

# Markscheme

May 2024

**Mathematics:  
Applications and interpretation**

**Higher level**

**Paper 1**

20 pages

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## Instructions to Examiners

### Abbreviations

- M** Marks awarded for attempting to use a correct **Method**.
- A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- R** Marks awarded for clear **Reasoning**.
- AG** Answer given in the question and so no marks are awarded.
- FT** Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

### Using the markscheme

#### 1 General

Award marks using the annotations as noted in the markscheme eg **M1**, **A2**.

#### 2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award **M0** followed by **A1**, as **A** mark(s) depend on the preceding **M** mark(s), if any.
- Where **M** and **A** marks are noted on the same line, e.g. **M1A1**, this usually means **M1** for an **attempt** to use an appropriate method (e.g. substitution into a formula) and **A1** for using the **correct** values.
- Where there are two or more **A** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award **A0A1A1**.
- Where the markscheme specifies **A3**, **M2** etc., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the **AG** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award **FT** marks as appropriate but do not award the final **A1** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... (incorrect decimal value)	No. Last part in question.	Award <b>A1</b> for the final mark (condone the incorrect further working)
2.	$\frac{35}{72}$	0.468111... (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award <b>A0</b> for the final mark (and full <b>FT</b> is available in subsequent parts)

### 3 Implied marks

Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

### 4 Follow through marks (only applied after an error is made)

Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then **FT** marks should be awarded for *their* correct answer, even when working is not present.

**For example:** following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is **(M1)A1**, it is possible to award full marks for *their* correct answer, **without working being seen**. For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further **A** marks can be awarded for work which uses the error, but **M** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (e.g. probability greater than 1,  $\sin \theta = 1.5$ , non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any **FT** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these **FT** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

## 5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread and do not award the first mark, even if this is an **M** mark, but award all others as appropriate.

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (e.g. probability greater than 1,  $\sin \theta = 1.5$ , non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- **MR** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

## 6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, etc.
- Alternative solutions for parts of questions are indicated by **EITHER . . . OR**.

## 7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, **M** marks and intermediate **A** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

## 8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a “correct” level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come “from the use of 3 sf values”.

**Simplification of final answers:** Candidates are advised to give final answers using good mathematical form. In general, for an **A** mark to be awarded, arithmetic should be completed, and

any values that lead to integers should be simplified; for example,  $\sqrt{\frac{25}{4}}$  should be written as  $\frac{5}{2}$ .

An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example,  $\frac{10}{4}$  may be left in this form or

written as  $\frac{5}{2}$ . However,  $\frac{10}{5}$  should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g.  $4e^{2x} \times e^{3x}$  should be simplified to  $4e^{5x}$ , and  $4e^{2x} \times e^{3x} - e^{4x} \times e^x$  should be simplified to  $3e^{5x}$ . Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so  $x(x+1)$  and  $x^2 + x$  are both acceptable.

**Please note:** intermediate **A** marks do NOT need to be simplified.

## 9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

## 10. Presentation of candidate work

**Crossed out work:** If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

**More than one solution:** Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a) **EITHER**

$$N = 96$$

$$PV = \mp 100000$$

$$FV = \pm 150000$$

$$P/Y = 12$$

$$C/Y = 12$$

(M1)(A1)

**OR**

$$N = 8$$

$$PV = \mp 100000$$

$$FV = \pm 150000$$

$$P/Y = 1$$

$$C/Y = 12$$

(M1)(A1)

**OR**

$$150000 = 100000 \left( 1 + \frac{I}{100 \times 12} \right)^{12 \times 8}$$

(M1)(A1)

**Note:** Award **M1** for an attempt to use a financial app (at least 3 entries, not necessarily correct) or an attempt to use a compound interest formula.  
Award **A1** for all entries correct in financial app or correct substitution in compound interest formula.

