

Markscheme

November 2021

Mathematics

On-screen examination

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IMPORTANT

A0 only use to award a zero mark for an answer that has no merit e.g. awarded for the candidate that has a wrong answer with no working

NR only use when the candidate has not made any response also stamp the response with **SEEN**

SEEN Seen; must be stamped on all blank response areas and on concatenated responses

- Bullet notation means award 1 mark – see example below

Example 1
.1 mark awarded and corresponding notes are aligned

b	.1 Show clear line of reasoning in the method	.1 45 & 49 seen OE eg, $49 = 45 + x$	2
	.2 4	.2 Accept $45 + X/10 = 4.9$ and Ans 4	

Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

- ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
- If an answer resulting from **ECF** is inappropriate (eg, negative distances or $\sin x > 1$) then subsequent marks should not be awarded.
- If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
- To award **ECF** marks for a question part, **there must be working present for that part**.
- ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- A misread (**MR**) is an error. **ECF** is normally awarded.

General points

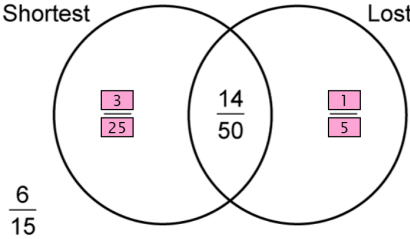
- As this is an international examination, accept all alternative forms of **notation**, for example 1.9 and 1,9 or 1 000 or 1.000. However **DO NOT ACCEPT** incorrect mathematical notation e.g. x^2 for x^2 unless noted otherwise in the MS.
- Accept notation errors in intermediate steps.
- Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradicts the correct answer**, then the last mark cannot be awarded.
- In the case when a correct result is obtained using incorrect seen method, do not award the mark for the result.
- Where candidates have written two solutions to a question, mark the first solution.
- In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** (or equivalent) e.g. $\frac{1}{2}$ **OR** 1/2 **OR** 1÷2 and $\frac{x}{2}$ **OR** x/2 **OR** x÷2
- In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.
- Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.
- Accept seeing equation not in-line.
- Calculator screenshots are accepted as working steps. And when a calculator screenshot is taken, accept not seeing the whole operation.
- In task 2 and 3 where the markscheme is set out in a table then, unless noted otherwise, awarding the highest mark in a category includes all the lower marks in that category. It is probably best to look for the top category mark answer and if you don't find it look at the next mark down.
- ACCEPT** using the correct values regardless their previous result

Question		Answers	Notes	Total
1	a	correct coordinate	$(-3, 4)$ or $c = -3$ and $d = 4$ or $x = -3, y = 4$	1
	b	correct coordinate	$(3,1)$	1
	c	.1 correct transformation .2 correct details	.1 Translation ACCEPT displacement .2 5 down OE or $(0,-5)$ or $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$	2

Question Table	Answers	Notes	Total
2	<p>AM 1 (using symbols implies the use of AM1)</p> <p>.1 correctly write one equation</p> <p>.2 correctly write the other equation</p> <p>.3 correctly step towards solving the two equations</p> <p>.4 correctly reduce to one equation in one unknown</p> <p>.5 correctly identify the c value being 30</p> <p>AM 2</p> <p>.1 subtracting the correct values to find height of three cartons</p> <p>.2 correctly height of three cartons seen</p> <p>.3 divide by 3 the height of three cartons</p> <p>.4 correctly divide by 3 the height of three cartons</p> <p>.5 correctly identify the c value being 30</p> <p>AM3 next page</p>	<p>AM 1</p> <p>.1 $t + 2c = 140$</p> <p>.2 $t - c = 50$</p> <p>.3 Attempt to subtract equations or add $\cdot 1$ to double of $\cdot 2$ equation OR substitute $t = 50 + c$ into the other equation</p> <p>.4 $3c = 90$ OR $3t = 240$</p> <p>.5 $c = 30$ or carton=30 , DO NOT ACCEPT their c value</p> <p>AM 2</p> <p>.1 $140 - 50$</p> <p>.2 90 seen</p> <p>.3 $90/3$</p> <p>.4 30</p> <p>.5 $c = 30$ or carton=30 , DO NOT ACCEPT their c value</p> <p>AM3 next page</p>	5

