 + e^{-t} - e^{-\sin 2t}$ for $0 \le t \le 2$.

(a) Find the velocity of the particle at $t = 2$. [1]

(b) Find the maximum velocity of the particle. [2]

(c) Find the acceleration of the particle at the instant it changes direction. [3]

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5. [Maximum mark: 5]

Consider a random variable X such that $X \sim \mathrm{B}(n, 0.25)$.

Determine the least value of n such that $\mathrm{P}(X \geq 1) > 0.99$.

6. [Maximum mark: 6]

The volume of a spherical bubble increases at a constant rate of $5\,\text{cm}^3\,\text{s}^{-1}$.

The initial volume of the bubble can be assumed to be zero.

Find the rate, in $\text{cm}\,\text{s}^{-1}$, at which the radius of the bubble is increasing when the volume of the bubble is $20\,\text{cm}^3$.

7. [Maximum mark: 5]

The curve $y = 4\ln(x-2)$ for $0 \le y \le 4$ is rotated $360°$ about the y-axis to form a solid of revolution.

Find the volume of the solid formed.

8. [Maximum mark: 10]

Let $z = 1 + \cos 2\theta + \mathrm{i}\sin 2\theta$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

(a) Show that

(i) $\arg z = \theta$;

(ii) $|z| = 2\cos\theta$. [7]

(b) Hence or otherwise, find the value of θ such that $\arg(z^2) = |z^3|$. [3]

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9. [Maximum mark: 8]

Consider the curve $y = \frac{x-4}{ax^2+bx+c}$, where a, b and c are non-zero constants.

The curve has a local minimum point at $(2, 1)$ and a vertical asymptote with equation $x = 1$.

Find the values of a, b and c.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 15]

A shop sells chocolates. The weight, in kilograms, of chocolates bought by a random customer can be modelled by a continuous random variable X with probability density function f defined by

$$f(x)=\begin{cases}\frac{6}{85}\left(4+3x-x^2\right), & 0.5\le x\le 3\\ 0, & \text{otherwise}\end{cases}.$$

(a) Find the mode of X. [2]

(b) Find $\mathrm{P}(1\le X\le 2)$. [2]

(c) Find the median of X. [3]

The shop sells chocolates to customers at $\$25$ per kilogram.

However, if the weight of chocolate bought by a customer is at least 0.75 kilograms, the shop sells chocolate at a discounted rate of $\$24$ per kilogram.

(d) Find the probability that a randomly selected customer spends at most $\$48$. [3]

(e) Find the expected amount spent per customer. Give your answer correct to the nearest cent. [5]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

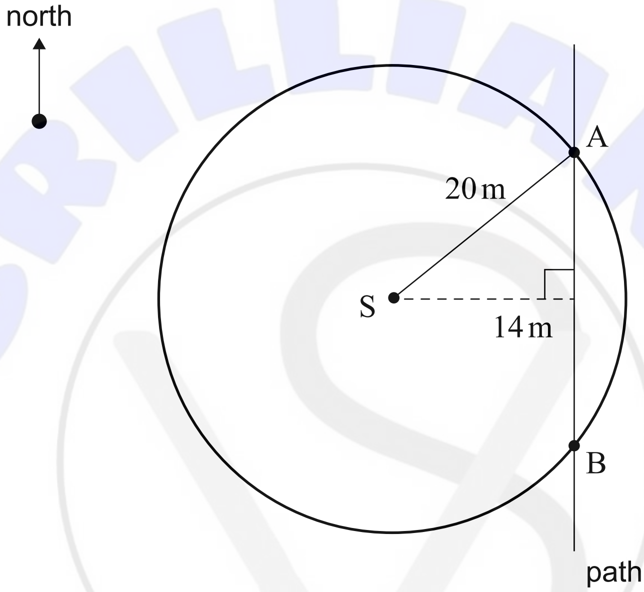
A rotating sprinkler is at a fixed point S.

It waters all points inside and on a circle of radius 20 metres.

Point S is 14 metres from the edge of a path which runs in a north-south direction.

The edge of the path intersects the circle at points A and B.

This information is shown in the following diagram.



(a) Show that $\mathrm{AB} = 28.57$, correct to four significant figures. [3]

The sprinkler rotates at a constant rate of one revolution every 16 seconds.

(b) Show that the sprinkler rotates through an angle of $\frac{\pi}{8}$ radians in one second. [1]

Let T seconds be the time that $[\mathrm{AB}]$ is watered in each revolution.

