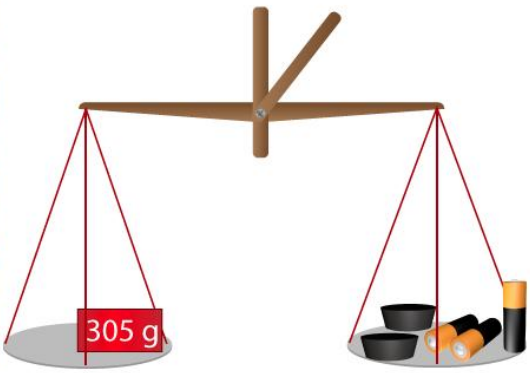
Question 1 (5 marks)

The basic scales as shown in the image use specific items to weigh fruit and vegetables at a market. The handler uses batteries and metal weights.

Image

Diagram



Key:

Metal weight

Battery

As shown in the diagram, the combined weight of **two** metal weights and **three** batteries is 305 grams (g).

One metal weight measures 100 g.

Question 1a (3 marks)

Using the information from the diagram, **find** the weight of one battery.

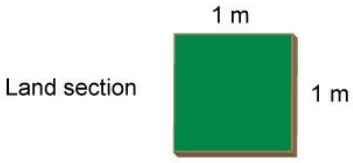
Weight of one battery

Question 1b (2 marks)

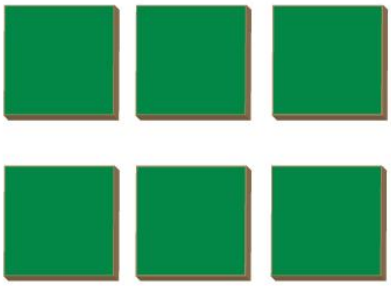
Ten similar sized tomatoes weigh the same as **three** metal weights and **two** batteries.
Calculate the weight in grams of one tomato.

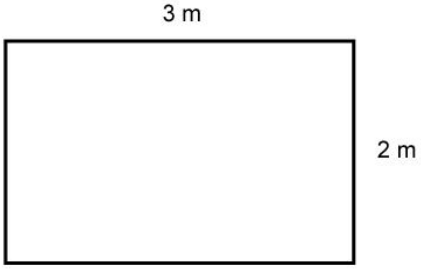
Question 2 (13 marks)

In order for a player to make progress in online games such as *Farmville* the player gains sections of land. The land sections usually take the form of 1 metre (m) squares as shown in the diagram below.



As more and more sections of land are gained they can be arranged to form rectangles as shown in the diagram below.





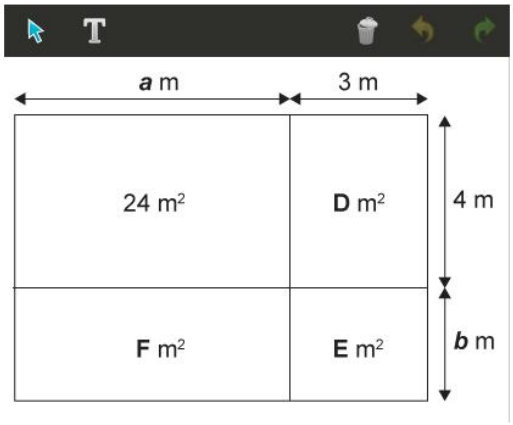
The video below shows an outline of land that has been gained and arranged into sections.

This media contains no audio

In the diagram below we have an online game that has made some progress. Land has been gained and arranged into sections. Use this diagram to answer parts (a) to (d).

Question 2a (1 mark)

You can annotate the diagram below if required.



Write down the area of the section labelled **D**.

B I U x_2 x^2 Ω Σ

Styles

Question 2b (1 mark)

Determine the value of the length labelled ***a***.

Question 2c (2 marks)

Write down an expression for the total area of the rectangle in terms of length ***b***.

Question 2d (4 marks)

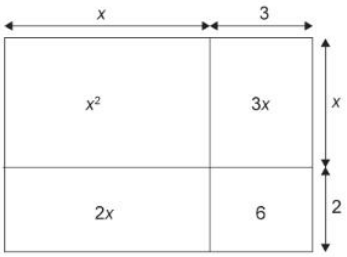
The total area is 63 m^2, **determine** the value of the length ***b*** and areas **F** and **E**.

Question 2e (5 marks)

Diagram | Formulas

The diagram below shows an outline of another piece of land that has been gained and arranged into sections with a total area of 210 m^2.

Express the area of the land as a quadratic equation in x and hence **find** the value of x.



Question 3 (12 marks)

The grades of 20 students in mathematics are shown in the table and pictogram below.

Table and pictogram | Formulas

Grade	1	2	3	4	5	6	7
Number of students	0	1	4	3	6	4	2

Grade	Number of students
1	
2	
3	
4	
5	
6	
7	

Scroll down to continue

Question 3a (1 mark)

Write down the modal grade.

B I U x_2 x^2 Ω Σ Styles

Question 3b (4 marks)

Show that the arithmetic mean grade of the students in mathematics is 4.7 .

Question 3c (2 marks)

Determine the median grade.

Question 3d (2 marks)

A student is selected at random from the 20 students. **Determine** the probability that the student achieved a grade of 6 or more.

Question 3e (3 marks)

The grades of these same 20 students in physics are shown in the table and pictogram below.

Table and pictogram | Formulas

Grade	1	2	3	4	5	6	7
Number of students	0	1	5	2	4	5	3

Grade	Number of students
1	
2	
3	
4	
5	
6	
7	

The students achieving a grade of 6 or more in mathematics **and** physics will be selected to participate in a competition.