	<p>AM 3</p> <p>.1 Seeing two numbers where the sum of one and the double of the other is 140</p> <p>.2 correct two numbers (80 and 30) in an operation involving 140</p> <p>.3 Seeing two numbers with difference 50</p> <p>.4 correct two numbers (80 and 30) in an operation involving 50</p> <p>.5 correctly identify the c value being 30</p>	<p>AM 3</p> <p>.1 Ex: 80 and 30 or 120 and 10 ACCEPT 80 and 60 or 120 and 20</p> <p>.2 $80+2 \times 30=140$ ACCEPT $140-60=80$ OE</p> <p>.3 Ex: 80 and 30 or 120 and 70 or $70+50=120$ OE</p> <p>.4 $80-30=50$ or $50+30=80$ OE</p> <p>.5 $c = 30$, or carton=30 , DO NOT ACCEPT their c value</p> <p>ACCEPT using the word "carton" or any symbol to represent it</p>	
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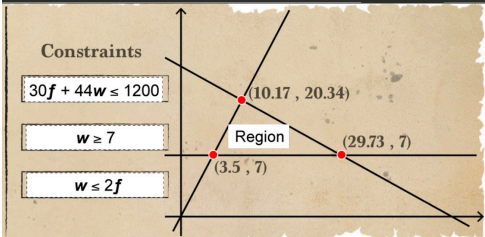
Question		Answers	Notes	Total	
		Note: If their probability used is greater than 1 then do not award the bullet point			
3	a	• $\frac{3}{5}$ or 0.6 OE		1	
	b	correctly write 3/5 and 3/10 and 2/3 in the appropriate place	<p>The diagram shows a probability tree. The first level branches into 'Shortest' with probability $\frac{2}{5}$ and 'Not shortest' with probability $\frac{3}{5}$. From 'Shortest', it branches into 'Lost' with probability $\frac{7}{10}$ and 'Not lost' with probability $\frac{3}{10}$. From 'Not shortest', it branches into 'Lost' with probability $\frac{1}{3}$ and 'Not lost' with probability $\frac{2}{3}$.</p>	1	
	c	.1 multiply the Shortest by their Not Lost .2 correctly calculate their result after multiplication of fractions	.1 $\frac{2}{5} \times \text{their } \frac{3}{10}$.2 $\text{their } \frac{3}{25}$ or 0.12 OE, DO NOT ACCEPT 0.1	2	
	d	.1 multiply their Not shortest by their Not Lost .2 add $\text{their } \frac{3}{25}$ to their .1 .3 correctly calculate their result after adding their multiplied fractions	.1 $\text{their } \frac{3}{5} \times \text{their } \frac{2}{3}$ or 0.4 seen .2 $\text{their } \frac{3}{25} + \text{their } \frac{3}{5} \times \text{their } \frac{2}{3}$.3 $\text{their } \frac{13}{25}$ or 0.52 OE, DO NOT ACCEPT 0.5	3	

<p>e</p>	<p>.1 correctly place their result from 3c on the diagram</p> <p>.2 multiply their Not shortest by $\frac{1}{3}$ OR subtract all from 1</p> <p>.3 correctly place their $\frac{1}{5}$ OE from .2 on the diagram</p>	<p>Shortest</p>  <p>Lost</p> <p>$\frac{6}{15}$</p> <p>.1 their $\frac{3}{25}$ OE correctly placed on the diagram</p> <p>.2 <i>their</i> $\frac{3}{5} \times \frac{1}{3}$ OE OR $1 - \frac{6}{15} - \text{their } \frac{3}{25} - \frac{14}{50}$</p> <p>.3 DO NOT award unless the total of probabilities is 1</p>	<p>3</p>
<p>f</p>	<p>.1 multiply the correct probability three times</p> <p>.2 correctly calculate their result after multiplying their probability three times</p>	<p>.1 $\frac{6}{15} \times \frac{6}{15} \times \frac{6}{15}$ OE</p> <p>.2 <i>their</i> $\frac{8}{125}$ or 0.064 OE, DO NOT ACCEPT 0.06</p>	<p>2</p>

