Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

1 May 2024

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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Please **do not** write on this page.

Answers written on this page
will not be marked.

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

The prices, in dollars, of 10 different garden chairs are:

$$79 \quad 139 \quad 255 \quad 99 \quad 50 \quad 209 \quad 229 \quad 193 \quad 69 \quad 49$$

(a) Find the range of the prices of the 10 chairs. [2]

(b) Use your graphic display calculator to find

(i) the mean price of the chairs.

(ii) the standard deviation of the price of the chairs. [3]

In a sale, the price of each of the 10 garden chairs is reduced by $\$20$.

(c) Write down

(i) the new mean.

(ii) the new standard deviation. [2]

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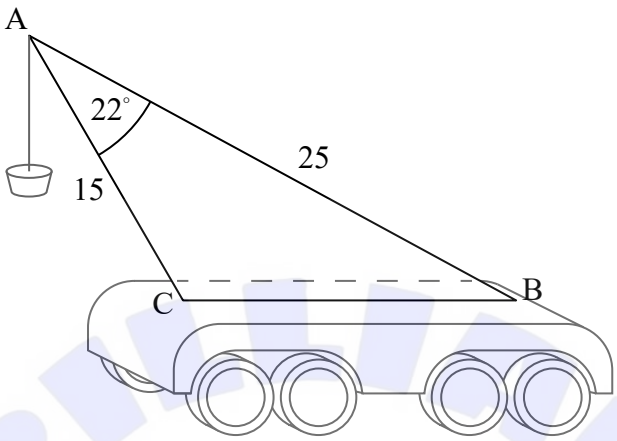
2. [Maximum mark: 6]

The diagram shows a toy crane.

diagram not to scale



$AB = 25\,\text{cm}$, $AC = 15\,\text{cm}$ and $B\hat{A}C = 22^\circ$.

(a) Calculate BC. [3]

(b) Given that $A\hat{B}C$ is acute, calculate $A\hat{B}C$. [3]

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3. [Maximum mark: 5]

Sunita sorts 300 peppers into sizes of small, medium or large. Some peppers are red, some are green, and some are yellow.

The following table shows her results.

	Small	**Medium**	**Large**
Red	18	31	46
Green	26	32	21
Yellow	34	66	26

Sunita wants to test, at the 5% significance level, whether the size of the peppers is independent of the colour.

(a) State the null and alternative hypotheses for this test. [1]

The critical value for this test is 9.49.

(b) (i) Calculate χ^2_{calc}.

(ii) State a conclusion to the test. Give a reason for your answer. [4]

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4. [Maximum mark: 6]

At a particular building site, the number of square metres of bricks, n, that can be laid in one working day varies directly with the number of bricklayers, B.

Five bricklayers can lay an area of $60\,\mathrm{m}^2$ of bricks in one working day.

(a) Calculate the area that can be laid by 7 bricklayers in one working day. [3]

At another building site, the time, t hours, it takes to lay bricks varies inversely with the number of bricklayers, B.

Five bricklayers take 8 hours to lay an area of $60\,\mathrm{m}^2$.

(b) Calculate how long it takes 12 bricklayers to lay an area of $60\,\mathrm{m}^2$. [3]

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5. [Maximum mark: 9]

Imani invests $\$3000$ in a bank that pays a nominal annual interest rate of 1.25% compounded monthly.

(a) Calculate the amount of money Imani will have in the bank at the end of 6 years. Give your answer correct to two decimal places. [3]

(b) Calculate the number of months it takes until Imani has at least $\$3550$ in the bank. [2]

Imani uses the $\$3550$ as a partial payment for a used car costing $\$22\,000$. For the remainder she takes out a loan from a bank.

(c) Write down the amount of money that Imani takes out as a loan. [1]

The loan is for 8 years and the nominal annual interest rate is 12.6% compounded monthly. Imani will pay the loan in fixed monthly instalments at the end of each month.

(d) Calculate the amount, correct to the nearest dollar, that Imani will have to pay the bank each month. [3]

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6. [Maximum mark: 6]

Jerry makes handcrafted chocolates. On average, 1 in 25 of the chocolates that Jerry makes is flawed. Whether or not a chocolate is flawed is independent of all other chocolates.

(a) In a batch of 20 chocolates, chosen at random, find the probability that

(i) two are flawed.

(ii) more than two are flawed. [4]

Jerry sells the perfect chocolates for 50 pesos each and the flawed ones for 15 pesos each.