Find the probability that a randomly selected student will participate in the competition.

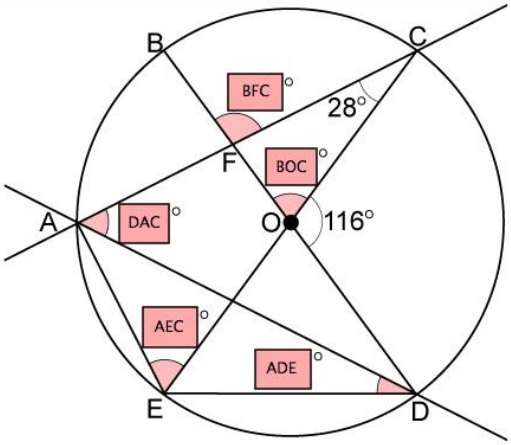
B I U x_2 x^2 Ω Σ Styles

Question 4 (7 marks)

The lines BD and CE pass through the centre (O) of the circle.

- **Determine** the value of the angle BOC.
- **Determine** the value of the angle DAC.
- **Determine** the value of the angle BFC.
- **Write down** the value of the angle ADE.
- **Determine** the value of the angle AEC.

To insert your answers on the diagram, click inside the box and replace the letters with your answers in the "Add label" box. A text tool is available for you to add working where required.

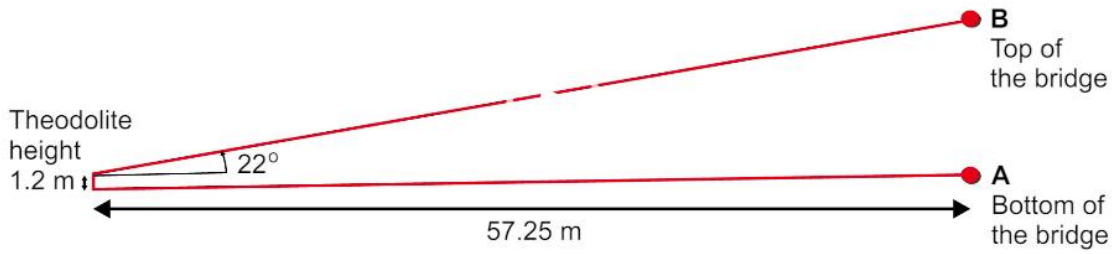


Question 5 (5 marks)

An engineer is examining a weak bridge from a safe distance. In order to make a calculation for the height of the bridge to the ground vertically below she uses a measuring instrument called a theodolite that allows her to measure angles accurately. The theodolite is set at a height of 1.2 metres (m). It is placed 57.25 m, to the nearest centimetre, from the point A at the bottom of the bridge. The angle of elevation from the horizontal to the top of the arch at B is measured at 22° to the nearest degree. This information is shown in the animation below.

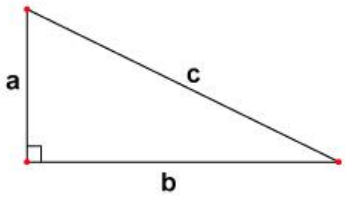
Animation | Formulas

This media contains no audio

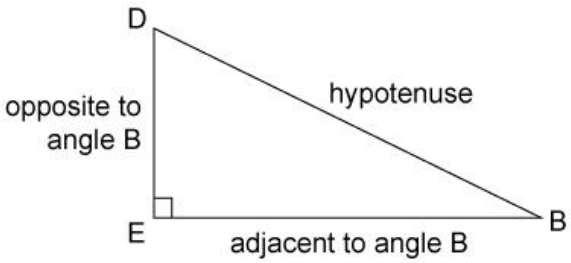


Pythagoras' Theorem

$$c^2 = a^2 + b^2$$



Trigonometric ratios



$$\sin B = \frac{\text{opposite to angle } B}{\text{hypotenuse}}$$

$$\cos B = \frac{\text{adjacent to angle } B}{\text{hypotenuse}}$$

$$\tan B = \frac{\text{opposite to angle } B}{\text{adjacent to angle } B}$$

The measurements are modelled in the diagram below which is a side view from the bridge to the theodolite. The canvas below has been provided for annotating if required.