**THEN**

$$I = 5.08 \text{ (5.07903...)}$$

**A1**

[3 marks]

(b)  $N = 120$

$$I\% = 6.1$$

$$PV = \mp 150000$$

$$PMT = \pm 1000$$

$$P/Y = 12$$

$$C/Y = 12$$

(M1)(A1)

**Note:** Award **M1** for an attempt to use a financial app (at least 3 entries, not necessarily correct). Award **A1** for all entries correct in financial app (condone missing +/- sign if the correct final answer is seen).

$$FV = (\$) 110867$$

**A1**

**Note:** Answer must be correct to nearest dollar to award the final **A1**. Award **(M1)(A1)A0** for an unsupported final answer to a greater degree of accuracy eg. (\$) 110866.70...  
Award **M1A1A0** for a truncated answer of 110866 if no working is shown.

[3 marks]

[Total: 6 marks]

2. (a) (i)  $r = 0.995$  (0.994705...) A2

**Note:** Award **A1** for 0.99.

- (ii)  $m = 10.6t + 43.9$  (10.6032... $t$  + 43.8780...) A1A1

**Note:** Second **A1** is for the correct variables.

[4 marks]

- (b) **EITHER**  
10.6032... $\times$ 1.5 (M1)

**OR**  
(10.6032...(t+1.5) + 43.8780...) – (10.6032...(t) + 43.8780...) (M1)

**THEN**  
15.9 (marks) (15.9048...) A1

**Note:** Accept 16.

[2 marks]

- (c) *Accept any valid reason* R1

e.g:

The students in the sample might not be of equal ability / she has not controlled for ability

She might have originally obtained close to full marks so an extra 15.9 would not be possible.

[1 mark]

[Total: 7 marks]

3. (a) (i) attempt to rearrange to isolate  $C$  (M1)  
e.g., subtracting 32 or dividing the equation by 1.8

$$C = \frac{5}{9}(F - 32) \left( C = \frac{F - 32}{1.8}, C = 0.556F - 17.8 \right) \quad \text{A1}$$

**Note:** If the answer is not written as an equation, award at most **M1A0**.

(ii)  $C = \left( \frac{77 - 32}{1.8} \right) 25$  (°C) A1

[3 marks]

- (b) (i)  $(1.8 \times 17 + 32 =)$  62.6 (°F) A1

(ii) recognizing that the "+32" does not affect the SD (M1)  
 $(1.8 \times 9 =)$  16.2 (°F) A1

**Note:** Award **M0A0** for  $1.8 \times 9 + 32$  (= 48.2).

[3 marks]

[Total 6 marks]

4. attempt to use Euler

$$y_{n+1} = y_n + 0.1 \log(x_n + y_n) \quad (\mathbf{A1})$$

$$y_1 (= 1 + 0.1 \times \log_{10}(1)) = 1 \quad (\mathbf{A1})$$

$$y_2 = 1.004139... \quad (\mathbf{A1})$$

**THEN**

when  $x = 2$        $y(2) \approx 1.61$  (1.60536...)      **A1**

**[Total: 4 marks]**

5. (a)  $y = 0.5x - 1$       **A1A1**

**Note:** Award **A1** for  $0.5x$  and **A1** for  $-1$  (or equivalent equation). Award at most **A1A0** if answer is not presented as an equation.