Question	Answers	Notes	Total
4 a	<p>AM1 (starting with SA) .1 correctly substitute in surface area equation .2 correctly calculate their value of h from a surface area equation</p> <p>.3 correctly substitute their 8 into volume of cylinder and volume of cone</p> <p>.4 add the two correct volumes AG 84π</p> <p>AM 2 (Starting with Volume) .1 correctly substitute in volume equation .2 correctly calculate their value of h from a volume equation</p> <p>.3 correctly substitute their 8 into SA of cylinder and SA of cone</p> <p>.4 add the two correct surface areas AG 63π</p> <p>AM 3 (Starting with an assumed h) .1 correctly assume $h=8$</p> <p>.2 correctly substitute their8 into both correct volumes formulas OR both correct surface area formulas</p> <p>.3 correctly show that the sum of volumes is 84π OR the sum of surface areas is 63π</p>	<p>AM1 (starting with Surface Area) .1 $63\pi = 3\pi \times 5 + 2\pi \times 3 \times h$ OE .2 ($h =$) their8 ACCEPT 8 seen after the correct surface area equation</p> <p>.3 $\pi \times 3^2 \times$ their8 OE and $\frac{1}{3}\pi \times 3^2 \times 4$ OE or their226.194...and their37.699... .3 ACCEPT [226 , 226.2] and [37.69 , 37.7]</p> <p>.4 $72\pi + 12\pi$ seen .4 ACCEPT (226.194...+ 37.699.. =) [263.7, 263.9] which is 84π WTTE</p> <p>AM 2 (Starting with Volume) .1 $84\pi = \frac{1}{3}\pi \times 3^2 \times 4 + \pi \times 3^2 \times h$ OE .2 ($h =$) their8 ACCEPT 8 seen after the correct volume equation .3 (SA=)$3\pi \times 5$ and $2\pi \times 3 \times$ their 8 or their47.123.....and their150.796... .3 ACCEPT [47 , 47.124] and [150.79 , 150.8]</p> <p>.4 $15\pi + 48\pi$ seen .4 ACCEPT (47.123...+ 150.796... =) [197.79, 198] which is 63π WTTE</p> <p>AM 3 (Starting with an assumed h) .1 8 seen .2 ($V =$)$\pi \times 3^2 \times$ their8 and $\frac{1}{3}\pi \times 3^2 \times 4$ OE OR (SA =)$3\pi \times 5$ and $2\pi \times 3 \times$ their8 OE .2 ACCEPT ($V =$) [226 , 226.2] and [37.69 , 37.7] OR (SA=) [47 , 47.124] and [150.79 , 150.8]</p> <p>.3 ($V =$)$72\pi + 12\pi$ seen OR (SA=)$15\pi + 48\pi$ seen .3 ACCEPT ($V =$226.194...+ 37.699.. =) [263.7, 263.9] which is 84π WTTE .3 ACCEPT (SA=47.123...+ 150.796... =) [197.79, 198] which is 63π WTTE</p>	4
	b	<p>.1 their$84\pi/10$ or 26.38...or $84\pi/8$ or 32.98... .2 $84\pi/80$ $84\pi/(8 \times 10)$ or 3.29.... .3 4</p>	3

