Question 1 (8 marks)

ABCD is a trapezium with the properties shown in the diagram below.

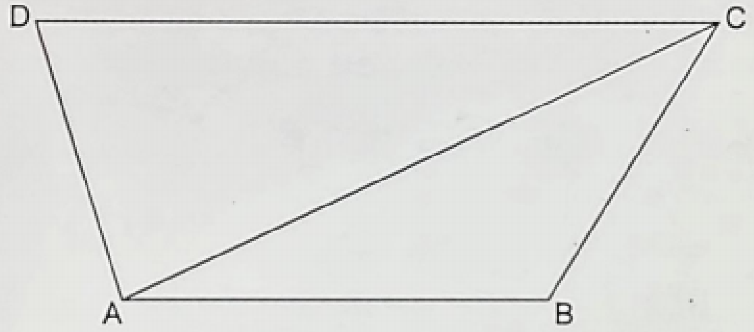
Diagram not to scale

$\overrightarrow{AB} = \begin{pmatrix} 4 \\ -0.5 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$

$|\overrightarrow{DC}|$ is double $|\overrightarrow{AB}|$

$\overrightarrow{DC}$ is parallel to $\overrightarrow{AB}$



Question 1a (2 marks)

Determine $\vec{BC}$, give your answer as a column vector.

B I U x_2 x^2 Ω Σ Styles

Question 1b (3 marks)

Determine $\vec{AD}$, give your answer as a column vector.

B I U x_2 x^2 Ω Σ Styles

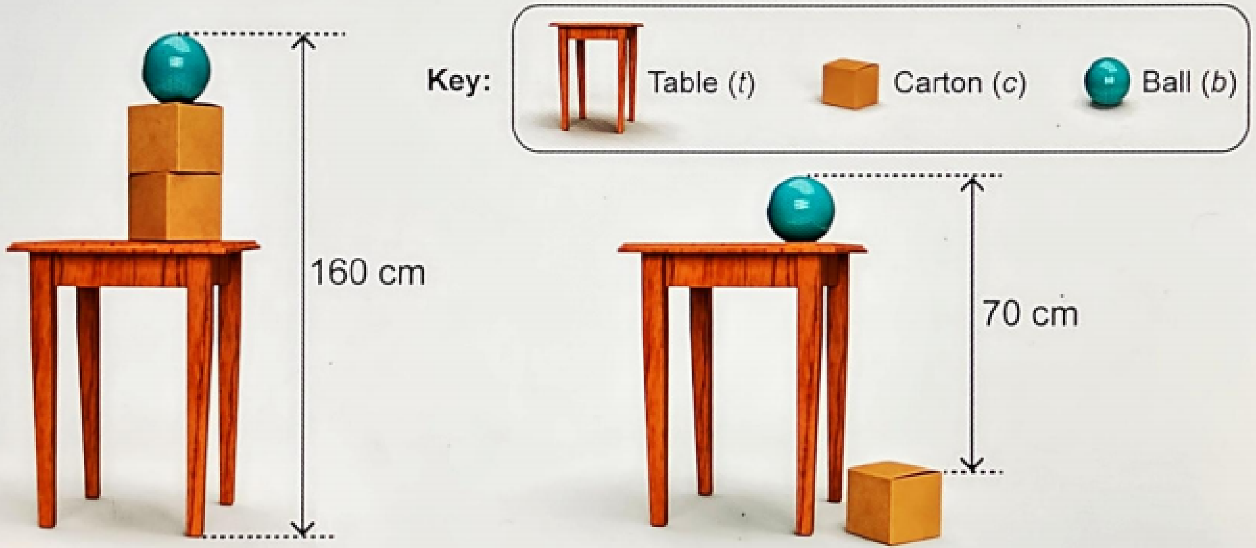
Question 1c (3 marks)

Hence, **show that** $\overrightarrow{AD}$ is perpendicular to $\overrightarrow{AC}$.

Question 2 (5 marks)

Using the information provided in the diagram below, **find** the height of a carton.

Diagram not to scale



Question 3 (10 marks)

In ancient times, Rome was the economic and cultural centre of Europe. Most big cities were linked by roads to Rome. Travellers often had to go from one city to another and choose between different roads before reaching Rome.

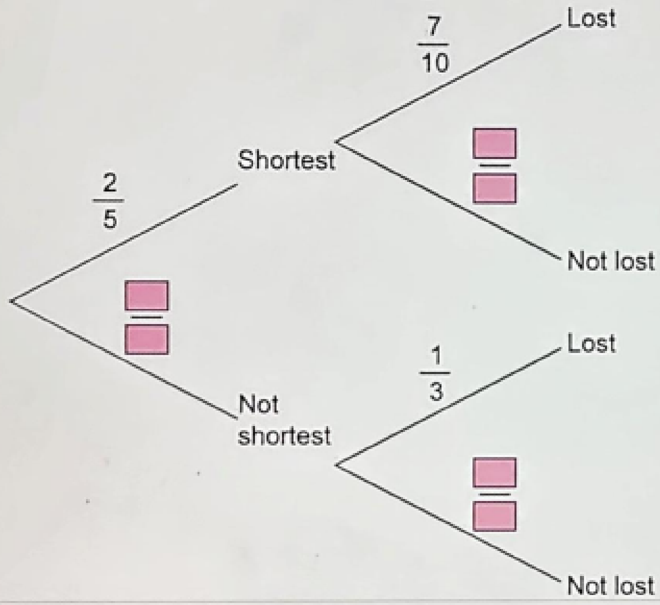
Question 3a (1 mark)

The probability that travellers choose the shortest road to Rome is $\frac{2}{5}$. **Write down** the probability that they will **not** choose the shortest road.

B I U x_2 x^2 Ω Σ Styles

Travellers could get lost on the different roads. The probabilities for getting lost are illustrated in the tree diagram below.

Write down the missing probabilities in the tree diagram.



Question 3c (3 marks)

Find the probability that a traveller will not get lost.

B I U Ω Σ

Styles

Question 3d (2 marks)

Given that a traveller did not get lost, **determine** the probability that he chose the shortest road.