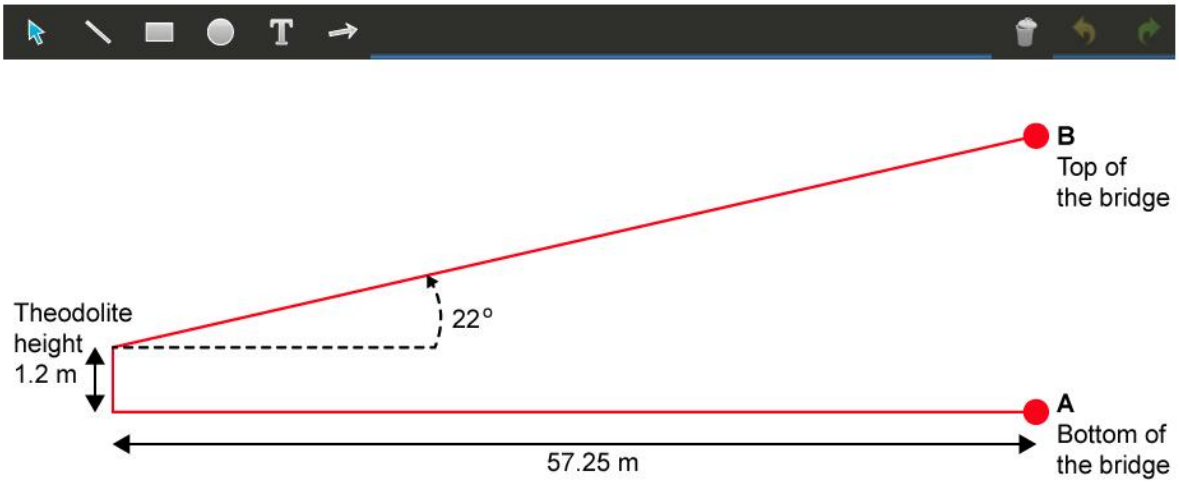


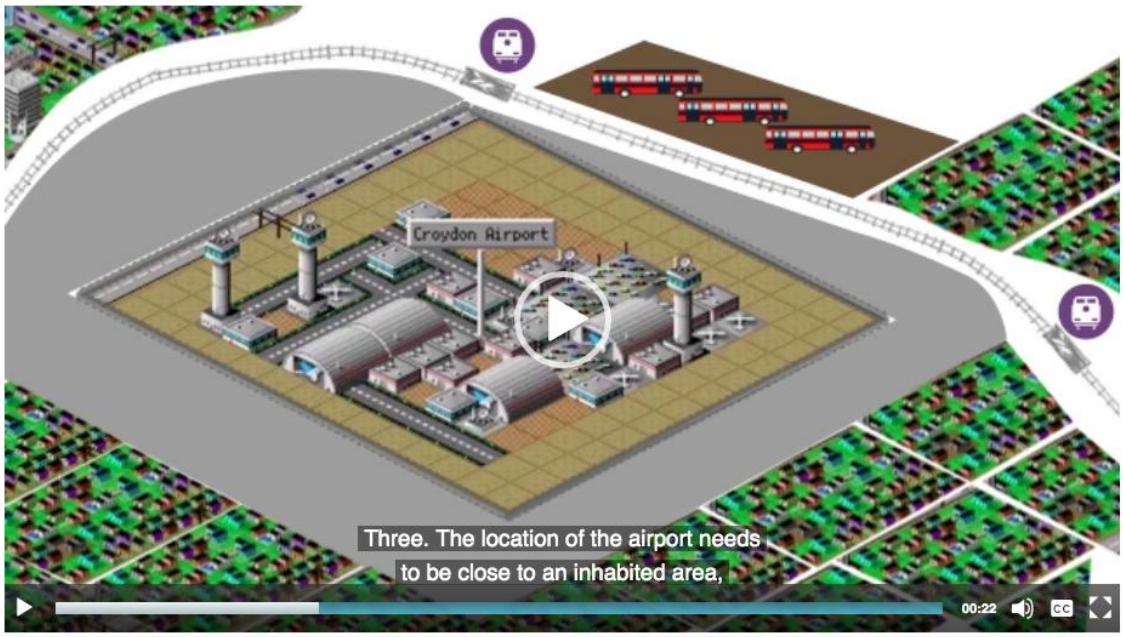
Diagram not to scale

Calculate the height from the top of the bridge at B to the ground vertically below at A to the nearest centimetre.

Question 6 (20 marks)

In this question you will interpret information to discuss the sustainability of a proposed airport development.

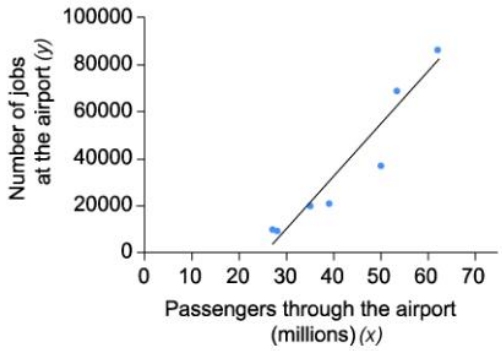
The video below provides some background information for this question.



Tab 1 | Tab 2 | Tab 3

Airport employment data for 2013

$y = 2110x - 52818$



The airport employment data for 2013 in Tab 1 shows the relationship between the number of jobs at the airport (y) and the number of passengers (x) through the airport in millions. The line of best fit is represented by the equation:

$$y = 2110x - 52818.$$

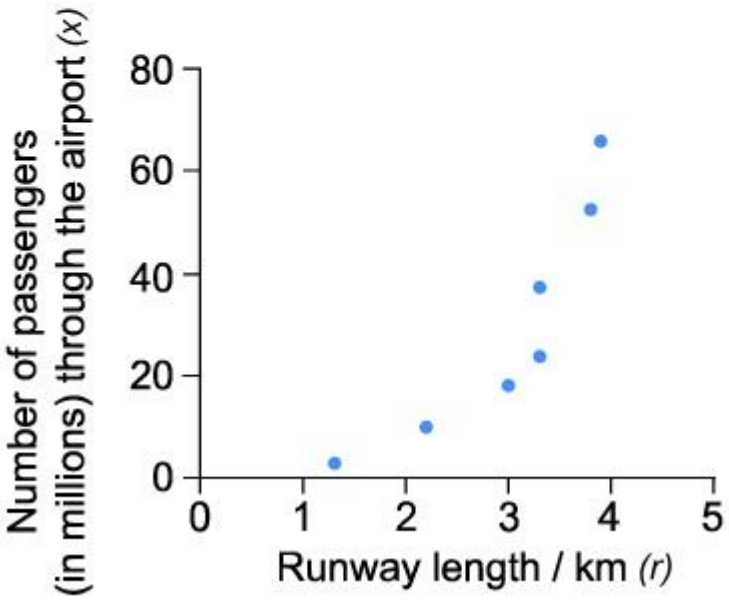
Question 6a (3 marks)

The number of jobs at Hong Kong International Airport was 65 000 in 2013. Use the equation of the line of best fit from Tab 1 to **estimate** the number of passengers passing through the airport. Round your answer to the nearest million.

