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**Mathematics: applications and interpretation**  
**Higher level**  
**Paper 1**

1 May 2024

**Zone A** afternoon | **Zone B** afternoon | **Zone C** afternoon

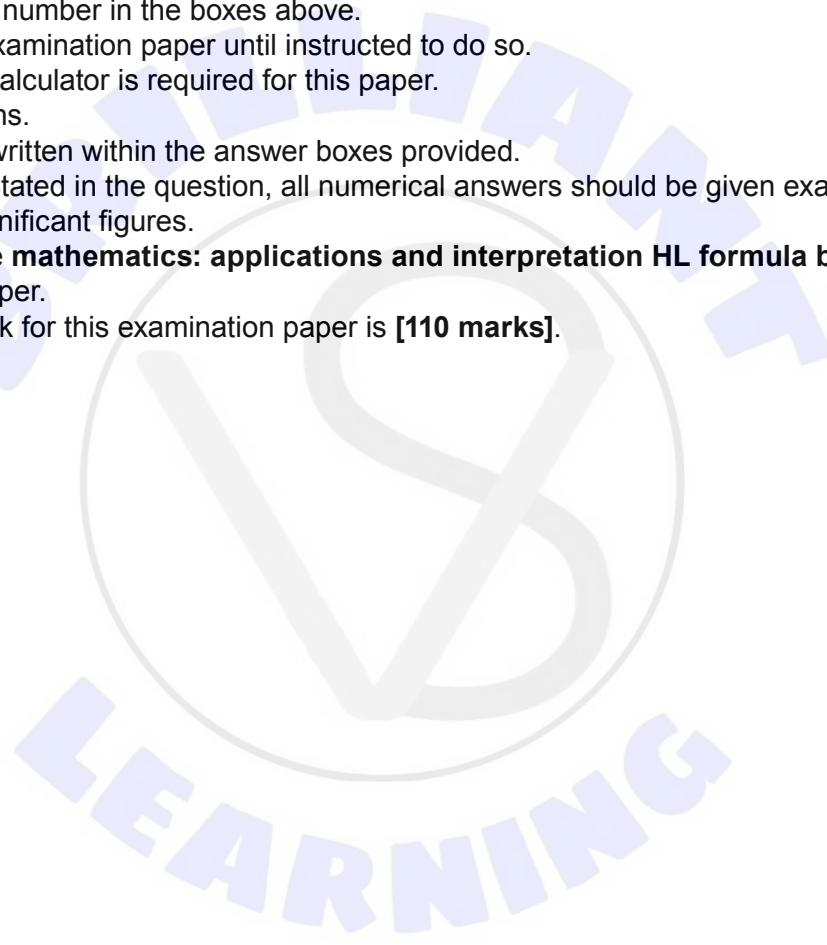
Candidate session number

2 hours

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**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.