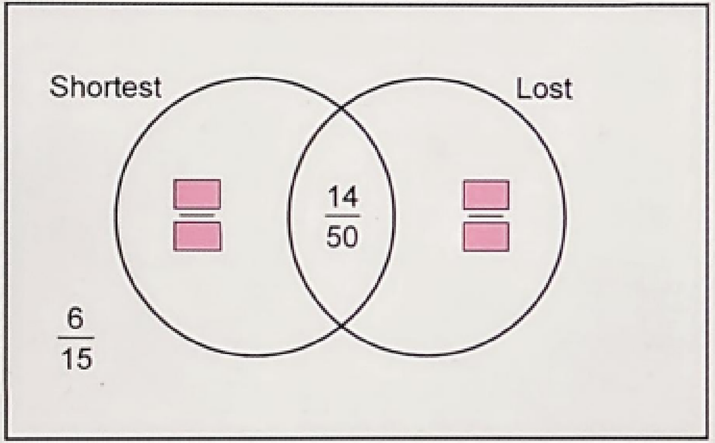
B I U Ω Σ

Styles

Question 3e (3 marks)

The probabilities from the tree diagram can be presented in a Venn diagram.

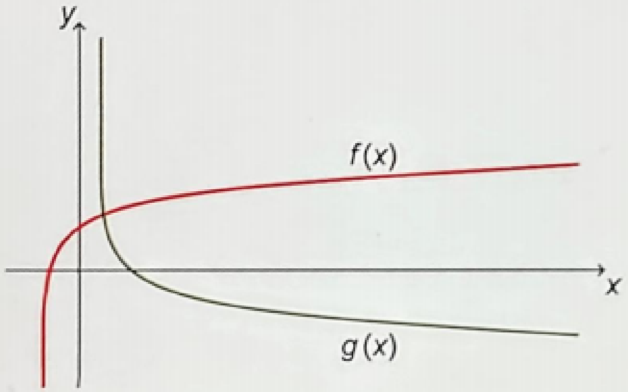
Using your previous answers, **determine** the missing values in the Venn diagram below.



Question 4 (11 marks)

The diagram below shows two logarithmic functions.

$$f(x) = \log_3(5x + 7) \quad \text{and} \quad g(x) = k\log_3(x - 1)$$



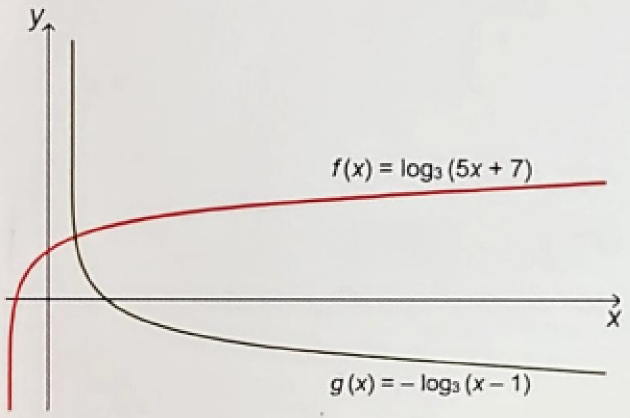
Question 4a (2 marks)

Given that the point (10, –2) lies on $g(x) = k \log_3 (x - 1)$, **show that** $k = -1$.

B I U x_2 x^2 Ω Σ Styles

The simulation below shows the vertical distance *d* between the two functions.

This media is interactive



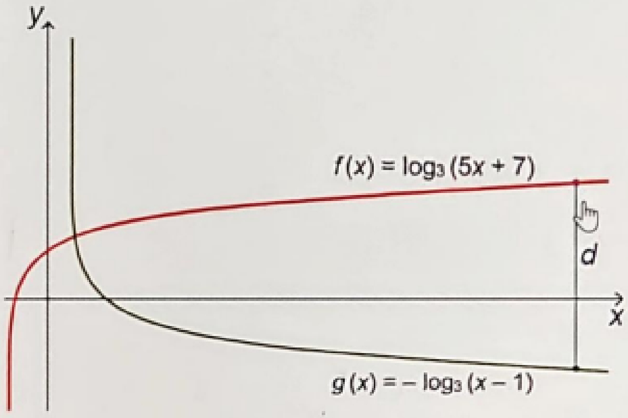
Question 4b (3 marks)

Find an expression for *d* in terms of *x*.
Give your answer in the form of
$\log_3(ax^2 + bx + c)$.

B I U Ω Σ
Styles

The simulation below shows the vertical distance d between the two functions.

This media is interactive



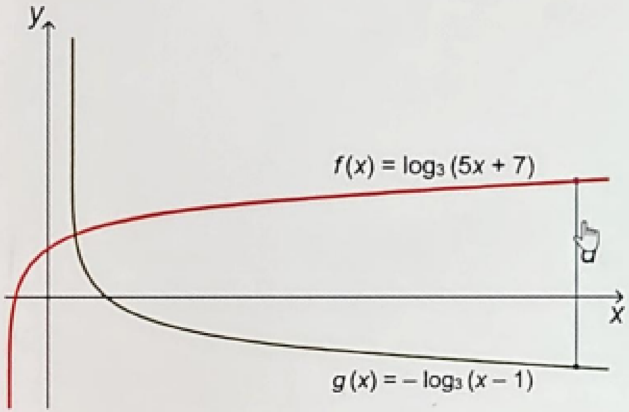
Question 4b (3 marks)

Find an expression for d in terms of x. Give your answer in the form of $\log_3(ax^2 + bx + c)$.

B I U Ω Σ
Styles

The simulation below shows the vertical distance *d* between the two functions.

This media is interactive



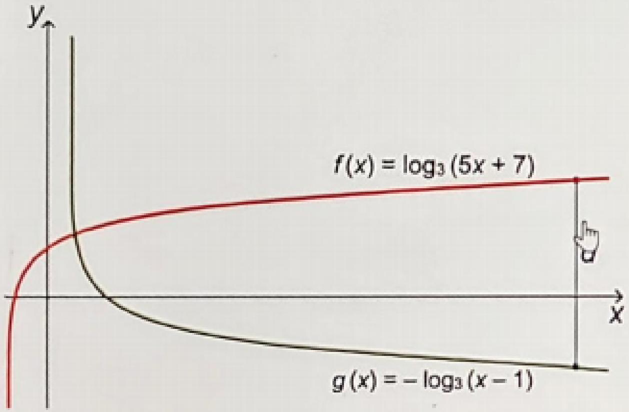
Question 4c (2 marks)

Hence, **find** the value of *d* when $x = 4$.