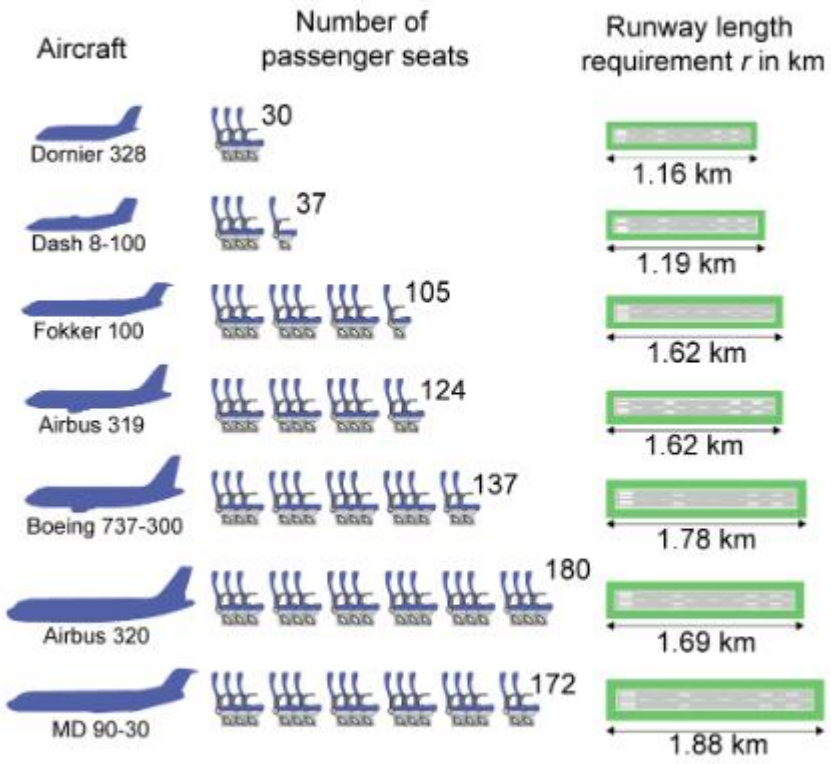
Tab 1 | Tab 2 | Tab 3

Airport runway data for 2013

$x = 0.7(3)^r$



Tab 1 | Tab 2 | Tab 3



Question 6b (3 marks)

The actual number of passengers through Hong Kong International Airport was 68 488 000. **Calculate** the percentage error of your rounded answer from part (a).

$$\text{Percentage error} = \left| \frac{\text{rounded value} - \text{actual value}}{\text{actual value}} \right| \times 100\ \%$$

Question 6c (2 marks)

According to economic predictions, 1 million more passengers through the airport would generate approximately 2000 more jobs. **Justify** how the line of best fit in Tab 1 supports this claim.

The airport runway data for 2013 in Tab 2 shows an exponential relationship between the longest airport runway length (r) in kilometres and the number of passengers (x) through the airport in millions and is modelled by the equation:

$$x = 0.7(3)^r$$

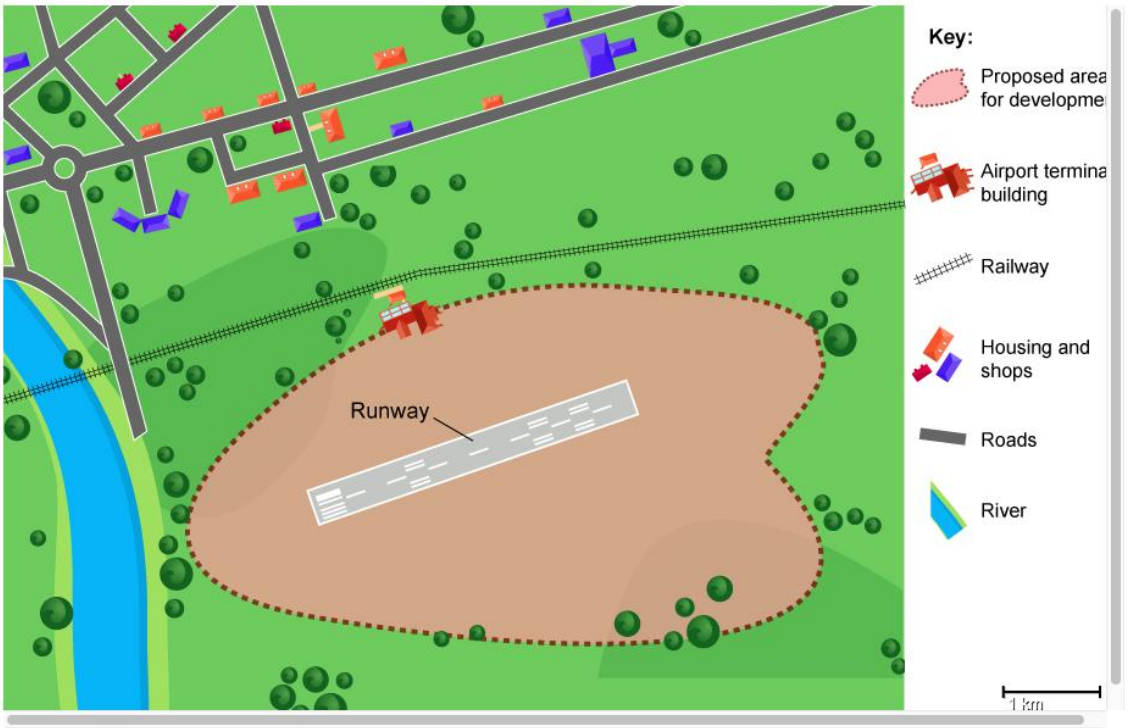
Question 6d (2 marks)

The longest runway length at Motu Mute Airport in Bora Bora is 1.5 km (correct to 2 significant figures).