(c) Find the value of T. [4]

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(Question 11 continued)

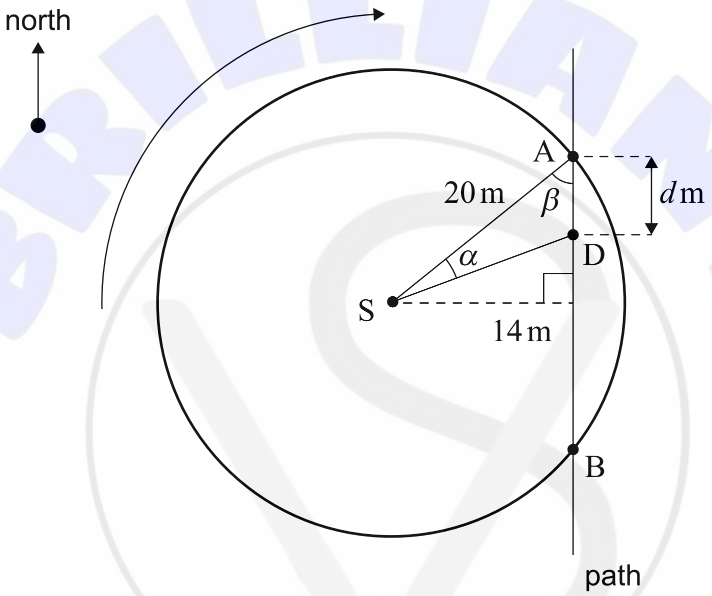
Consider one clockwise revolution of the sprinkler.

At $t = 0$, the water crosses the edge of the path at A.

At time t seconds, the water crosses the edge of the path at a movable point D which is a distance d metres south of point A.

Let $\alpha = \text{A}\hat{\text{S}}\text{D}$ and $\beta = \text{S}\hat{\text{A}}\text{B}$, where α, β are measured in radians.

This information is shown in the following diagram.



(d) Write down an expression for α in terms of t. [1]

It is known that $\beta = 0.7754$ radians, correct to four significant figures.

(e) By using the sine rule in ΔASD, show that the distance, d, at time t, can be modelled by

$$d(t) = \frac{20\sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)}.$$ [3]

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(Question 11 continued)

A turtle walks south along the edge of the path.

At time t seconds, the turtle's distance, g metres south of A, can be modelled by

$$g(t) = 0.05t^2 + 1.1t + 18\text{, where } t \geq 0.$$

(f) At $t = 0$, state how far south the turtle is from A. [1]

Let w represent the distance between the turtle and point D at time t seconds.

(g) (i) Use the expressions for $g(t)$ and $d(t)$ to write down an expression for w in terms of t.

(ii) Hence find when and where on the path the water first reaches the turtle. [4]

Do **not** write solutions on this page.

12. [Maximum mark: 21]

Consider the differential equation $\frac{dy}{dx} - y\operatorname{cosec} 2x = \sqrt{\tan x}$, where $0 < x < \frac{\pi}{2}$ and $y = \frac{\pi}{4}$ at $x = \frac{\pi}{4}$.

(a) Use Euler's method with step length $\frac{\pi}{12}$ to find an approximate value of y when $x = \frac{5\pi}{12}$.

Give your answer correct to three significant figures. [3]

(b) Show that $\frac{d}{dx}\left(\frac{1}{2}\ln(\cot x)\right) = -\operatorname{cosec} 2x$. [4]

(c) Show that $\sqrt{\cot x}$ is an integrating factor for this differential equation. [4]

(d) Hence, by solving the differential equation, show that $y = x\sqrt{\tan x}$. [5]

(e) Consider the curve $y = x\sqrt{\tan x}$ for $0 < x < \frac{\pi}{2}$ and the Euler's method approximation calculated in part (a).

(i) Find the y-coordinate at $x = \frac{5\pi}{12}$. Give your answer correct to three significant figures.

(ii) By considering the gradient of the curve, suggest a reason why Euler's method does not give a good approximation for the y-coordinate at $x = \frac{5\pi}{12}$.

(iii) State why this approximation is less than the y-coordinate at $x = \frac{5\pi}{12}$. [3]

(f) By considering $\frac{dy}{dx} - y\operatorname{cosec} 2x = \sqrt{\tan x}$, deduce that the curve $y = x\sqrt{\tan x}$ has a positive gradient for $0 < x < \frac{\pi}{2}$. [2]

Please **do not** write on this page.

Answers written on this page
will not be marked.