B I U Ω Σ
Styles

The simulation below shows the vertical distance *d* between the two functions.

This media is interactive



Question 4d (4 marks)

Find the value of *x* when $d = 0$. Give your answer to 1 decimal place.

Question 5 (15 marks)

Video Script

Air traffic control provides an essential service for flight path safety.

The air traffic control officer directs pilots to fly at specific coordinates, in order to ensure that no airplane is at the same place at the same time.

The minimum vertical distance between two airplanes must be 300 metres.

The minimum distance between two airplanes flying at the same altitude must be 9200 metres.

In this question, you will make calculations for airplanes flying between two cities and consider the safety implications of the flights.

Several flights take place between Seoul, Tokyo and Jakarta each day. Airplanes cross each other in the sky several times a day.

Origin	Destination	Departure time	Flight time / minutes	Distance / km	Average speed km/h
Seoul	Tokyo	07:00	140		500
Tokyo	Jakarta	11:00		5760	900

Question 5a (2 marks)

Show that the distance between Seoul and Tokyo is 1170 km, correct to three significant figures.

B I U x₂ x² Ω Σ

Styles

Question 5b (2 marks)

Determine the flight time from Tokyo to Jakarta. Give your answer in minutes.

B I U x₂ x² Ω Σ

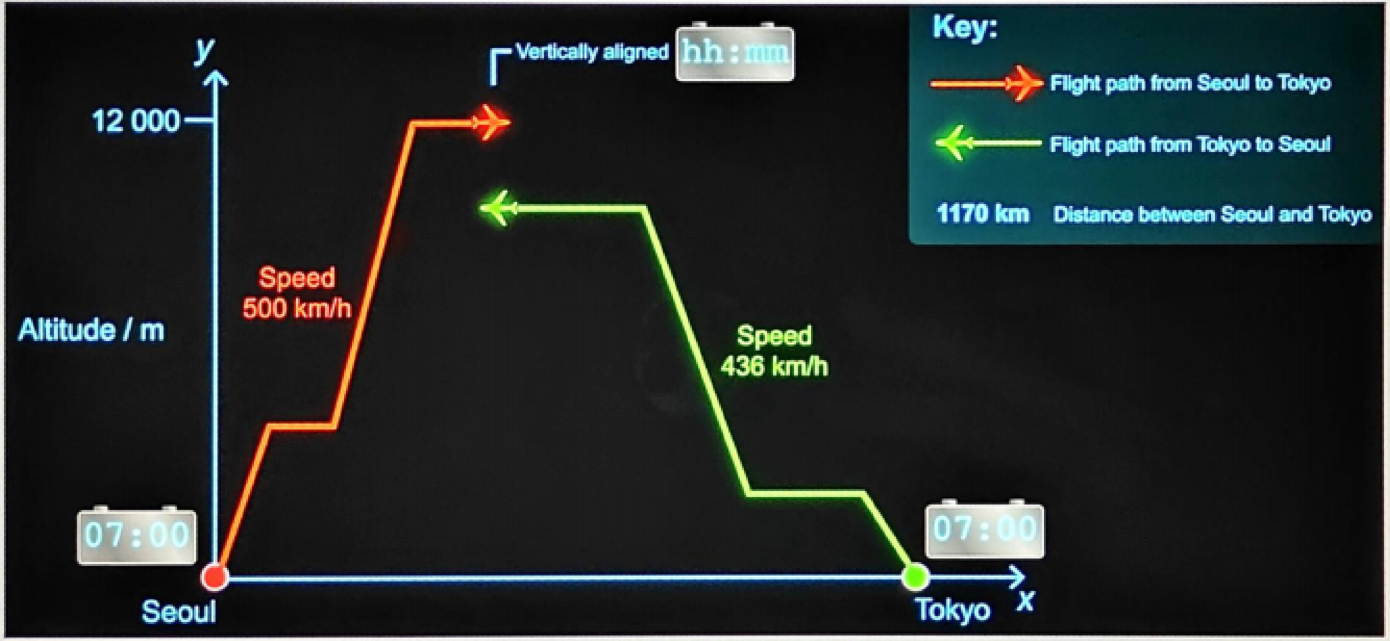
Styles

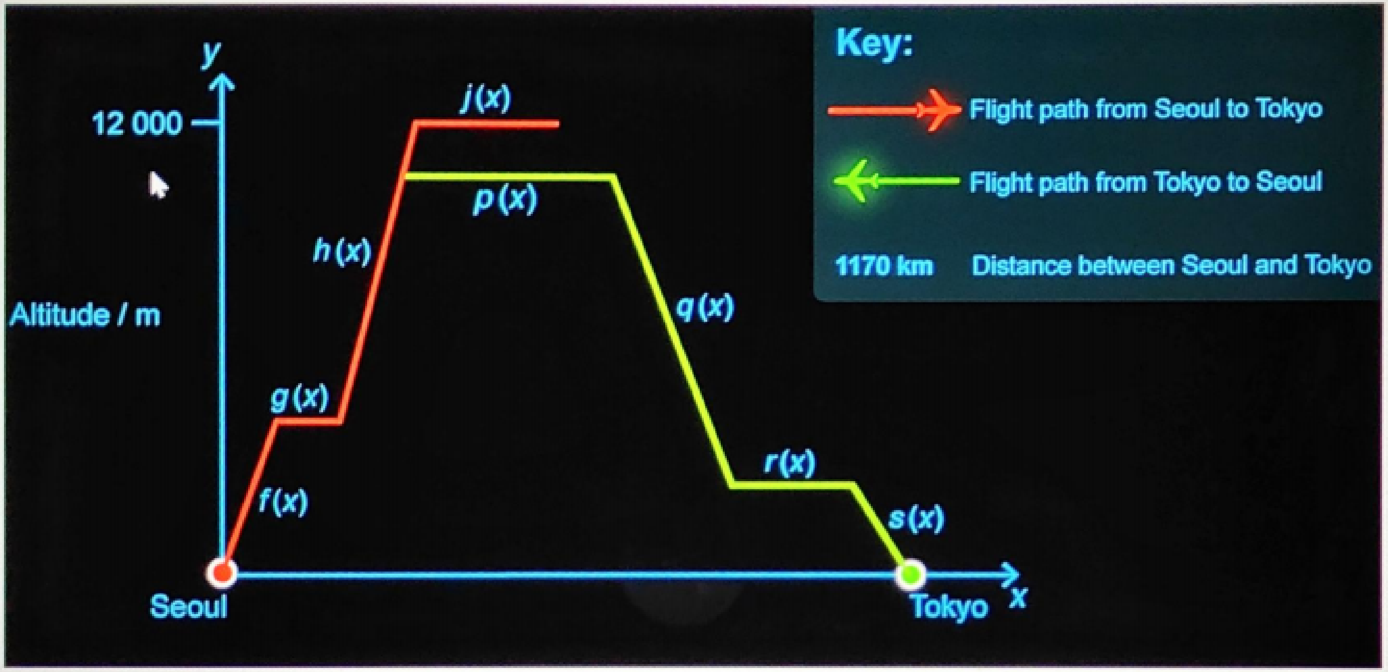
Calculate the time at which they will be vertically aligned. Give your answer to the nearest minute.