Use the equation from Tab 2 to **determine** the number of passengers x, through Motu Mute Airport to the nearest million.

Question 6e (10 marks)

Here is a map for a proposed airport development. A measurement tool is provided if required.



The local government claims the airport development will create around 18 000 jobs.
You should write a report in which you **discuss** the claim made by the local government.

Your report will be assessed on the mathematical evidence you provide in your discussion and you should consider the future of the proposed airport using the data provided in the tabs. In your answer you should:

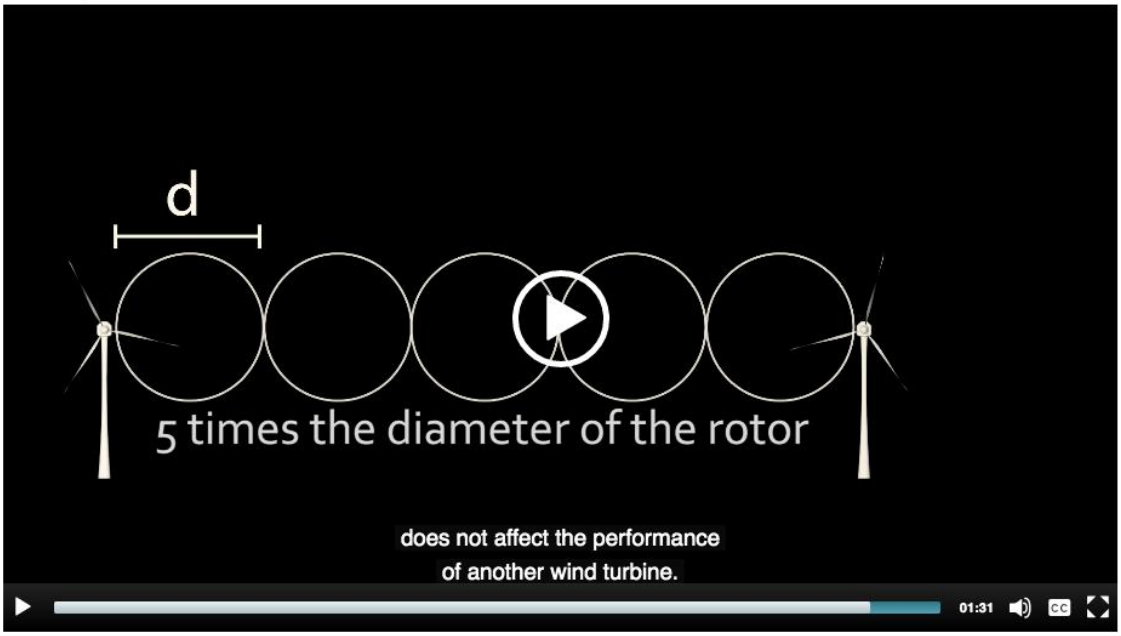
- identify the relevant mathematical information for the opportunities available by the proposed airport
- make appropriate calculations to provide evidence to support your report
- consider the accuracy of your predictions
- comment on the sustainability of the airport.

Question 7 (19 marks)

In this question you will obtain relevant information and use reasoning methods in order to design a sustainable wind farm.

The video below provides some background information for this question.

Video | Formulas



Video | Formulas

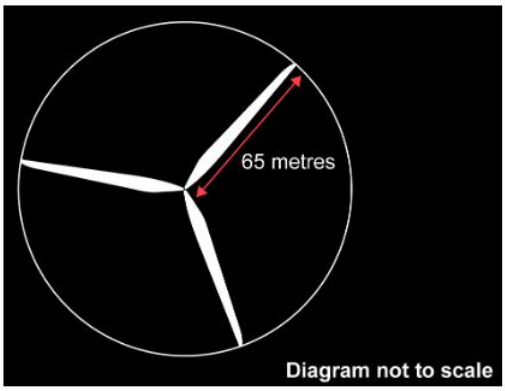
Cicumference of a circle

$C = 2\pi r$, where r is the radius

Area of a circle

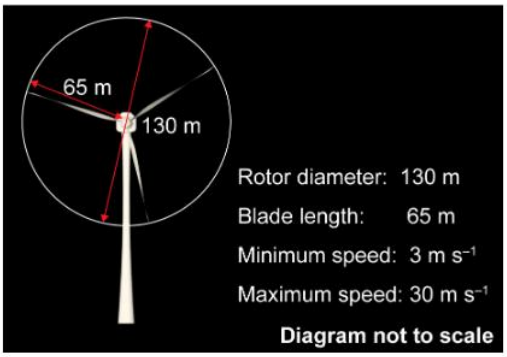
$A = \pi r^2$, where r is the radius

Question 7a (2 marks)



Determine the exact area of the circle covered by the rotors as they turn. Give your answer in terms of π.