**[2 marks]**

(b)  $(6.857, 2.429)$       **A1A1**

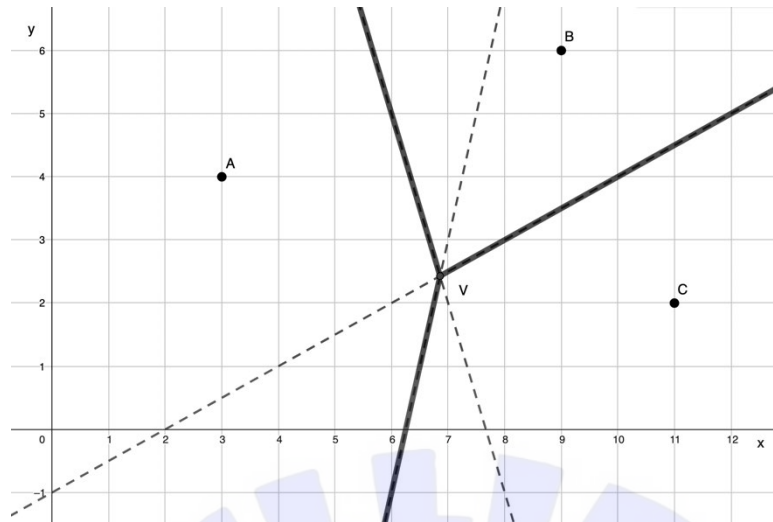
**Note:** If both answers are not correct to 4 sig figs, award at most **A1A0**.

Accept  $x = 6.857$ ,  $y = 2.429$ .

Award **A1A0** for  $(\frac{48}{7}, \frac{17}{7})$ . Award **A0A1** for  $(2.429, 6.857)$ .

**[2 marks]**

(c)



**A2**

**Note:** Award marks as shown in the table below. Condone edges that do not extend to the sides of the graph or beyond the  $x$ -axis.

Correct edges	Incorrect edges	Marks
3	0	<b>A2</b>
3	1	<b>A1A0</b>
3	2 or more	<b>A0A0</b>
2	0	<b>A1A0</b>
2	1	<b>A1A0</b>
2	2 or more	<b>A0A0</b>
1	0	<b>A1A0</b>
1	1 or more	<b>A0A0</b>

**[2 marks]**  
**[Total 6 marks]**

6. (a) attempt to substitute into geometric sequence formula for twelfth term **OR** at least three correct terms of the sequence **(M1)**  
 $u_{12} = 40 \times 1.1^{12-1}$  **OR** 40, 44, 48.4, ...  
 114 (114.124...) **A1**

[2 marks]

- (b) (i) attempt to substitute into geometric series formula **OR** a sum of at least the first three terms **(M1)**  
 $S_{12} = \frac{40(1.1^{12} - 1)}{1.1 - 1}$  **OR**  $\sum_1^{12} (40 \times 1.1^{n-1})$  **OR** 40 + 44 + 48.4 + ...

**Note:** Award **M1** for  $u_1 = 40$  and  $r = 1.1$  seen as part of a geometric series formula, or **M1** for sigma notation and their  $u_n$  formula (condone missing limits), or **M1** for the sum of at least the **correct** first three terms of the sequence.

$S_{12} = 855$  (855.371...) **A1**

- (ii) finding  $S_{24} = 3539.89...$  or attempt to find the sum between  $u_{13}$  and  $u_{24}$  **(M1)**

**Note:** Award **M1** for  $S_{24} = 3539.89...$  or sigma notation that includes correct limits and their  $u_n$  formula or a substituted geometric series formula that includes 125.537... and  $n = 12$  or a list of terms that includes at least the 13<sup>th</sup> term and the 24<sup>th</sup> term.

$3539.89... - 855.371...$  **OR**  $\sum_{13}^{24} (40 \times 1.1^{n-1})$  **OR**  
 $(S_{13 \text{ to } 24} =) \frac{125.537... (1.1^{12} - 1)}{1.1 - 1}$  **OR** 125.537 + ... + 358.172... **(A1)**

**Note:** Accept a calculation using  $u_{13} = 125$  or 126.

2680 (2684.52..., 2684, 2685) **A1**

**Note:** For  $u_{13} = 125$ , the sum is 2673.03... and for  $u_{13} = 126$ , the sum is 2694.41...

[5 marks]  
 [Total: 7 marks]

7. (a) (i)  $m = ah^3$  (M1)  
 $64 = a \times 0.8^3 \Rightarrow a = 125$  (A1)

$m = 125h^3$  A1

**Note:** The final answer must be written as  $m$  in terms of  $h$  to award the final A1.

(ii)  $m = (125(0.75)^3) = 52.7 \text{ (kg) (52.7343...)} \quad \text{A1}$

[4 marks]