B I U x_2 x^2 Ω Σ Styles

Question 5c (5 marks)

An airplane departs from Seoul to Tokyo at 07:00 with an average speed of 500 km/h. At the same time, another airplane departs from Tokyo to Seoul with an average speed of 436 km/h.





©

To avoid collision, the minimum vertical distance between two airplanes must be 300 metres. The path $f(x)$ and $h(x)$ are parallel.

	Function	Domain
f	$f(x) = 60x$	$0 \leq x \leq 40$
g	$g(x) = 2400$	$40 \leq x \leq 80$
h		$80 \leq x \leq 240$
j	$j(x) = 12\,000$	$240 \leq x \leq 700$

Question 5d (3 marks)

Find the equation of $h(x)$.

B I U x₂ x² Ω Σ

	Function	Domain
p	$p(x) =$	$233 \leq x \leq 700$
q	$q(x) = -40x + 39\,580$	$700 \leq x \leq 951$
r	$r(x) = 1540$	$951 \leq x \leq 1100$
s	$s(x) = -22x + 25\,740$	$1100 \leq x \leq 1170$

Question 5e (3 marks)

Deduce that the airplanes are at a safe vertical distance apart.

B I U x₂ x² Ω Σ

Question 6 (20 marks)

Video Script

During the early 19th century Santa Fe became Americas first great international trading city. Traders would travel by wagon from Missouri to Santa Fe to buy and sell goods.

At Cimarron traders had to choose between two paths: The Cimarron route and the Mountain route.

The Mountain route followed the Missouri river. This route was hard going. Fortunately there was plenty of food and water along the way. Traders had a lot of space in the wagon to carry trading goods.

The Cimarron route was the most favoured route as it was easy going. However water and food were scarce. On this route, traders would have to use more space in the wagon to carry a supply of food and water.

In this question you will compare and contrast the two routes.

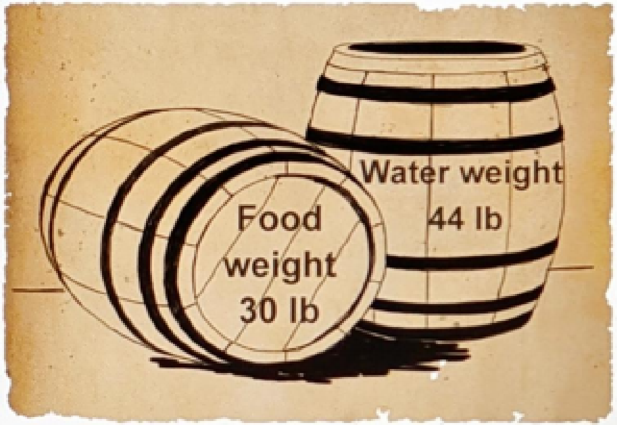
Glossary for scarce: not easily available

Two families travelled from Missouri to Santa Fe.

Question 6a (2 marks)

Family Fry chose the Cimarron route.

Since the Cimarron route is a desert, family Fry had to carry a supply of food and water for their trip.



They had the following constraints:

- The total weight of the supplies was **not more** than 1200 lb
- They needed **at least** 7 water barrels
- The number of water barrels was **at most** twice the number of food barrels.

Select the **three** inequalities that satisfy the constraints above.

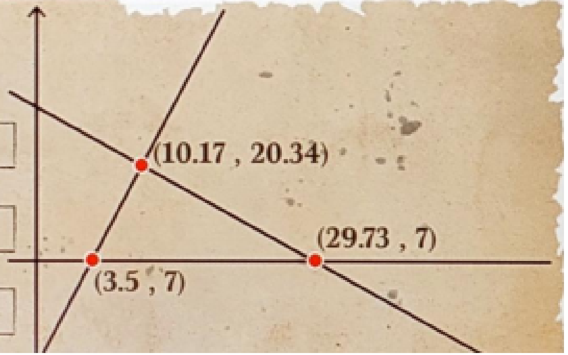
Identify the region that satisfies the constraints above by dragging the “Region” icon into the correct place on the graph.

f represents the number of food barrels and ***w*** represents the number of water barrels.