Question 7b (5 marks)



The wind turbine efficient power P_E in Watts (W) is calculated as about 45 % of its available power P_A. This is calculated by the formulas:

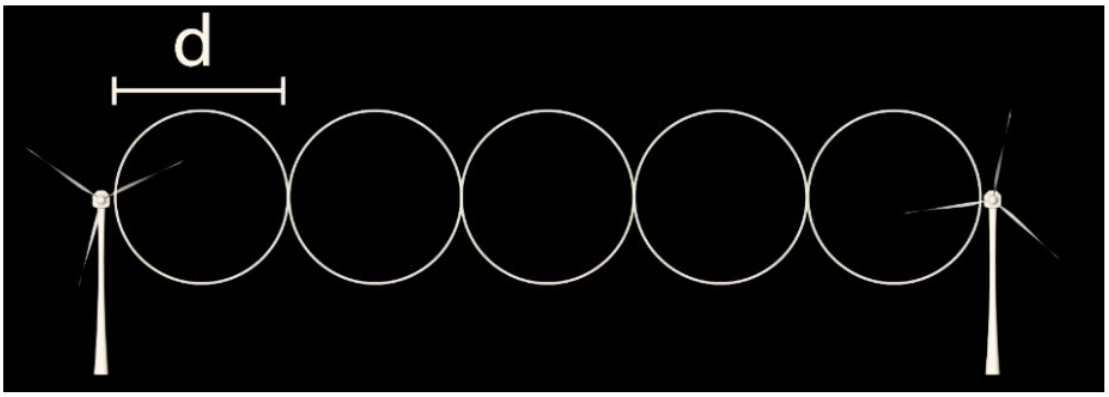
$$P_E = 0.45P_A \text{ where } P_A = 0.6AU^3$$

U is wind speed in metres per second (m s^{-1}).

A is the area of the circle covered by the rotors as they turn.

Calculate, to the nearest kilowatt (kW), the **maximum** efficient power P_E, which is supplied from a maximum wind speed U of 30 m s^{-1}. (Note: 1 kW = 1000 W)

Wind turbines need to be placed quite far apart so that the air turbulence from one wind turbine does not affect another. The wind turbines are spaced out at a distance based on the diameter of the rotors. For many wind turbines the distance between the bases of the towers is 5 times the diameter of the rotors *d*. We will call this the turning zone.



You are provided with a rectangular area to plan a wind farm using turbines with rotor diameter 130 metres (m). The rectangle has length 4000 m and width 1800 m.

Question 7c (2 marks)

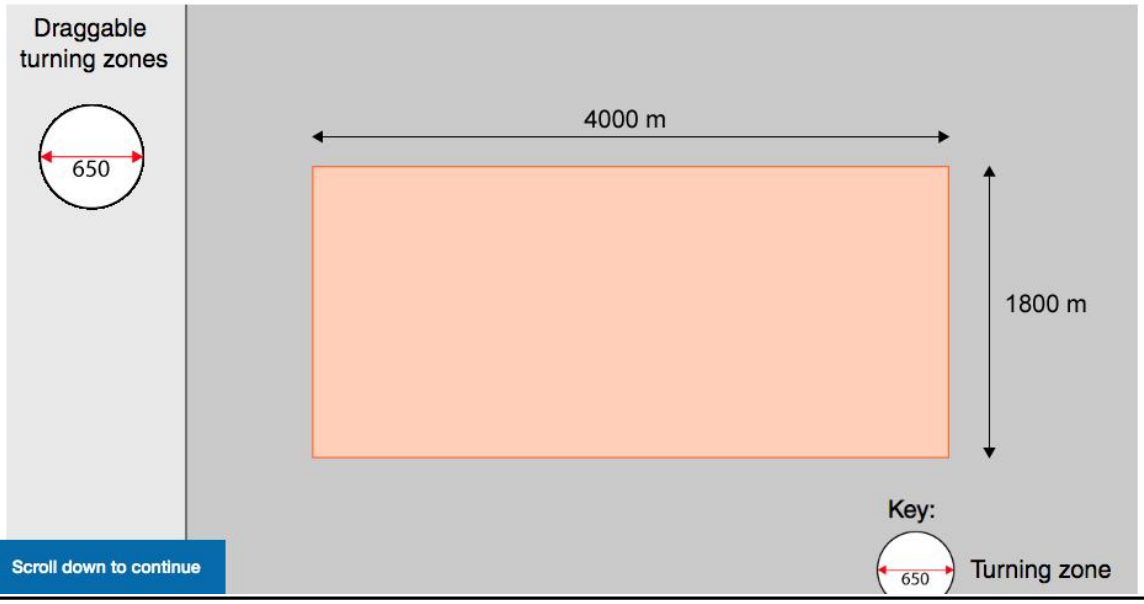
Show that the radius of the circular zone (turning zone) needed for one wind turbine is 325 m.

Question 7d (10 marks)

Design a wind farm using these 130 m diameter turbines in the 4000 m by 1800 m rectangular area provided below. The wind average speed is 30 m s^{-1}. The draggable turning zones below can be placed in the area provided.

In your answer you should:

- identify your strategy for the most efficient wind turbine arrangement
- determine the maximum number of wind turbines that can fit inside this rectangular area
- estimate the maximum efficient power output of your designed wind farm
- consider the sustainability features of the planned wind farm
- justify the degree of accuracy of your estimate.