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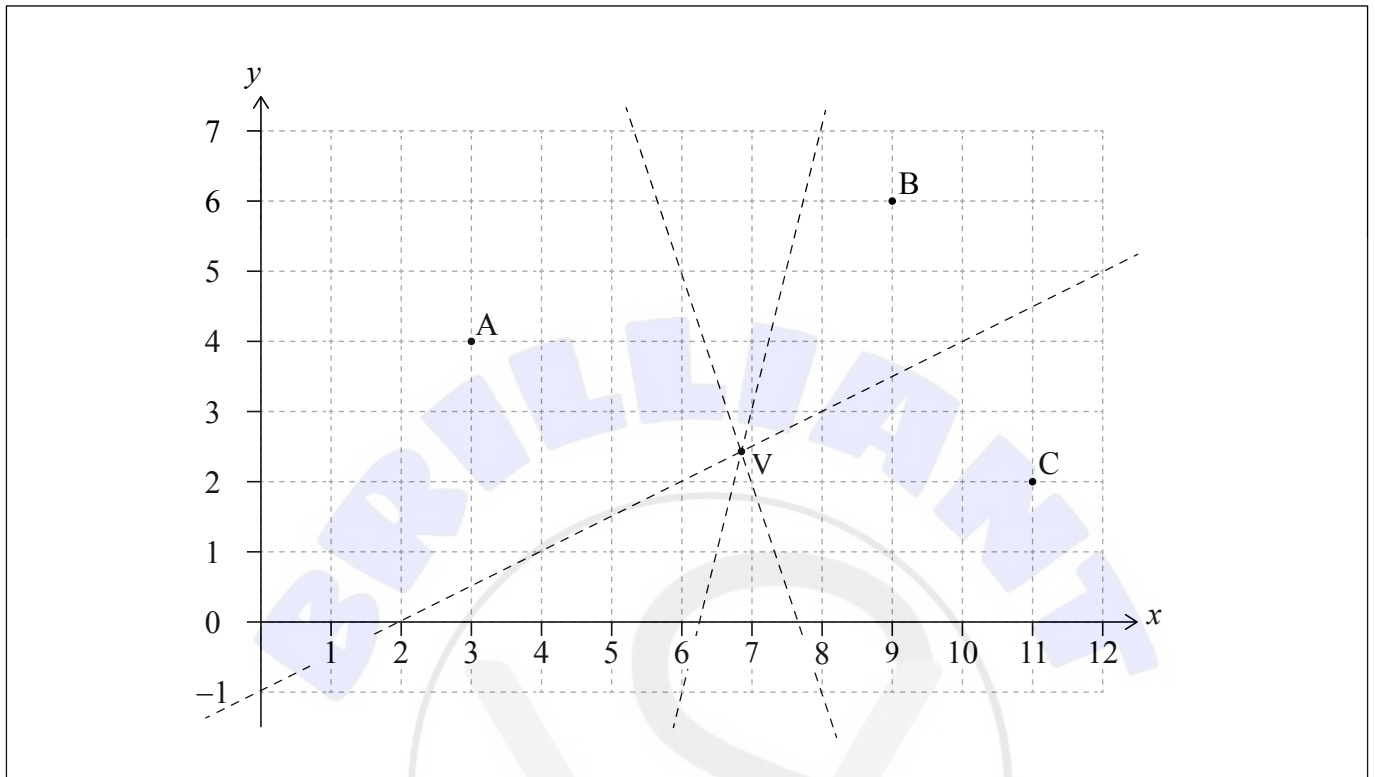
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5. [Maximum mark: 6]

Points A(3, 4), B(9, 6) and C(11, 2) are shown on the following diagram, along with the perpendicular bisectors of [AB], [AC] and [BC].



The perpendicular bisector of [BC] intercepts the axes at coordinates (0, -1) and (2, 0).

- (a) Write down the equation of the perpendicular bisector of [BC]. [2]

The equation of the perpendicular bisector of [AB] is  $y = -3x + 23$ .

- (b) Find the coordinates of point V where the perpendicular bisectors meet. Give your answer to four significant figures. [2]

A Voronoi diagram is constructed with points A, B and C as the three sites.

- (c) Draw, clearly, the edges of the Voronoi diagram on the given diagram. [2]

(This question continues on the following page)





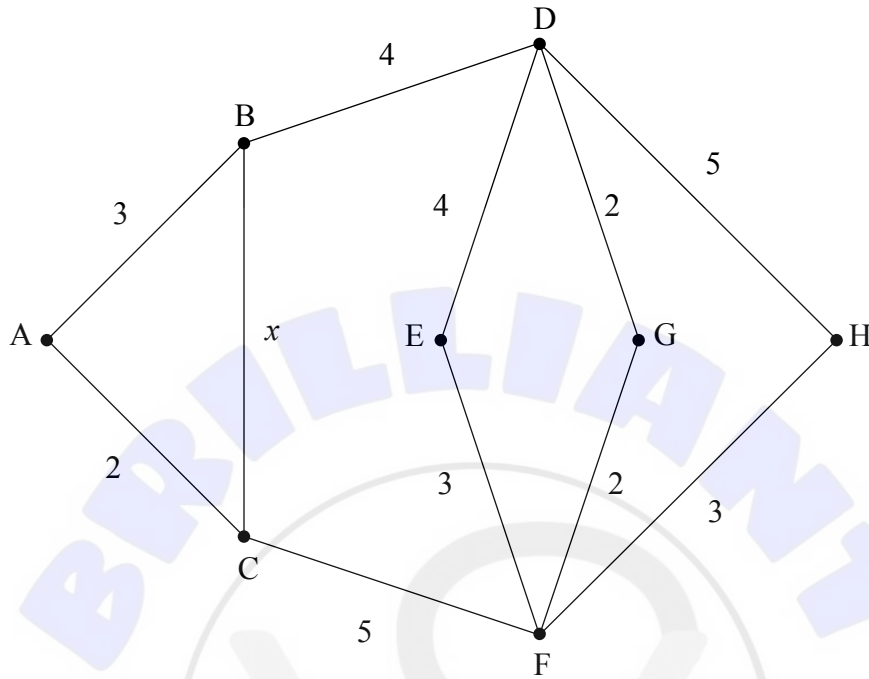




8. [Maximum mark: 5]

The weights on the following graph represent the lengths of different roads in kilometres.

diagram not to scale



(a) Write down the vertices with odd degree. [1]

The total length of the roads is  $33 + x$  km.