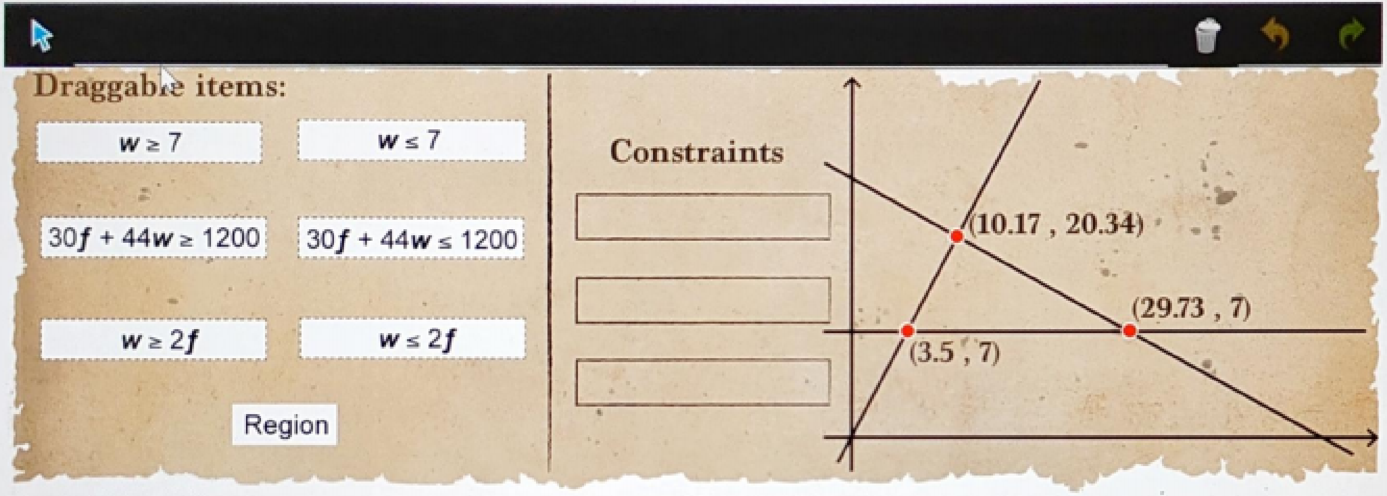
Draggable items:

$w \geq 7$	$w \leq 7$
$30f + 44w \geq 1200$	$30f + 44w \leq 1200$
$w \geq 2f$	$w \leq 2f$

Constraints



f represents the number of food barrels and w represents the number of water barrels.



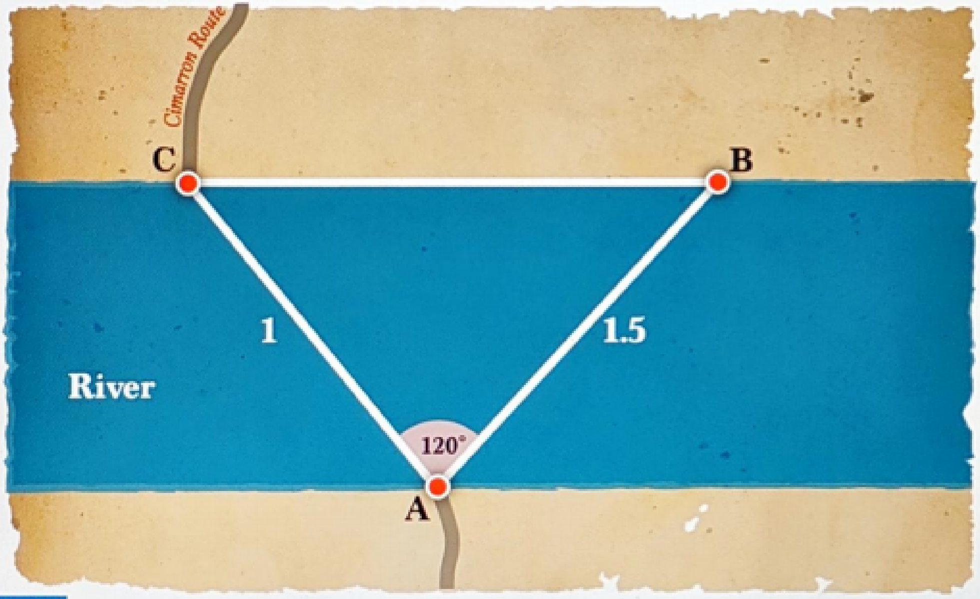
Question 6b (3 marks)

Using the information in the graph above, **find** the maximum weight of food and water the wagon can carry.

B *I* U x_2 x^2 Ω Σ Styles

To access the Cimarron route (C), the family had to cross the Mississippi river from A. The bridge across the river to the Cimarron route from A to C was broken so the family had to cross the river to B.

Diagram not to scale



Question 6c (4 marks)

Calculate the distance from B to C.

Question 6d (1 mark)

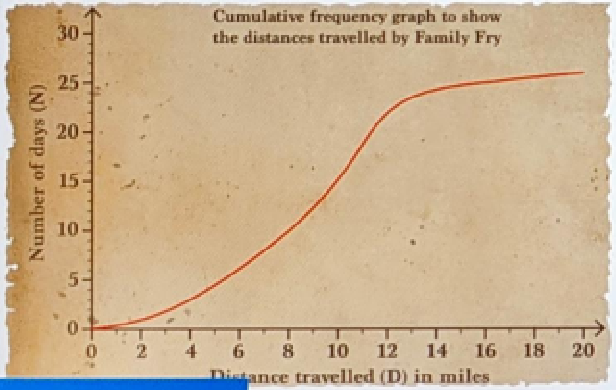
Hence, **determine** how far the family travelled to get from A to the start of the Cimarron route.

Question 6e (10 marks)

Family Fry chose the Cimarron route. A summary of their journey is illustrated in the cumulative frequency graph below, however some of the data has been lost.

Cimarron route

This media is interactive

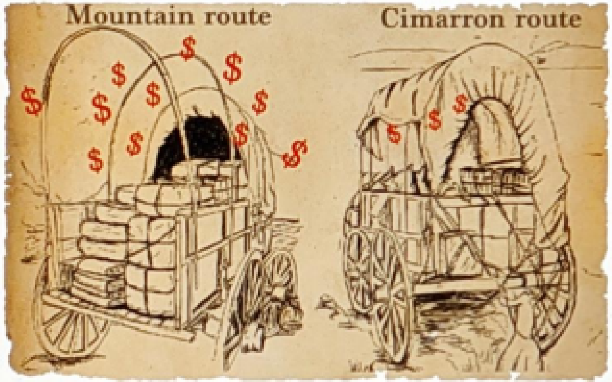


Distance travelled (D) in miles	Number of days (N)
0 ≤ D < 4	3
4 ≤ D < 8	
8 ≤ D < 12	12
12 ≤ D < 16	3

Measures of central tendency for the distance travelled by Family Fry			Total number of days	Estimate for the total distance travelled
Modal class	Estimate for the median	Estimate for the mean		

Scroll through the images to reveal more information on the two routes.