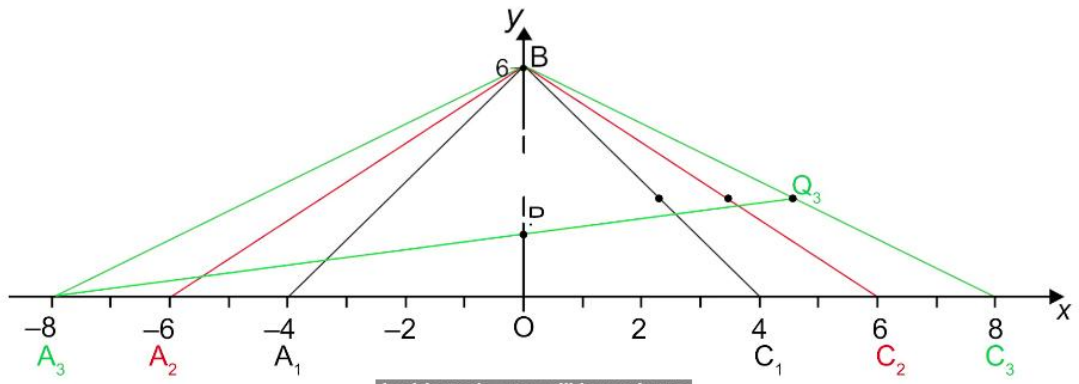
Scroll down to continue

Question 8 (39 marks)

In this question you will investigate relationships in cable-stayed bridges.

The video below provides some background information for this question.

Video | Formulas



In this task, you will investigate some of the mathematical relationships

Video | Formulas

Coordinates of the midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2) $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Equation of a straight line

$y = mx + c; \quad ax + by + d = 0$

Gradient formula

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Tab 1 Tab 2

The coordinates of the points C, A and Q as the cables move along the *x* axis are shown in the table.

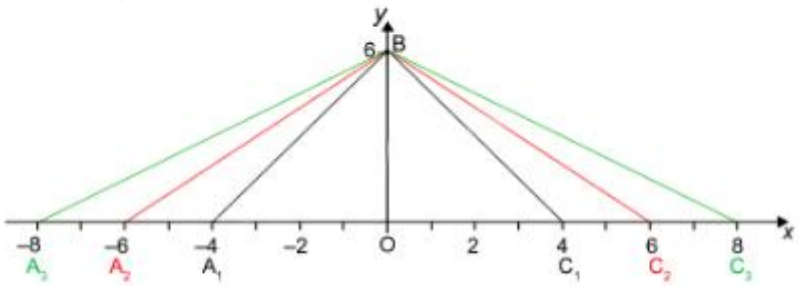
n	Coordinates C	Coordinates A	Coordinates Q
1	(4,0)	(-4,0)	(2,3)
2	(6,0)	(-6,0)	(3,3)
3	(8,0)	(-8,0)	(4,3)
4	(10,0)	(-10,0)	
5	(12,0)	(-12,0)	
6	(14,0)	(-14,0)	

Question 8a (2 marks)

Write down the relationship between the coordinates of C and the coordinates of A.

B I U x_2 x^2 Ω Σ Styles

Tab 1 Tab 2



Question 8b (1 mark)

Write down the relationship between X_Q, the *x* coordinates of the midpoint Q, and X_C, the *x* coordinates of C.

Question 8c (3 marks)

Predict the coordinates of the remaining midpoints Q_4, Q_5, Q_6 and write your answers in the table.

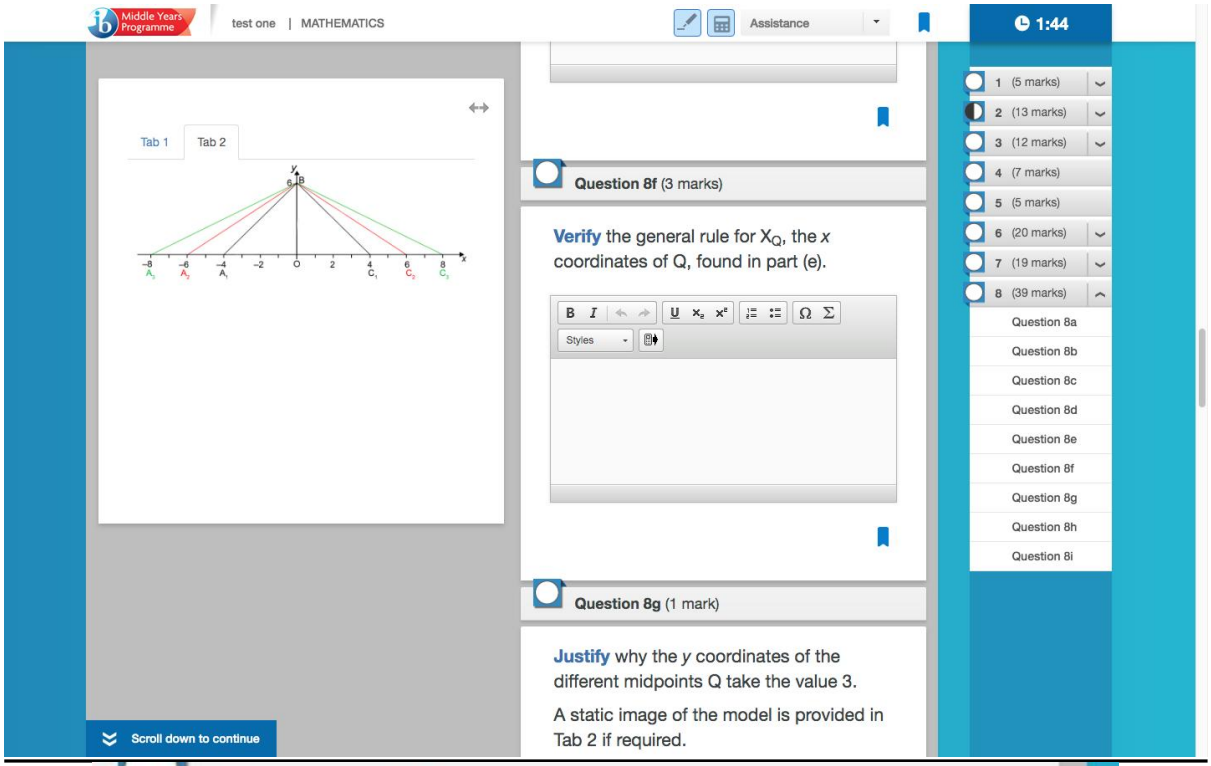
n	Coordinates C	Coordinates A	Coordinates Q
1	(4,0)	(-4,0)	(2,3)
2	(6,0)	(-6,0)	(3,3)
3	(8,0)	(-8,0)	(4,3)
4	(10,0)	(-10,0)	
5	(12,0)	(-12,0)	
6	(14,0)	(-14,0)	