(b) Find **two** expressions, in terms of  $x$ , for the shortest distance required to walk along all of the paths, beginning and ending at the same vertex. Include in your answer the interval of values of  $x$  for which each expression is valid. [4]

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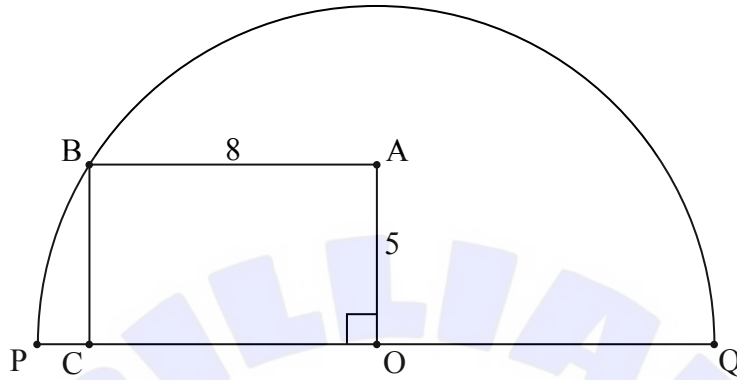




10. [Maximum mark: 5]

The following diagram shows a semicircle with centre  $O$  and diameter  $PQ$ . A rectangle  $OABC$  is also shown, such that  $AB = 8$  and  $OA = 5$ .

diagram not to scale



Find the length of the arc  $BQ$ .

Area for student response with horizontal dotted lines.











15. [Maximum mark: 6]

A system of differential equations of the form  $\frac{dx}{dt} = ax + by$ ,  $\frac{dy}{dt} = cx + dy$  has

eigenvalues  $\lambda = -1$  and  $\lambda = 2$  with corresponding eigenvectors  $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ .

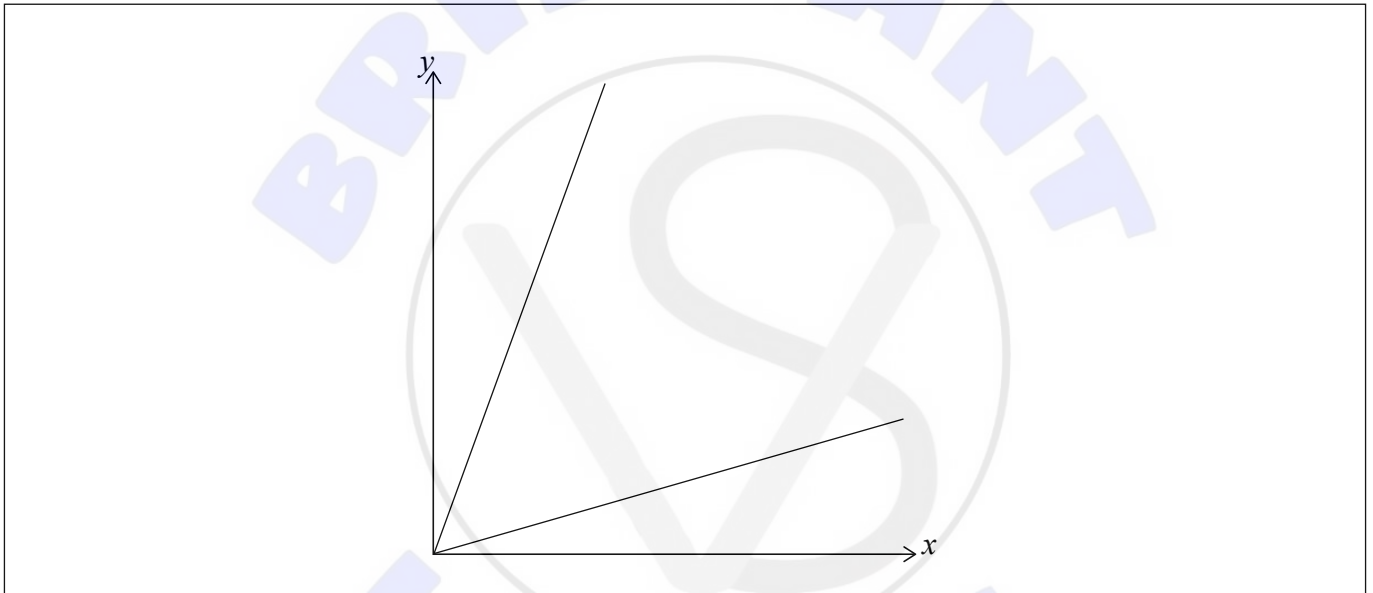
The following incomplete phase portrait for this system, with  $x, y \geq 0$ , shows lines through  $(0, 0)$  parallel to the eigenvectors.