A lot
of trading goods to sell



Not a lot
of trading goods to sell

Scroll through the images to reveal more information on the two routes.

Family Kane chose the Mountain route. A summary of their journey is illustrated in the table below.

Mountain Route

Measures of central tendency for the distance travelled by Family Kane			Total number of days	Estimate for the total distance travelled
Modal class	Estimate for the median	Estimate for the mean	39	502
$12 \leq D < 16$	13.5	12.87		

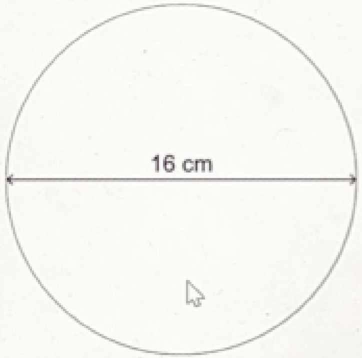
Analyse the routes taken by the two families. In your answer, you should consider:

- **three** relevant factors
- measures of central tendency for each route
- the total distance and number of days for each route
- similarities and differences between the two routes
- the degree of accuracy in the context of the question.

Styles

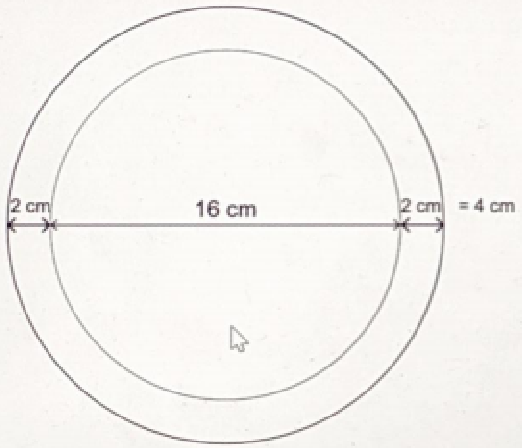
Question 7 (31 marks)

Here is a circle with diameter of 16 cm



Question 7 (31 marks)

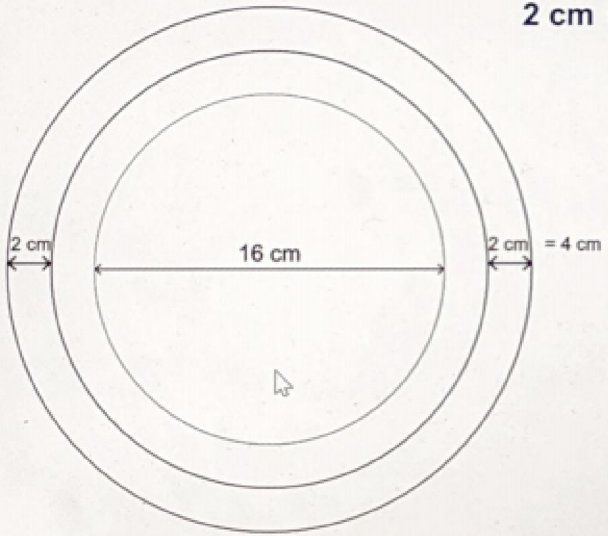
Larger circles are added around the circle.



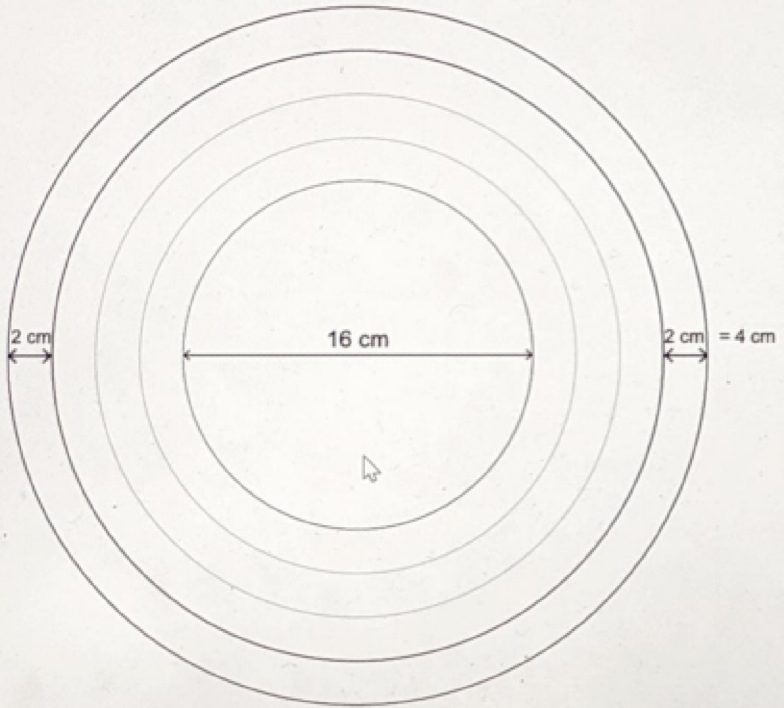
Question 7 (31 marks)

Each time the diameter of the circles increases by 4 cm.

2 cm + 2 cm



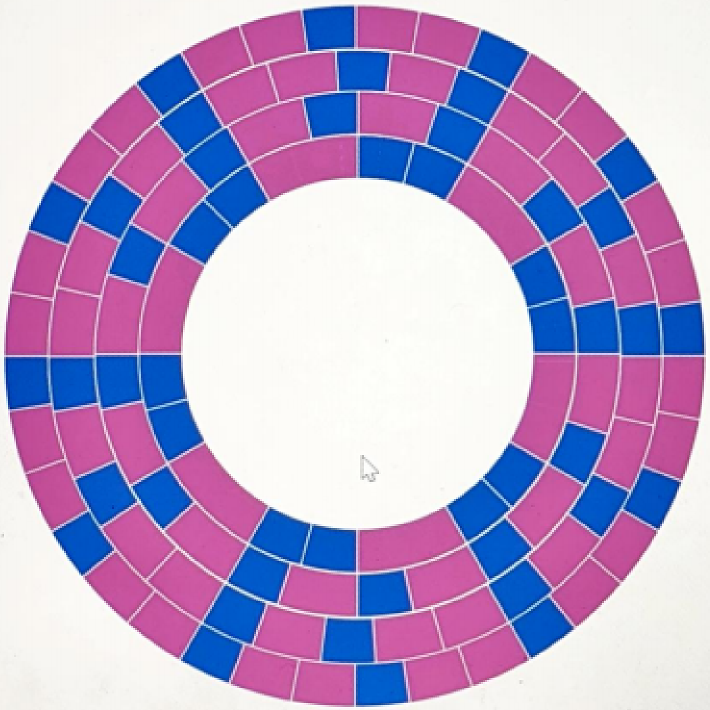
Question 7 (31 marks)



