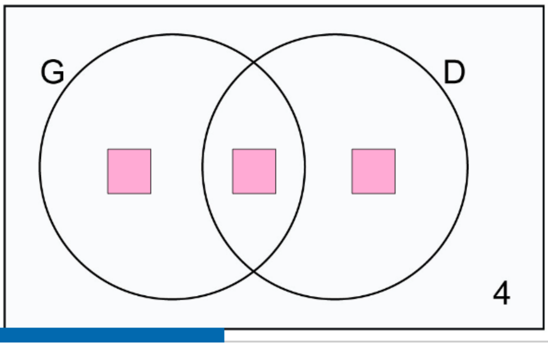
Question 1 (6 marks)

Question 1a (2 marks)

There are 24 students in a class. Of these students:

- 15 play the guitar (G)
- 12 play the drums (D)
- 4 play neither.

Determine the missing values in the Venn diagram below.



Question 1b (1 mark)

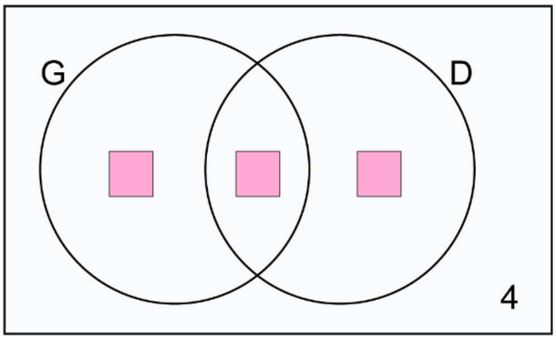
A student is selected at random. Given that the student plays at least one musical instrument, **determine** the probability that this musical instrument is guitar.

Styles

There are 24 students in a class. Of these students:

- 15 play the guitar (G)
- 12 play the drums (D)
- 4 play neither.

Determine the missing values in the Venn diagram below.



Scroll down to continue

Question 1c (3 marks)

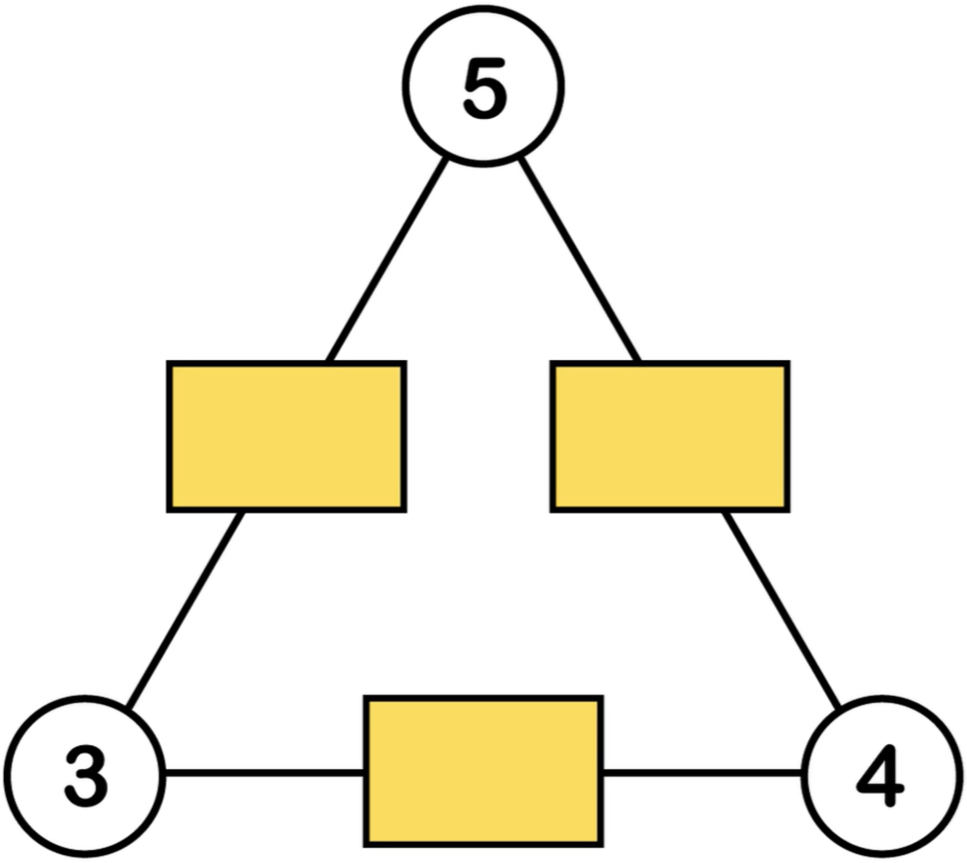
Three students are selected at random. **Determine** the probability that the three students play at least one musical instrument.