Question	Answers	Notes	Total
5 a	<p>AM 1 (using x when substituting into Pythagoras implies the use of AM1)</p> <p>.1 correctly substitute into Pythagoras' Theorem</p> <p>.2 correctly expand and simplify their quadratic equation</p> <p>.3 correctly solve their quadratic equation (ACCEPT even if positive value only) OR correctly factorize their quadratic equation OR correctly substitute their a,b,c into the quadratic formula</p> <p>.4 correctly identify the x value being 8</p> <p>AM 2 (Using a value for x when substituting into Pythagoras implies the use of AM2)</p> <p>.1 correctly substitute their value into Pythagoras' Theorem or realising the Pythagorean triple 8,15,17</p> <p>.2 correctly expand the left-hand side when $x = 8$</p> <p>.3 correctly expand right-hand side</p> <p>.4 correctly identify the x value being 8</p>	<p>AM 1</p> <p>.1 $x^2 + (x + 7)^2 = 17^2$, ACCEPT missing brackets</p> <p>.2 $2x^2 + 14x + 49 = 289$ OE</p> <p>.3 $(x + 15)(x - 8) = 0$ OR $\frac{-7 \pm \sqrt{7^2 - 4(1)(-120)}}{2(1)}$ OE</p> <p>.4 8 , DO NOT ACCEPT their x value</p> <p>AM 2</p> <p>.1 $8^2 + (8 + 7)^2 = 17^2$ or 8,15,17</p> <p>.1 ACCEPT missing brackets and ACCEPT any value for x used</p> <p>.2 $64 + 225$ seen</p> <p>.3 289 seen</p> <p>.4 8 , DO NOT ACCEPT their x value</p>	4
b	<p>.1 correctly substitute their x into area of triangle formula</p> <p>.2 correctly calculate their result after substitution into area formula</p>	<p>.1 $\frac{1}{2} \times \text{their } 8 \times (\text{their } 8 + 7)$ ACCEPT missing brackets</p> <p>.2 Their 60</p>	2

Question		Answers	Notes	Total
6	a	.1 correctly substitute in distance formula OR add distances .2 correct distance before rounding 1170 AG	.1 $(D =) 500 \times \frac{140}{60}$ OE OR $500 + 500 + \frac{20}{60} \times 500$ OE .1 ACCEPT $(D =) 500 \times 2.3(\dots)$.1 ACCEPT $(\text{speed} =) \frac{1170}{(140 / 60)}$ or $\frac{1170}{2.3(\dots)}$.2 1166.66 ACCEPT [1165,1167]	2
	b	.1 correctly divide 5760 by 900 .2 correctly convert their time to minutes	.1 6.4 .2 $(\text{their } 6.4 \times 60) = \text{their } 384$	2
	c	AM1 using distance as unknown .1 correctly write one time in terms of x (distance) .2 correctly write the other time in terms of x (distance) .3 correctly calculate the value of x (distance) .4 correctly calculate the time in hours .5 correctly write their time after 7:00 AM2 using time as unknown .1 correctly write one distance expression .2 correctly write the other distance expression .3 equate the correct expressions .4 correctly calculate the time in hours .5 correctly write their time after 7:00	AM1 .1 $x/500$ or $(1170-x)/436$ ACCEPT a number/500 or a number/ 436 .2 $x/500$ and $(1170-x)/436$.3 $(x =) 625$.4 $(625/500 =) 1.25$.5 their 8 :15 AM2 .1 $(d =) 436t$ or $(d =) 500t$ ACCEPT $436x$ a number or $500x$ a number .2 $1170 - 500t$ or $1170 - 436t$.3 $436t = 1170 - 500t$ or $500t = 1170 - 436t$.4 $(t =) 1.25$.5 their 8 :15	5
	d	.1 evidence of correct gradient .2 correctly substitute (80,2400) into $h(x) = 60x + c$.3 correct $h(x)$.1 60 seen as gradient .2 $2400 = 60(80) + c$.3 $h(x) = 60x - 2400$	3
	e	.1 correctly substitute 700 into $q(x)$.2 correct $p(x)$.3 correct deduction after subtracting their 11580 from 12000	.1 $(q(x) =) -40(700) + 39580$.2 $(p(x) =) 11580$.3 (Safe) because their $420 > 300$. ACCEPT their 420 being the result of any calculation they make DO NOT ACCEPT their 420 if less than 300	3