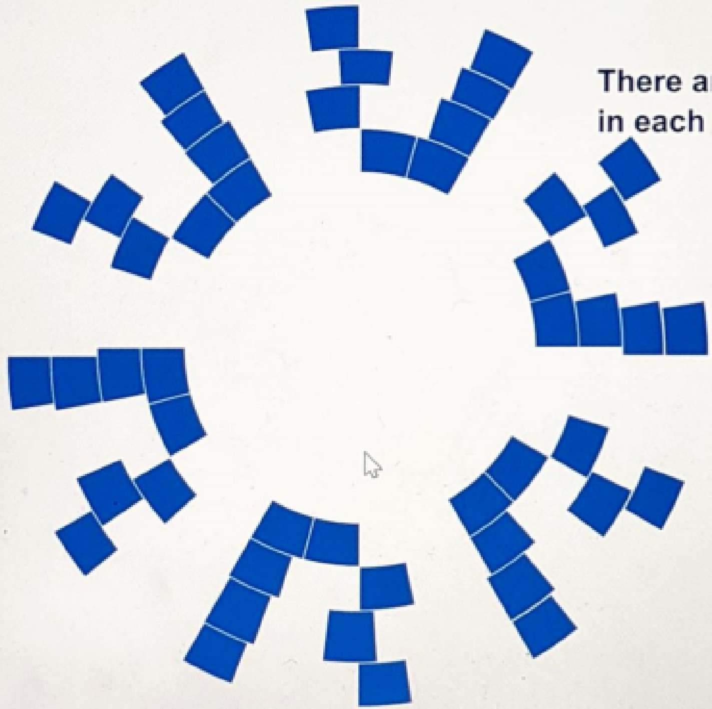
Question 7 (31 marks)

The rings between the circles are filled with blue and pink tiles.

Question 7 (31 marks)



Question 7 (31 marks)



There are exactly 12 blue tiles in each ring.

Question 7 (31 marks)

$\frac{3\pi}{2}$

$\frac{3\pi}{2}$

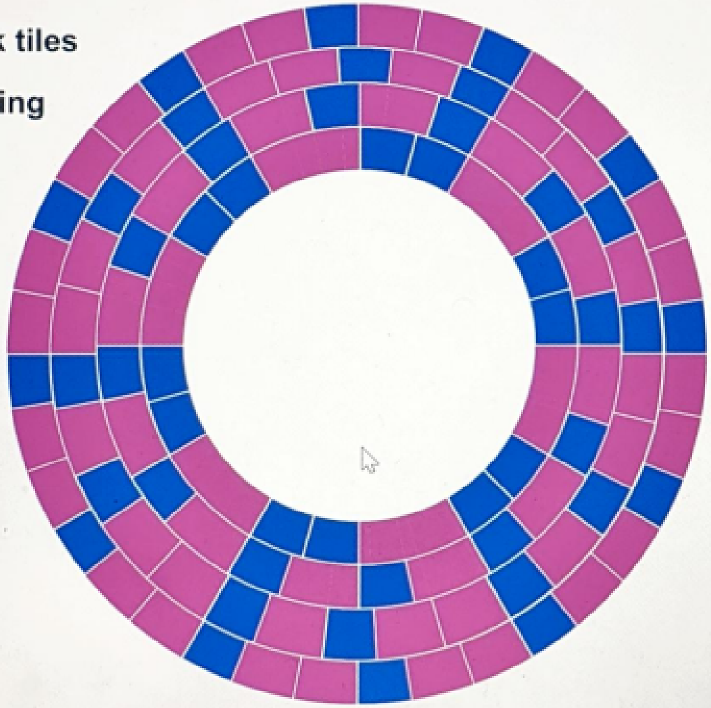
$\frac{3\pi}{2}$

$\frac{3\pi}{2}$

Question 7 (31 marks)