(b) **EITHER (finding the height of the lion)**

$(220 = 125h^3)$   
 $h = \sqrt[3]{1.76} \text{ (1.20736...)} \quad \text{(A1)}$

$E = k_1h^2 \text{ (seen anywhere)} \quad \text{(A1)}$

$k = \frac{(\sqrt[3]{1.76})^2}{0.8^2} \quad \text{(M1)}$   
 $= 2.28 \text{ (2.27769...)} \quad \text{A1}$

**OR (finding a formula for E in terms of m)**

$(m = 125h^3)$   
 $E = k_1h^2 \quad \text{(A1)}$

$E = k_2m^{\frac{2}{3}} \quad \text{(A1)}$

$k = \frac{220^{\frac{2}{3}}}{64^{\frac{2}{3}}} \quad \text{(M1)}$

$= 2.28 \text{ (2.27769...)} \quad \text{A1}$

[4 marks]  
[Total 8 marks]

8. (a) B and C A1  
[1 mark]

(b) correct intervals seen ( $x \leq 5$  (or  $x < 5$ ) AND  $x \geq 5$  (or  $x > 5$ )) A1

**Note:** The case of  $x = 5$  must be included for this **A1** to be awarded.

attempt to add edges to  $33+x$  (M1)

(If  $x < 5$  (or  $x \leq 5$ ) then repeat BC and) length is  $33+2x$  A1

(If  $x > 5$  (or  $x \geq 5$ ) then repeat AB and AC and) length is  $(33+x+5)=38+x$  A1

**Note:** If the intervals are not explicit, award at most **A0(M1)A1A1**.

[4 marks]  
[Total 5 marks]

9. (a) attempt to integrate by substitution or inspection (M1)  
 $4 \ln|2x+3|+c$  OR  $4 \ln|x+1.5|+c$  A1A1

**Note:** Award **M1** for  $\ln(2x+3)$  or  $\ln(x+1.5)$ , **A1** for the 4 and **A1** for  $c$ . The **A** marks can only be awarded if the **M** mark is awarded. Condone absence of modulus signs.

[3 marks]

(b) recognizing that area is  $[4 \ln(2x+3)]_0^6$  (M1)

$= 4 \ln(15) - 4 \ln(3)$  (A1)

use of log laws for their expression (M1)

$= 4 \ln(5) (= 2 \ln(25) = 1 \ln(625))$  A1

**Note:** Award **(M1)A0M0A0** for an unsupported final answer of 6.43775...  
Award at most **(M1)A1FTM0A0** if their answer from part (a) does not include  $\ln$ .

[4 marks]  
[Total 7 marks]

10. (recognition that OB is a radius)

$$(\text{radius} =) \sqrt{5^2 + 8^2} (= \sqrt{89})$$

(A1)

**EITHER (finding angle BOQ)**

correct calculation for finding  $\hat{B}OA$

(A1)

$$\hat{B}OA = \arctan\left(\frac{8}{5}\right) \quad \text{OR} \quad \tan \hat{B}OA = \frac{8}{5}$$

expressing  $\hat{B}OQ$  as  $90 + \hat{B}OA$

(M1)

$$\hat{B}OQ = 90 + \arctan\left(\frac{8}{5}\right) \quad \text{OR} \quad \hat{B}OQ = \frac{\pi}{2} + \arctan\left(\frac{8}{5}\right)$$

$$(\hat{B}OQ =) 147.994^\circ \dots \quad \text{OR} \quad 2.58299 \dots$$

substituting *their* radius and angle BOQ correctly into arc length formula

(M1)

$$(\text{arc BQ} =) \frac{90 + \arctan\left(\frac{8}{5}\right)}{360} \times 2\pi(\sqrt{5^2 + 8^2}) \quad \text{OR} \quad \left(\frac{\pi}{2} + \arctan\left(\frac{8}{5}\right)\right) \times (\sqrt{5^2 + 8^2})$$