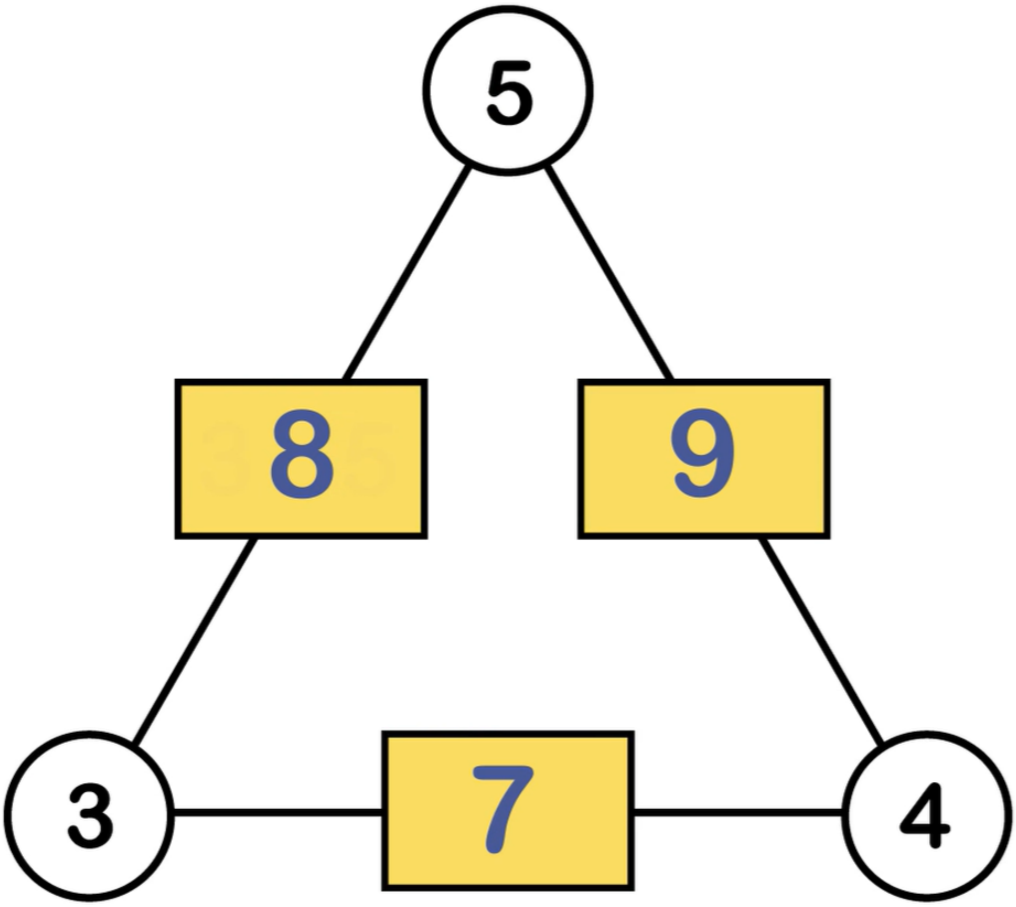
Question 2 (6 marks)

An arithmagon is a mathematical puzzle where each number in a box is the sum of the two circled numbers adjacent to it.