Question	Answers	Notes	Total
7 a	.1 correctly place two inequalities .2 correctly place the third inequality and region.	 <p>DO ACCETPT ECF for their region from their constraints DO NOT ACCEPT their region placed in-between regions</p>	2
b	.1 select 10 and 20 .2 correctly substitute their 10 and their 20 .3 correctly calculate their maximum weight.	.1 10 and 20 seen .2 their $10 \times (30) + \text{their } 20 \times (44)$.2 ACCEPT their 10 and their 20 only if they are whole numbers or they are (10.17, 20.34) .3 Their 1180 (lb) ACCEPT their 1180 only if less than 1200	3
c	<p>AM1</p> .1 substitute correctly into a trig ratio .2 correct algebraic step .3 correctly calculate AB (or x) <p>AM2</p> .1 correctly calculate AC .2 correctly substitute in Pythagoras .3 correctly calculate AB (or x)	<p>AM1</p> .1 $\sin 53 = \frac{x}{1.25}$ or $\cos 37 = \frac{x}{1.25}$.2 $(x =) 1.25 \sin 53$ or $(x =) 1.25 \cos 37$.3 [0.998... , 1] <p>AM2</p> .1 (AC =) $1.25 \cos 53$ or $0.75(22..)$ seen .2 $(x^2 =) 1.25^2 - 0.75^2(22..)$ OE .3 [0.998... , 1]	3
d	correctly add their AB to 1.25	Their [2.248... , 2.25]	1

e	Mark	1	2	3	4	
	Identify factors (F)	Explicitly state two factors from: <ul style="list-style-type: none"> - Length of route - Number of days for the trip - Availability of food and water - Amount of goods they are able to carry for trading - Terrain features (mountain or crossing river,..etc) Ignore additional irrelevant factors DO NOT ACCEPT factors embedded in working	Explicitly state three factors			
	Calculations (L)	Correct two values 7 days for $4 \leq D < 8$ Modal class ACCEPT $8 < D < 12$ or 8 to 12 or 8-12 Estimate Median = 9 Estimate mean = 8.769.. ACCEPT 8.8 Total number of days = 26 Estimate for total distance travelled = 228 In all, allow ecf from their number of days for $4 \leq D < 8$	Correct three values	Correct four values	Correct six values	10
	Comparison (C)	Compare statistical values: Correctly compare at least two statistical values Example: Mean and number of days and total distance are less in Cimarron route MUST compare using a word like less, more, on the other hand, while,..etc OR State at least three statistical values for each route without explicit word for comparison	Compare statistical values: Correctly compare at least two statistical values Example: Mean and number of days and total distance are less in Cimarron route MUST compare using a word like less, more, on the other hand, while,..etc OR State at least three statistical values for each route without explicit word for comparison			

		<p>OR</p> <p>Correctly compare nature of the two routes Example: comparing the roughness of the two routes OR Realise that the objective of the journey is to trade and the mountain route allows more space for trading goods</p>	<p>AND</p> <p>Correctly compare nature of the two routes Example: comparing the roughness of the two routes OR Realise that the objective of the journey is to trade and the mountain route allows more space for trading goods</p>			
	<p>Justify accuracy (A)</p>	<p>inaccurate with weak justification</p> <p>Concerning the maths Inaccurate because rounding used OR these are approximate calculations and not accurate OR mean and median are just estimates</p> <p>OR</p> <p>Concerning the context Anything related to hazards or things unaccounted for that families may face OR because families cannot report exact distances every day OR we cannot know for sure how they measure their distances travelled</p> <p>DO NOT ACCEPT: my results are accurate with any reason WTTE</p>	<p>inaccurate with good justification</p> <p>Concerning the maths The mean and total distance travelled are just estimates <u>since we are using mid-interval class.</u> OR The <u>use of mid-class</u> in calculations makes it an estimate</p> <p>OR</p> <p>Concerning the maths: mean and median are just estimates AND Concerning the context Anything related to hazards or things unaccounted for that families may face OR because families cannot report exact distances every day OR we cannot know for sure how they measure their distances travelled</p>			

Distance travelled (D) in miles	Number of days (N)	Measures of central tendency for the distance travelled by Family Fry			Total number of days	Estimate for the total distance travelled
$0 \leq D < 4$	3	Modal class	Estimate for the median	Estimate for the mean	26	228
$4 \leq D < 8$	7	$8 < D < 12$	9	8.769...		
$8 \leq D < 12$	12					
$12 \leq D < 16$	3					
$16 \leq D < 20$	1					

Mountain Route

Measures of central tendency for the distance travelled by Family Kane			Total number of days	Estimate for the total distance travelled
Modal class	Estimate for the median	Estimate for the mean	39	502
$12 \leq D < 16$	13.5	12.87		