$$24.4 \text{ (m)} \quad (24.3679 \dots)$$

A1

**OR (finding angle BOP)**

correct calculation for finding angle  $\hat{B}OP$

(A1)

$$\hat{B}OP = \arctan\left(\frac{5}{8}\right) \quad \text{OR} \quad \tan \hat{B}OP = \frac{5}{8}$$

substituting *their* radius and  $\hat{B}OP$  correctly into arc length formula

(M1)

$$(\text{arc BP} =) \frac{\arctan\left(\frac{5}{8}\right)}{360} \times 2\pi(\sqrt{5^2 + 8^2})$$

subtracting *their* arc BP from arc PQ

(M1)

$$(\text{arc BQ} =) \pi\sqrt{5^2 + 8^2} - \frac{\arctan\left(\frac{5}{8}\right)}{360} \times 2\pi(\sqrt{5^2 + 8^2})$$

$$24.4 \text{ (m)} \quad (24.3679 \dots)$$

A1

[Total: 5 marks]

11. (a)  $\begin{pmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{pmatrix}$  **A1A1**

**Note:** Award **A1** for selecting the correct matrix, **A1** for substituting  $2\alpha$  into a rotation matrix  
Award **A1A0** for clockwise rotation. These marks can be awarded independently.  
Condone the use of a different symbol.

**[2 marks]**

(b)  $\begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$   
 $= \begin{pmatrix} \cos^2 \alpha - \sin^2 \alpha & -2 \cos \alpha \sin \alpha \\ 2 \cos \alpha \sin \alpha & \cos^2 \alpha - \sin^2 \alpha \end{pmatrix}$  **M1A1**

**Note:** Award **M1** for an attempt to multiply matrices, e.g. at least one entry correct.

**[2 marks]**

(c) (i) (Because matrix multiplication represents the composition of transformations)  
 two rotations of  $\alpha$  are equivalent to a rotation of  $2\alpha$  **R1**  
 so the two matrices are equal (so each of the entries are also equal) **R1**  
 $\sin(2\alpha) = 2 \sin(\alpha) \cos(\alpha)$  **AG**

(ii)  $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$  **A1**

replacing  $\cos^2 \alpha$  with  $1 - \sin^2 \alpha$  **M1**

$= 1 - \sin^2 \alpha - \sin^2 \alpha$

$= 1 - 2 \sin^2 \alpha$  **AG**

**[4 marks]**

**[Total 8 marks]**

12. (a)  $40 = 100 - 100 \times 2^{-t}$  (A1)  
 0.737 (hours) (0.736965...) A1

**Note:** Accept 44.2 minutes.

[2 marks]

- (b) values of  $P$  are 50, 75, 87.5, 93.75 (A1)

$$SS_{res} = (50 - 48)^2 + (75 - 74)^2 + (87.5 - 86)^2 + (93.75 - 91)^2 \quad (M1)(A1)$$

$$= 14.8 \quad (14.8125) \quad A1$$

[4 marks]

- (c) (i) The sum of the square residuals is smaller so it is a better fit R1

- (ii) Accept a valid argument in favour of model  $P$  or against the quadratic model. R1

*e.g. any one of the following:*  
 quadratic has no asymptote  
 quadratic will begin to go down  
 quadratic will become negative  
 quadratic might not go through (0, 0)  
 model  $P$  is the manufacturer's model

[2 marks]  
 [Total 8 marks]

13. (a) (i)  $(|z_1| = \sqrt{4^2 + 5^2} =) \quad 6.40 \quad (6.40312\dots, \sqrt{41}) \quad A1$

- (ii)  $(\arg(z_1) =) \quad 0.896 \left( 0.896055\dots, 51.3401\dots^\circ, \arctan\left(\frac{5}{4}\right) \right) \quad A1$

[2 marks]