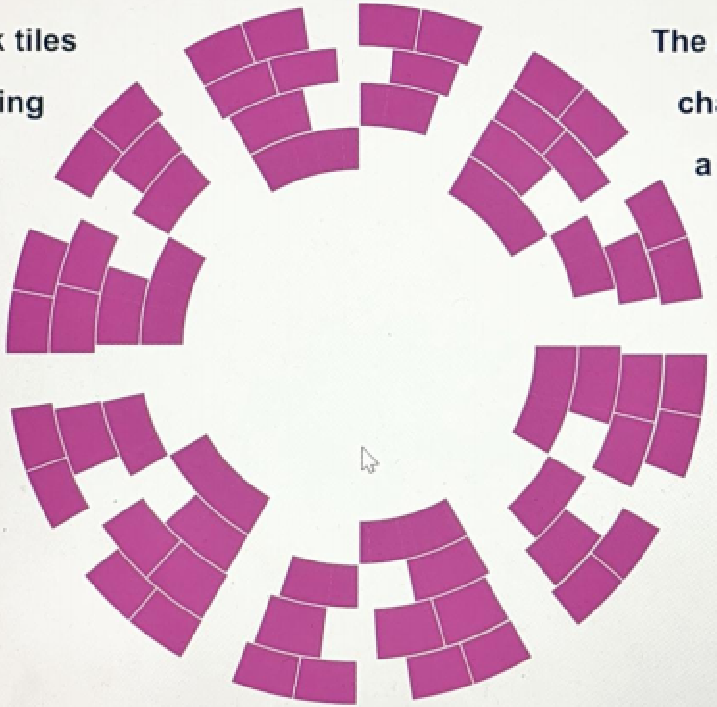
The number of pink tiles

increases in each ring



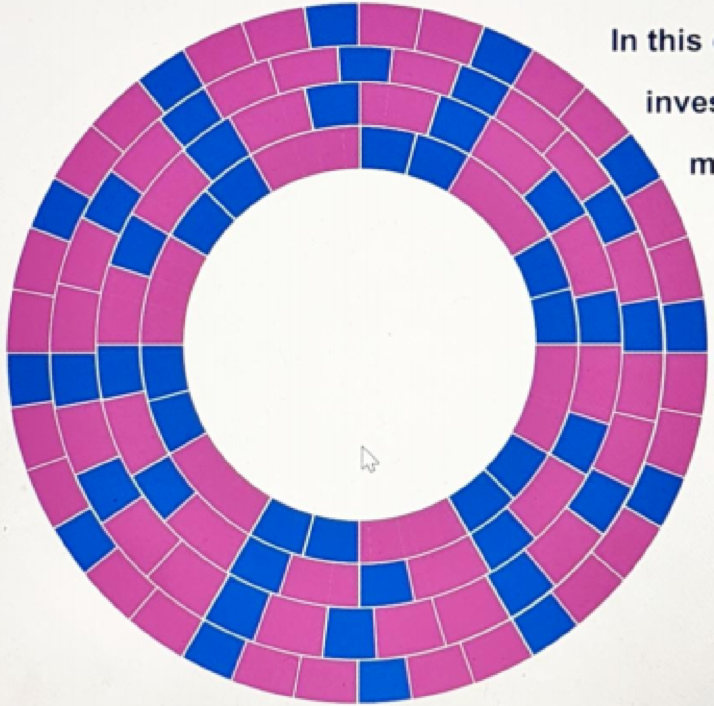
Question 7 (31 marks)

The number of pink tiles increases in each ring



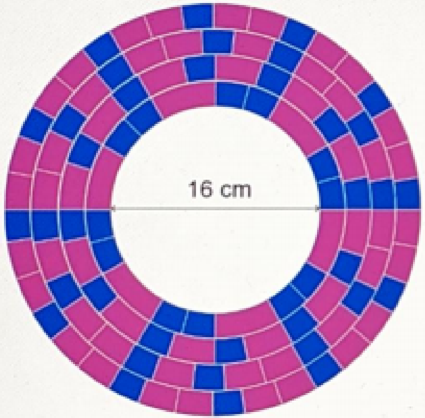
The area of the pink tiles changes every time a ring is added.

Question 7 (31 marks)



In this question you will investigate the patterns made by the rings of circles and tiles

Interact with the stage control to see the different rings.



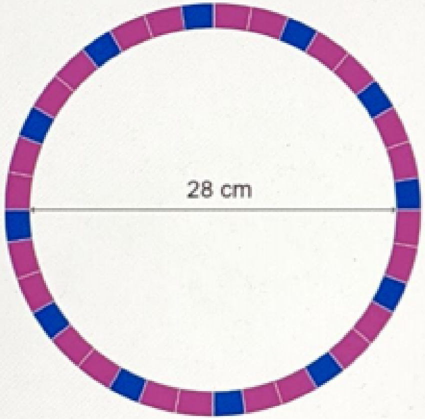
Stage control

Ring: [redacted]

Ring (n)	Area of the ring (R)

Interact with the stage control to see the different rings.

Ring 4



Stage control

Ring: 4

Ring (n)	Area of the ring (R)
1	36π
2	44π
3	52π
4	60π

Question 7a (1 mark)

Ring (n)	Area of the ring (R)
1	36π
2	44π
3	52π
4	60π
5	
6	

Write down the missing values in the table up to ring 6.

Question 7b (1 mark)

Describe in words **one** pattern you see in the table for the area (R).

B I U x_2 x^2 Ω Σ

Styles

Ring (n)	Area of the ring (R)
1	36π
2	44π
3	52π
4	60π
5	
6	

Question 7c (2 marks)

Write down a general rule for R in terms of n.

B *I* U x_2 x^2 Ω Σ
Styles

Ring (n)	Area of the ring (R)
1	36π
2	44π
3	52π
4	60π
5	
6	

B I U x_2 x^2 Ω Σ

Styles

Question 7d (3 marks)

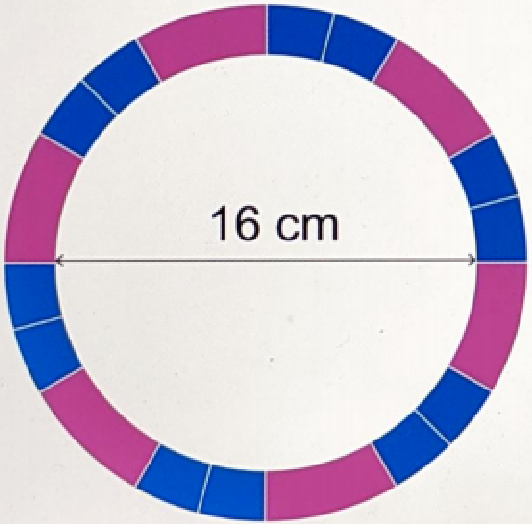
Verify your general rule for R.

B I U x_2 x^2 Ω Σ

Question 7e (2 marks)

Ring 1 includes 12 blue tiles and 6 pink tiles.

In the first ring the area of one pink tile is double the area of one blue tile.



Show that the area of a blue tile on the first ring is $\frac{3}{2}\pi$.

B I U Ω Σ

Styles

Question 7f (22 marks)

There are always 12 blue tiles and the area of each blue tile is always $\frac{3}{2}\pi$.

Investigate the values in the table to find a relationship for the area (A) of the pink tile in terms of *n*. In your answer, you should:

- predict more values and record these in the table
- describe in words **two** patterns for column A
- find a general rule for A in terms of *n*
- test your general rule for A
- verify and justify your general rule for A
- ensure that you communicate all your working appropriately.

Ring (n)	Area of the ring (R)	Number of pink tiles (P)	Area of a pink tile (A)		
1	36π	6	$\frac{18}{6}\pi$		
2	44π	12	$\frac{26}{12}\pi$		
3	52π	18	$\frac{34}{18}\pi$		
4	60π	24	$\frac{42}{24}\pi$		
5					
6					