Question		Answers	Notes	Total														
8	a	correctly place 96π and 108π	<table border="1"> <thead> <tr> <th>Ring (n)</th> <th>Circumference of the circle (C)</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>48π</td> </tr> <tr> <td>2</td> <td>60π</td> </tr> <tr> <td>3</td> <td>72π</td> </tr> <tr> <td>4</td> <td>84π</td> </tr> <tr> <td>5</td> <td>96π</td> </tr> <tr> <td>6</td> <td>108π</td> </tr> </tbody> </table>	Ring (n)	Circumference of the circle (C)	1	48π	2	60π	3	72π	4	84π	5	96π	6	108π	1
	Ring (n)	Circumference of the circle (C)																
	1	48π																
	2	60π																
	3	72π																
	4	84π																
	5	96π																
6	108π																	
b	correctly describe one pattern for C in words with correct terminology	Examples of suitable patterns and acceptable terminology: Increases by 12π or 12π increases by 12 and the pi is always there or regardless pi WTTE DO NOT ACCEPT Increasing by 12 Multiples of 12 even numbers	1															
c	.1 the correct general rule with or without π .2 the correct simplified general rule with correct notation	.1 (C=) $36\pi + 12n\pi$ ACCEPT (C=) $36\pi + 12 \times n\pi$ or (c=) $36 + 12n$ Or $C = 36 + 12n \times \pi$ or (C=) $36\pi + 12n$.2 $C = 36\pi + 12n\pi$ ACCEPT $c = 48\pi + (n - 1)12\pi$ or $C = (36 + 12n) \times \pi$ DO NOT ACCEPT description in words	2															
d	.1 correctly substitute $n \geq 5$ into their general rule .2 correctly calculate their value of C after substituting $n \geq 5$.3 recognize that their correctly calculated value of C is the same as their predicted value	.1 Ex : $36\pi + 12 \times 5\pi$.2 96π (for $n = 5$) .3 Same as value I predicted in table (and we find the candidate has 96π in the table for $n = 5$) OR same as when we continue the pattern and explains how 96π is obtained from pattern of adding 12π to 84π	3															
e	.1 correct circumference for length of 12 blue tiles .2 subtract from 48π .3 divide the correct simplified difference by 6 AG 3π	.1 $12 \times \frac{(5\pi)}{2}$ or 30π or $94.2(\dots)$ seen .2 $48\pi - 12 \times \frac{(5\pi)}{2}$ or $48\pi - 30\pi$ or 18π or $56.5(\dots)$ seen .3 $\frac{18\pi}{6}$ ACCEPT showing that $\frac{56.5(\dots)}{6} = 9.4(\dots)$ and $3\pi = 9.4(\dots)$ OE																

8		f				22
Mark	1	2	3	4	5	
Predictions (P)	<p>Correctly predict three terms for P or L</p> <p>ACCEPT whether in the table or in the response box ACCEPT typing errors like seeing $11/5\pi$ for $\frac{11}{5}\pi$ or using pi instead of π or missing the pi</p>	<p>Correctly predict four terms for P and L</p> <p>ACCEPT whether in the table or in the response box ACCEPT typing errors like seeing $11/5\pi$ for $\frac{11}{5}\pi$ or using pi instead of π or missing the pi</p>				
Description (D)	<p>Attempt to describe a pattern in words for L</p> <p>Ex: numerators are odd numbers The up number increases by 2 The lower number increases by 1</p> <p>OR</p> <p>Attempt to describe a rule in words for L</p> <p>OR</p> <p>A correct pattern described as general rule for P</p> <p>DO NOT ACCEPT L is increasing DO NOT ACCEPT any description for P in words</p>	<p>Attempt to describe pattern for L as general rule</p> <p>Ex: Correct general rule for numerator $(1+2n)$ Or Correct general rule for denominator (n)</p> <p>OR</p> <p>One correct pattern described in words for L</p> <p>Ex: Numerator increases by 2 Denominator increases by 1 Difference between numerator and denominator is 2, 3, 4,...</p>	<p>Correctly describe the pattern for L as a general rule</p> <p>Rule: $L = \frac{6\pi(1+2n)}{6n}$ or $L = \frac{\pi(1+2n)}{n}$</p> <p>ACCEPT if the π is missing and penalize in notation ACCEPT rule for numerator=1+2n and rule for denominator n and penalize in notation</p> <p>OR</p> <p>Two correct patterns described in words for L (one for numerator and the other for denominator)</p> <p>OR</p> <p>Correct general rule for numerator AND Two attempts to describe pattern in words</p>	<p>Correctly describe the pattern for L as a general rule AND one correct pattern described in words for L</p> <p>OR</p> <p>Attempt to describe pattern for L as general rule AND two correct patterns described in words for L (one for numerator and the other for denominator)</p> <p>ACCEPT if the π is missing and penalize in notation ACCEPT rule for numerator=1+2n and rule for</p>	<p>Correctly describe the pattern for L as a general rule AND two correct patterns described in words for L (one for numerator and the other for denominator)</p> <p>ACCEPT rule for numerator=1+2n and rule for denominator n and penalize in notation</p> <p>ACCEPT if the π is missing and penalize in notation ACCEPT rule for numerator=1+2</p>	