Question 8d (2 marks)

Determine the general rule for X_C, the x coordinates of C, in terms of n.

Question 8e (1 mark)

Hence, write down the general rule for X_Q, the x coordinates Q, in terms of n.



Question 8f (3 marks)

Verify the general rule for X_Q, the x coordinates of Q, found in part (e).

Question 8g (1 mark)

Justify why the y coordinates of the different midpoints Q take the value 3.

A static image of the model is provided in Tab 2 if required.

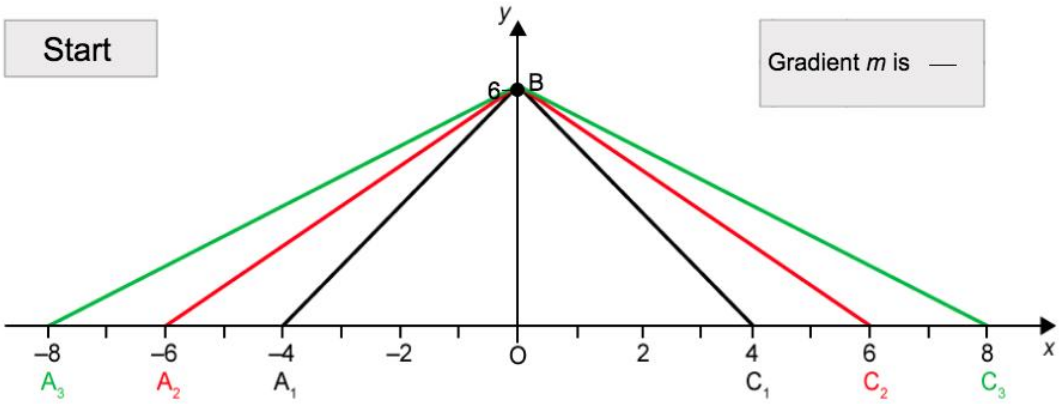
Question 8h (2 marks)

Using the animation and table below, **show** that the gradient from A_1 to Q_1 is $\frac{3}{6}$.

Click on "Start" and "Next" to see the gradient animated.

Start

Gradient *m* is —



n	1	2	3	4
C	(4,0)	(6,0)	(8,0)	(10,0)
A	(-4,0)	(-6,0)	(-8,0)	(-10,0)
Q	(2,3)	(3,3)	(4,3)	(5,3)

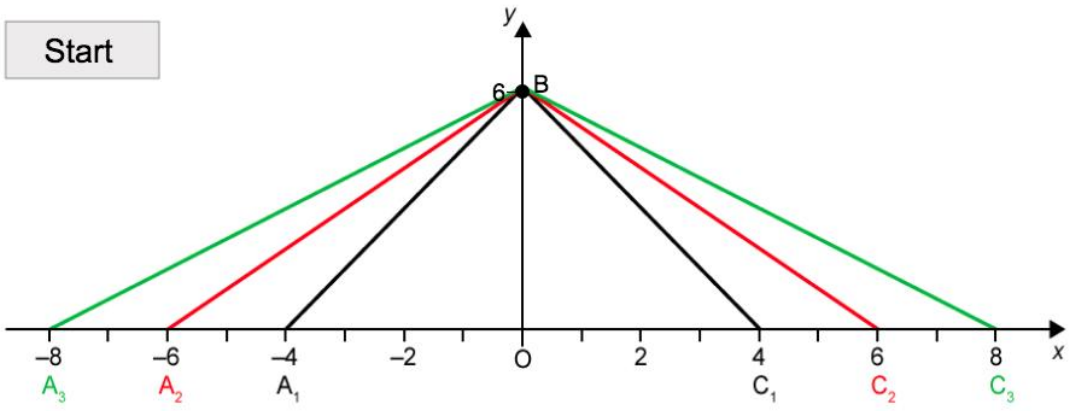
Question 8i (24 marks)

Investigate the equation of the line AQ for all line segments of AQ by considering the equation of a line or otherwise. In your answer you should:

- describe any patterns you see for the gradient of AQ and for the coordinates of P
- find a general rule for the gradient of AQ in terms of *n*
- hence find the general equation of all lines AQ
- test your general rule for the gradient of AQ
- test your general equation of all lines of AQ
- prove or verify and justify your general rule and general equation
- ensure that you communicate the above appropriately.

Click on "Start" and "Next" to animate the line segments of AQ.

Start



The canvas and table below have been provided for annotating if required.

To insert your answer into the table, click in the table cell and write your answer in the "Add label" box.

n	1	2	3	4	5	6
C_n	(4,0)	(6,0)	(8,0)	(10,0)		
A_n	(-4,0)	(-6,0)	(-8,0)	(-10,0)		
Q_n	(2,3)	(3,3)	(4,3)	(5,3)		
P	(0,2)					
m = *gradient*	$\frac{3}{6}$