- (b) angle in triangle is  $2 - 0.896055\dots$  OR  $114.591^\circ - 51.3401\dots^\circ$  (A1)  
 use of area of triangle formula (M1)

$$\frac{1}{2} \times 6.40312\dots \times 3 \times \sin(2 - 0.896055\dots) \quad (A1)$$

$$8.58 \quad (8.57688\dots) \quad A1$$

**Note:** Accept methods that use Cartesian form or vector product.

[4 marks]  
 [Total 6 marks]

14. (a)  $x_B = 4\sqrt{2(t-3)}$  ( $t \geq 3$ ) **A1A1**

**Note:** Award **A1** for multiplying by 2 and **A1** for  $t-3$ . Award **A1A0** for  $4\sqrt{2t-3}$ .

[2 marks]

(b) equating their  $x_B$  to  $4\sqrt{t}$  **(M1)**

$$4\sqrt{2(t-3)} = 4\sqrt{t}$$

$$t = 6 \text{ (seconds)}$$

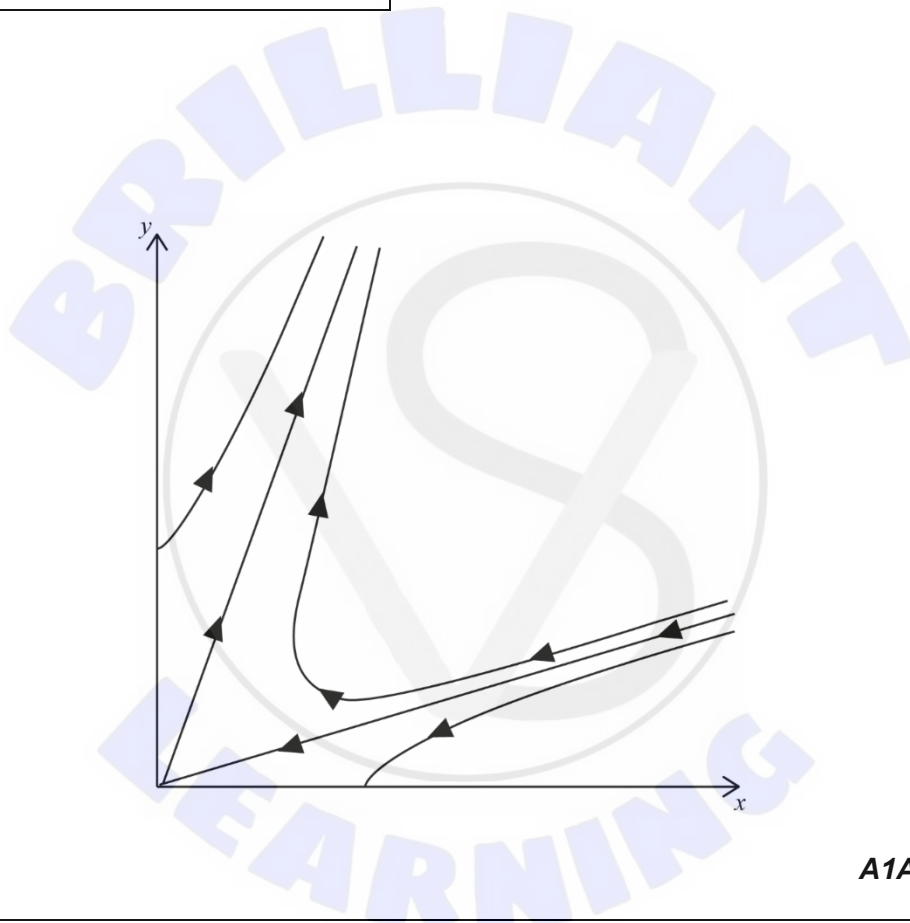
**A1**

**Note:** Do not **FT** from part (a) to part (b).

[2 marks]

[Total 4 marks]

15. (a) (i)&(ii)



**A1A1A1**

**Note:** Award **A1** for correct directions on eigenvectors, **A1** for correct trajectories, **A1** for correct arrows on trajectories.