Here is an example with integers.

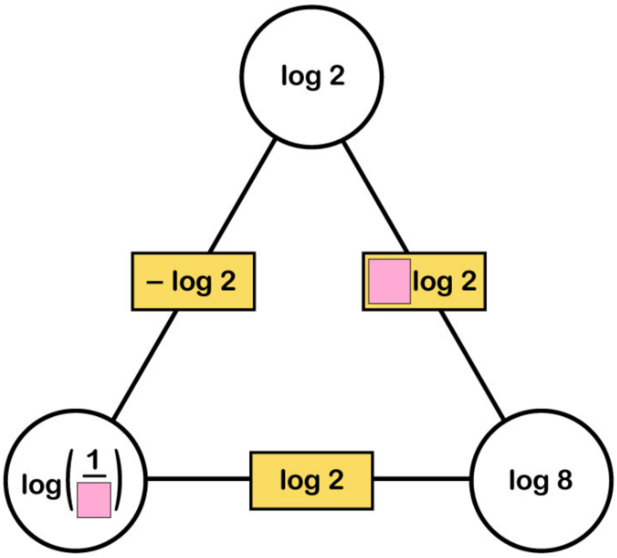
3



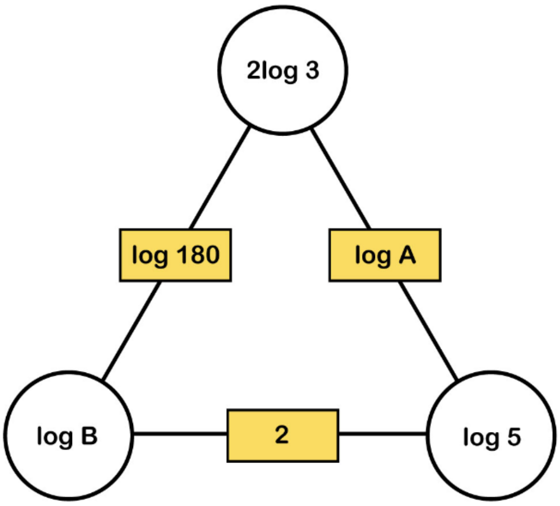

5
8
9
3
7
4

Question 2a (2 marks)

Determine the missing values in the arithmagon below.

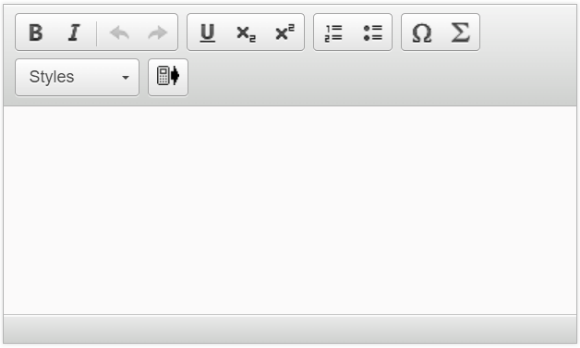


Below is another arithmagon created using logarithms.



Question 2b (2 marks)

Determine the value of A.



Question 2c (2 marks)

Determine the value of B.