(a) On the phase portrait

(i) show the direction of motion along the eigenvectors.

(ii) sketch one trajectory in each of the three regions.

[3]



In the system described above,  $x$  and  $y$  are the population sizes of two species, X and Y. The population of Y is vulnerable, so it will be increased by adding more animals from a different area. Currently,  $x = 252$  and  $y = 60$ .

(b) Find the minimum number of new animals from species Y that need to be added for the population not to reduce to 0 over time.

[3]

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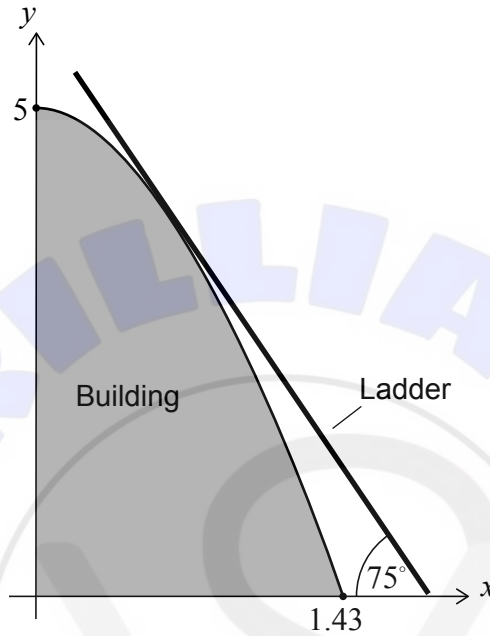
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16. [Maximum mark: 8]

The cross section of the side of a building can be modelled by a curve with equation  $y = 5 \cos(1.1x)$ ,  $0 \leq x \leq 1.43$ , as shown in the following diagram. Distances are measured in metres.

diagram not to scale



A builder leans a straight ladder against the building to do repairs. For safety reasons, the angle between the ladder and the horizontal ground must be  $75^\circ$ .

Find the height above the ground at which the ladder touches the building.

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17. [Maximum mark: 9]

Phoebe opens a coffee shop, near to a well-established Apollo coffee shop.

After being open for a few months, Phoebe notices that

- 10% of customers who preferred the Apollo coffee shop in one month preferred her coffee shop the following month.
- 25% of customers who preferred her coffee shop in one month preferred the Apollo coffee shop the following month.

She decides to show these changes in the following transition matrix.

$$\begin{pmatrix} 0.9 & 0.25 \\ 0.1 & 0.75 \end{pmatrix}$$

The two eigenvalues for this matrix are 1 and 0.65. An eigenvector corresponding to the eigenvalue of 1 is  $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ .

- (a) Find an eigenvector corresponding to the eigenvalue of 0.65. [2]

A diagonal matrix of eigenvalues is  $D = \begin{pmatrix} 0.65 & 0 \\ 0 & 1 \end{pmatrix}$ .

- (b) Write down an expression for  $D^n$ , giving your answer as a  $2 \times 2$  matrix in terms of  $n$ . [1]

When Phoebe’s coffee shop first opened, the Apollo shop had 7000 customers the previous month.

- (c) Assuming all 7000 customers continue to go to one of these coffee shops, find an expression for the number that will favour Phoebe’s coffee shop after  $n$  months. [6]

**(This question continues on the following page)**







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**References:**

7. Saddako, n.d. *Cheetah (Acinonyx jubatus) Running - stock photo*. [image online] Available at: <https://www.gettyimages.co.uk/detail/photo/cheetah-running-royalty-free-image/523244194?phrase=cheetah+speed&adppopup=true> [Accessed 2 May 2023]. Source adapted.

GlobalP, n.d. *Lion, Panthera leo, 8 years old, standing - stock photo*. [image online] Available at: <https://www.gettyimages.co.uk/detail/photo/lion-panthera-leo-8-years-old-standing-royalty-free-image/134976936?phrase=Lion+standing&adppopup=true> [Accessed 2 May 2023]. Source adapted.

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