(b) Calculate the expected number of pesos Jerry makes from selling a batch of 20 randomly selected chocolates. [2]

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7. [Maximum mark: 7]

The pH scale is a measure of the acidity of a solution. Its value is given by the formula

$$\text{pH} = -\log_{10}[\text{H}^+],$$

where $[\text{H}^+]$ is the concentration of hydrogen ions in the solution (measured in moles per litre).

(a) Calculate the pH value if the concentration of hydrogen ions is 0.0003. [2]

The pH of milk is 6.6.

(b) Calculate the concentration of hydrogen ions in milk. [2]

The strength of an acid is measured by its concentration of hydrogen ions.

A lemon has a pH value of 2 and a tomato has a pH value of 4.5.

(c) Calculate how many times stronger the acid in a lemon is when compared to the acid in a tomato. [3]

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8. [Maximum mark: 7]

Gustav plays a game in which he first tosses an unbiased coin and then rolls an unbiased six-sided die.

If the coin shows tails, the score on the die is Gustav's final number of points.

If the coin shows heads, one is added to the score on the die for Gustav's final number of points.

(a) Find the probability that Gustav's final number of points is 7. [2]

(b) Complete the following table. [3]

Final number of points	1	2	3	4	5	6	7
Probability							

(c) Calculate the expected value of Gustav's final number of points. [2]

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9. [Maximum mark: 6]

A marathon is a race over a distance of $42.195\,\text{km}$. Two runners, Eefje and Shumay, are training to run a marathon.

The two runners train in different ways:

- Eefje runs $5\,\text{km}$ on the first day of training and then increases the distance she runs by $2\,\text{km}$ on each subsequent day.
- Shumay runs $5\,\text{km}$ on the first day of training and then increases the distance she runs by $13\,\%$ on each subsequent day.

Determine which runner will be the first to run the distance of a marathon on a particular day of their training, and state on which day of their training this will occur.

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10. [Maximum mark: 7]

The gradient of the **normal** to the curve $y = ax^2 + bx - 10$ at the point $\mathrm{T}(2,\ 4)$ is $-\frac{1}{3}$.

Calculate the value of a and the value of b.

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11. [Maximum mark: 6]

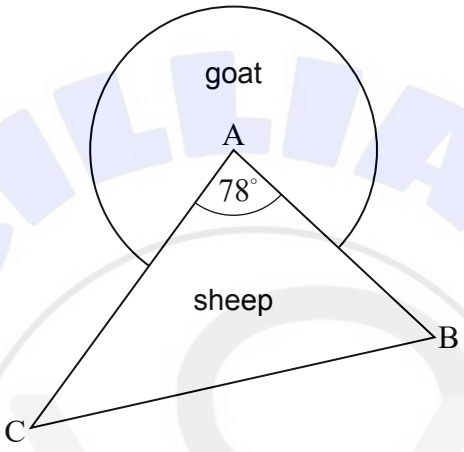
A sheep is in a field in the shape of a triangle, ABC.

$AC = 21$ metres, $AB = 15$ metres and $C\hat{A}B = 78^\circ$.

A goat is in an adjacent field in the shape of a sector of a circle with centre, A, and radius 8 metres.

The fields are shown in the diagram.

diagram not to scale



Determine which animal, the sheep or the goat, is in the field with the larger area, and state how many extra square metres are in this larger field.

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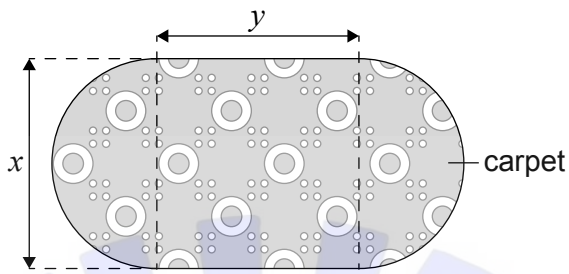
12. [Maximum mark: 8]

A company is designing a new carpet. The intended design of the carpet is in the shape of a rectangle with a semi-circle at each end.

The width of the rectangle is y metres and the diameter of each semi-circle is x metres, with $x > 0$ and $y \geq 0$.



The company has decided that the perimeter of the carpet will be 20 metres and would like to maximize its area.

(a) Find an expression for the perimeter in terms of x and y. [1]

(b) Show that the area, $A\,\text{m}^2$, of the carpet can be expressed as $A = 10x - \frac{\pi x^2}{4}$. [3]

(c) Find $\frac{\text{d}A}{\text{d}x}$. [2]

(d) Hence find the **exact** value of x for which the area is a maximum. [2]

(This question continues on the following page)

(Question 12 continued)

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