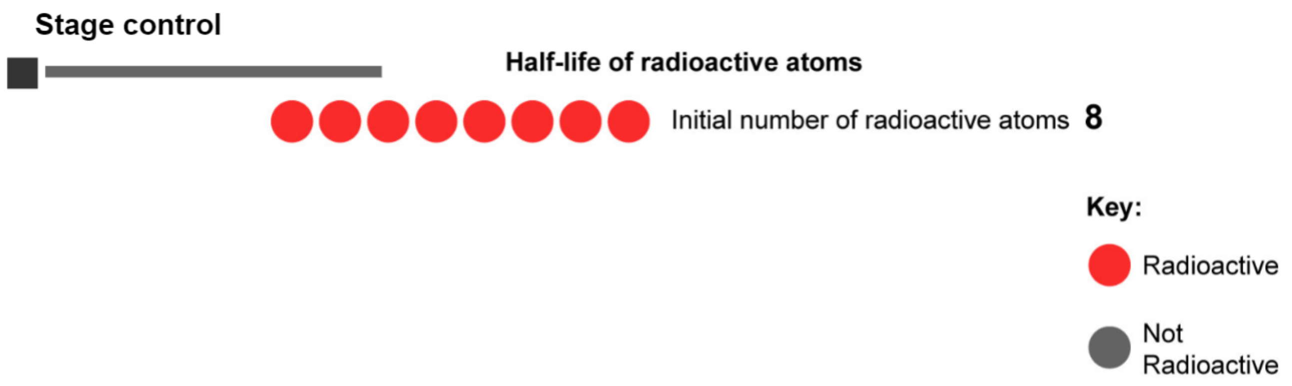
Styles

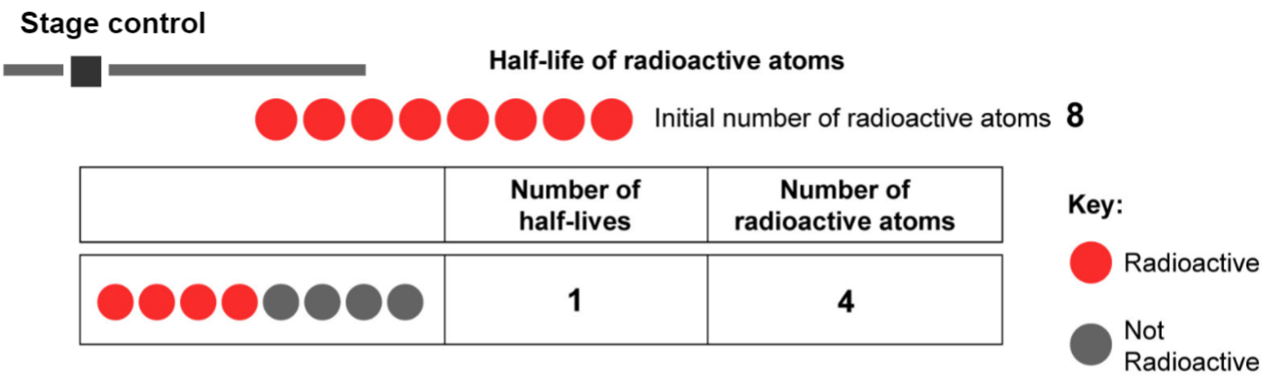
Question 3 (7 marks)

Scientists study the half-life of radioactive materials. The half-life of a radioactive material is the time it takes to half the number of radioactive atoms.

The simulation below is an example when the initial number of radioactive atoms is 8.

Interact with the slide controller to illustrate the half-life of radioactive atoms.


Stage control
Half-life of radioactive atoms
Initial number of radioactive atoms 8
Key:
Radioactive
Not Radioactive


Stage control
Half-life of radioactive atoms
Initial number of radioactive atoms 8
Number of half-lives
Number of radioactive atoms
1
4
Key:
Radioactive
Not Radioactive

Stage control

Half-life of radioactive atoms

 Initial number of radioactive atoms **8**

	Number of half-lives	Number of radioactive atoms
	1	4
	2	2
	3	1

Key:

Now the initial number of radioactive atoms is 512. The table below illustrates the number of radioactive atoms as time passes.

Number of half-lives (n)	Time passed in minutes (T)	Number of radioactive atoms (R)
1	110	256
2	220	128
3	330	64

Question 3a (1 mark)

The time passed (T) forms an arithmetic sequence with first term 110. **Write down** an expression for T in terms of n.