			OR Attempt to describe pattern for L as general rule AND one correct pattern described in words for L	denominator n and penalize in notation	n and rule for denominator n and penalize in notation
<p>Testing (T)</p> <p>ACCEPT transforming into decimals when testing</p> <p>ACCEPT testing without π</p>	<p>Attempt to test their general rule for L using $n \leq 4$</p> <p>Ex: Substitute in their general rule value of $n \leq 4$</p> <p>OR</p> <p>Correctly test their described pattern or their rule (e.g. recursive rule)</p>	<p>Correctly test their general rule for L only in terms of n using $n \leq 4$</p> <p>Ex: Correctly calculate their value for L in their general rule using $n \leq 4$</p> <p>AND Recognise that their correctly calculated value for L is the same as the given value.</p> <p>ACCEPT seeing their correctly calculated value for A and the given value in the table being equal</p>			
<p>Verifying (V)</p> <p>ACCEPT transforming into decimals when verifying</p> <p>ACCEPT verifying without π</p>	<p>Attempt to verify their general rule for L using $n \geq 5$</p> <p>Ex: substitute in their general rule value of $n \geq 5$</p> <p>OR</p> <p>Correctly verify their described pattern or their rule (e.g. recursive rule)</p>	<p>Correctly calculate their value for L in their general rule only in terms of n using $n \geq 5$</p>	<p>Correctly calculate their value for L in their general rule only in terms of n using $n \geq 5$</p> <p>AND Recognise that their correctly calculated value for L is the same as their predicted value obtained by continuing the pattern</p> <p>ACCEPT seeing their correctly calculated value for L and their predicted value in the table being equal</p>		

<p>Justify/proof (J)</p>	<p>Attempt to justify any of their described patterns or their general rule Ex: Attempt to use the arithmetic sequence OR Substitute at least two other values of n in L and say they are the same or the rule works (regardless the π)</p>	<p>Justify their general rule correctly Ex: Use the arithmetic sequence for the numerator to show the rule.</p>	<p>Attempt to justify geometrically the general rule Ex: Attempt to equate their C rule (from 8c) to the sum of $\frac{5\pi}{2} \times 12$ (or 30π) and their rule for P times their rule for L their $(36\pi + 12n\pi) = 30\pi + \text{their } 6n \times \frac{1+2n}{n} \pi$</p>	<p>Correctly justify geometrically the general rule Ex: Show that the correct general rule for C is equal the sum of $\frac{5\pi}{2} \times 12$ (or 30π) and the correct general rule for P times the correct general rule for L $36\pi + 12n\pi = 30\pi + 6n \times \frac{1+2n}{n} \pi$</p>	
<p>Notation and terminology (N)</p>	<p>Correct notation of <u>their</u> general rule Ex: rule for numerator $L = (2n + 1)\pi$ OR The notation of <u>the general</u> rule includes errors Ex: $(L =) \frac{6\pi(1 + 2x)}{6x}$ or $(L =) \frac{\pi(1 + 2x)}{x}$ or $(L =) \frac{1 + 2n}{n}$ OR $L = 6 \times \pi(1 + 2 * n) / 6n$ or $L = (1 + 2 * n) / n$</p>	<p>Correct notation of <u>the general</u> rule for L $L = \frac{6\pi(1 + 2n)}{6n}$ or $L = \frac{\pi(1 + 2n)}{n}$ OR The notation of <u>the general</u> rule includes errors AND Correctly describe a pattern in words for L DO NOT ACCEPT if they don't have a rule</p>	<p>Correct notation of <u>the general</u> rule for L AND Correctly describe a pattern in words for L</p>		