[3 marks]

(b) for Y not to die out  $y > \frac{1}{3}x$  **(R1)**

as  $x = 252$ ,  $y > 84$  **(M1)**

(minimum number of new animals is) 25 **A1**

**Note:** Award **(R1)(M1)A0** for an unsupported 24.

[3 marks]

[Total 6 marks]

16. attempt to find gradient (M1)

**EITHER**

gradient of tangent =  $-\tan 75^\circ$  ( $= -3.73205\dots, -2 - \sqrt{3}$ ) (A1)(A1)

**Note:** Award **A1** for negative and **A1** for  $\tan 75^\circ$  (or equivalent).

**OR**

gradient of tangent =  $\tan 105^\circ$  ( $= -3.73205\dots$ ) (A2)

**THEN**

$\frac{dy}{dx} = -5.5 \sin(1.1x)$  (A1)

**Note:** Award **(A1)** for a labelled sketch of the derivative function.

equating derivative to their gradient (M1)

$-5.5 \sin(1.1x) = -3.73205\dots$  **OR** line on graph

$x = 0.677993\dots$  (A1)

**Note:** Award **(A1)(M1)A0** for an answer of  $x = 38.8$ , from calculator being in degrees.

Award **A0M1A0** if " $\frac{d}{dx}(5 \cos(1.1x)) = -3.73205\dots$ " seen, but leading to an incorrect  $x$ -value.

height =  $5 \cos(1.1 \times 0.677993\dots)$  (M1)

$= 3.67$  (m) (3.67274...) A1

**[Total 8 marks]**

17. (a)  $\begin{pmatrix} 0.25 & 0.25 \\ 0.1 & 0.1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$  (M1)

**Note:** Accept equivalent methods including only using one line of the matrix.

$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$  (or any multiple) A1

[2 marks]

(b)  $D^n = \begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix}$  A1

[1 mark]

(c)  $\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}^{-1}$  (M1)

**EITHER**  
multiplying by the initial state (M1)

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 7000 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2000 \\ 1000 \end{pmatrix}$  (A1)

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2000 \times 0.65^n \\ 1000 \end{pmatrix}$  (A1)

$\begin{pmatrix} 2000 \times 0.65^n + 5000 \\ -2000 \times 0.65^n + 2000 \end{pmatrix}$  (A1)

**Note:** Award **A0** if either term in the matrix is incorrect.

**OR**

$$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}^{-1} = \frac{1}{7} \begin{pmatrix} 2 & -5 \\ 1 & 1 \end{pmatrix}$$

**A1**

$$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.65^n & 5 \\ -0.65^n & 2 \end{pmatrix}$$

**A1**

**Note:** The preceding **A1** marks can be awarded independently.

$$\frac{1}{7} \begin{pmatrix} 5 + 2 \times 0.65^n & 5 - 5 \times 0.65^n \\ 2 - 2 \times 0.65^n & 2 + 5 \times 0.65^n \end{pmatrix}$$

**A1**

**Note:** Award **A0** if any term in the matrix is incorrect.

multiplying by the initial state

**(M1)**

$$\frac{1}{7} \begin{pmatrix} 5 + 2 \times 0.65^n & 5 - 5 \times 0.65^n \\ 2 - 2 \times 0.65^n & 2 + 5 \times 0.65^n \end{pmatrix} \begin{pmatrix} 7000 \\ 0 \end{pmatrix}$$

**THEN**

$$2000 - 2000 \times 0.65^n \quad (= 2000(1 - 0.65^n))$$

**A1**

**Note:** For the final **A1**, follow through within the question part from the bottom-left entry of their 2x2 matrix or the bottom entry of their 2x1 matrix but only if “in terms of  $n$ ”.

If any mistake in the matrices is seen that DOES NOT affect the correct final answer, do not award the penultimate **A1** mark.

**[6 marks]**  
**[Total 9 marks]**