Styles

Now the initial number of radioactive atoms is 512. The table below illustrates the number of radioactive atoms as time passes.

Number of half-lives (n)	Time passed in minutes (T)	Number of radioactive atoms (R)
1	110	256
2	220	128
3	330	64

The number of radioactive atoms, R, forms a geometric sequence with

common ratio $\frac{1}{2}$ .The number of

radioactive atoms after the 1st half-life is 256.

Question 3b (2 marks)

Determine the number of radioactive atoms after the 6th half-life.

Now the initial number of radioactive atoms is 512. The table below illustrates the number of radioactive atoms as time passes.

Number of half-lives (n)	Time passed in minutes (T)	Number of radioactive atoms (R)
1	110	256
2	220	128
3	330	64

Question 3c (4 marks)

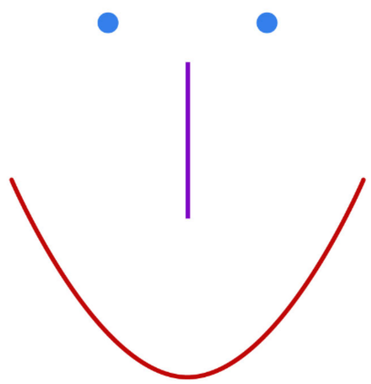
Calculate the time passed, T, when there is just 1 radioactive atom remaining.

Styles

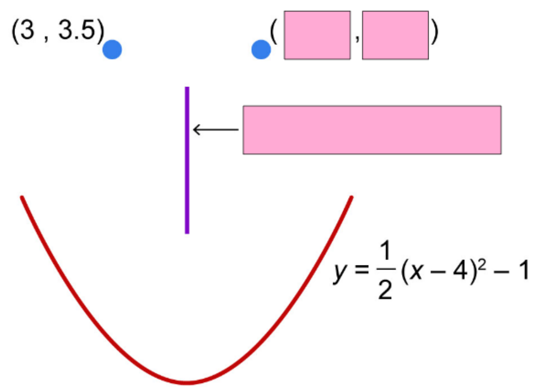
Question 4 (9 marks)

Question 4a (2 marks)

A **symmetrical** smiley face is plotted using two points, a line, and a parabola.



Determine the missing coordinates and the equation of the line.



Question 4b (1 mark)

The domain of the parabola is $a \leq x \leq 6$

Determine the value of *a*.

Question 4c (2 marks)

By expanding the brackets, **show that** $y = \frac{1}{2}(x - 4)^2 - 1$ can be written as

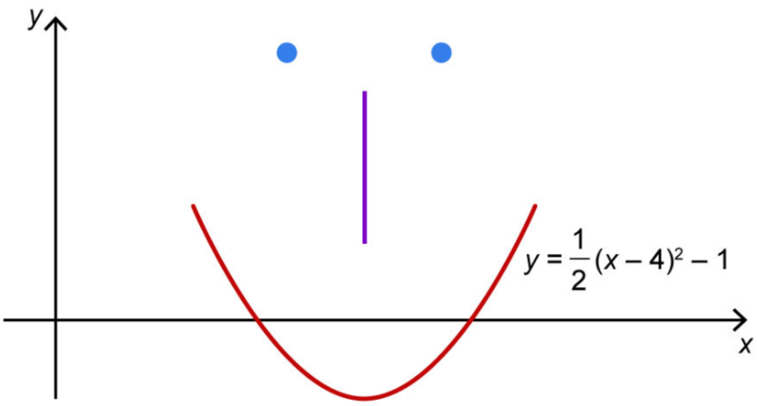
$y = \frac{1}{2}x^2 - 4x + 7$.

Question 4d (4 marks)

Hence or otherwise, **calculate** the coordinates of the *x*-intercepts of the parabola

$y = \frac{1}{2}(x - 4)^2 - 1.$

Give your answer to the nearest one decimal place.