	<p>The rule for L is $\frac{6n(1+2n)}{6n}$</p> <p>or</p> <p>The rule for numerator is $2n+1$ and rule for denominator is n</p> <p>OR</p> <p>Correctly describe a pattern in words for L</p> <p>DO NOT ACCEPT if they don't have any rules and they don't describe any patterns</p>				
<p>Communication (L) can be awarded even there are errors in their descriptions and working</p>	<p>At least three from the following are seen:</p> <ul style="list-style-type: none"> - describe a pattern or rule in words - write a general rule - test their general rule or pattern - verify their general rule or pattern - justify their general rule or pattern 	<p>At least four of the following are seen:</p> <ul style="list-style-type: none"> - describe a pattern or rule in words - write a general rule - test their general rule or pattern - verify their general rule or pattern - justify their general rule or pattern <p>AND</p> <p>For coherence, they identify the processes correctly. At least one from the following:</p> <ul style="list-style-type: none"> - test - verify - justify <p>Ex:</p> <p>-For test: they say "test" and they test using value(s) of $n \leq 4$ only</p> <p>-For verify:</p>	<p>DO NOT ACCEPT if D3 and J2 not awarded</p> <p>At least four of the following are seen:</p> <ul style="list-style-type: none"> - describe a pattern or rule in words - write <u>the general rule</u> - test <u>the general rule</u> - verify <u>the general rule</u> - justify <u>the general rule</u> <p>AND</p> <p>For coherence, they identify the processes correctly. At least two from the following:</p> <ul style="list-style-type: none"> - test - verify - justify <p>Ex:</p> <p>-For test: they say "test" and they test using value(s) of $n \leq 4$ only</p>		

		<p>they say "verify" and they verify using value(s) of $n \geq 5$ only</p> <p>-For test and for verify: they say 'test and verify' and they test using value(s) of $n \leq 4$ and then verify using value(s) of $n \geq 5$</p> <p>-For justify: they say "justify" or "my rule works because" WTTE and their justification is seen</p> <p>-For justify: they substitute at least two values of n and say "the rule justified" or "it works" WTTE</p> <p>- For justify: They assume quadratic model (or 2nd diff 8 OE) and get values of coefficient(s) using any method</p> <p>- For justify: They justify <u>the general rule</u> for A geometrically</p>	<p>-For verify: they say "verify" and they verify using value(s) of $n \geq 5$ only</p> <p>-For test and for verify: they say 'test and verify' and they test using value(s) of $n \leq 4$ and then verify using value(s) of $n \geq 5$</p> <p>-For justify: they say "justify" or "my rule works because" WTTE and their justification is seen</p> <p>- For justify: They assume quadratic model (or 2nd diff 8 OE) and get values of coefficient(s) using any method</p> <p>- For justify: They justify <u>the general</u> rule for A geometrically</p>		
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Predictions

Ring (n)	Circumference of the circle (C)	Number of pink tiles (P)	Arc length of the pink tile (L)
1	48π	6	3π
2	60π	12	$\frac{5}{2}\pi$
3	72π	18	$\frac{7}{3}\pi$
4	84π	24	$\frac{9}{4}\pi$
5	96π	30	$\frac{11}{5}\pi$
6	108π	36	$\frac{13}{6}\pi$
7	120π	42	$\frac{15}{7}\pi$

Rules

$$C = 12\pi(n + 3) \text{ or } C = 12\pi n + 36\pi \quad P = 6n \quad L = \frac{6\pi(1 + 2n)}{6n} \text{ or } L = \frac{\pi(1 + 2n)}{n}$$