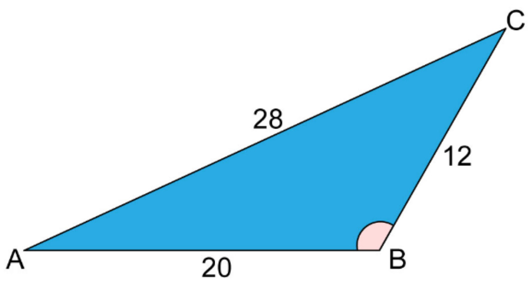
B I U Styles

Question 5 (10 marks)

Question 5a (3 marks)

The diagram shows the triangle ABC.

Diagram not to scale



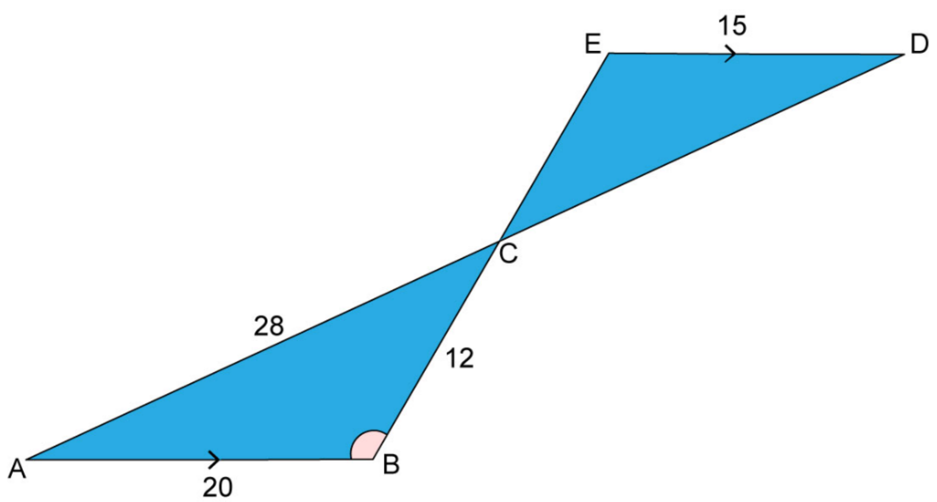
Find the size of the angle ABC.

Question 5b (2 marks)

AC and BC are extended to form the triangle DEC such that:

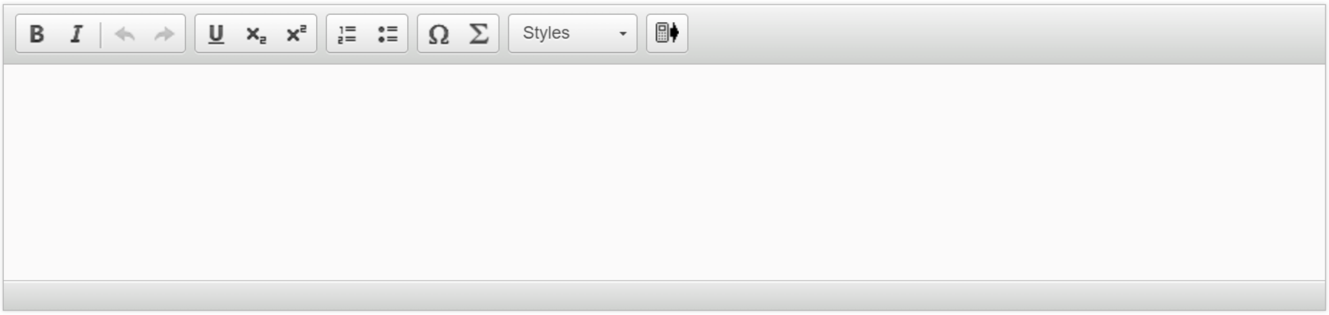
- AB is parallel to ED
- ABC and DEC are similar triangles.

Diagram not to scale



A 20 B

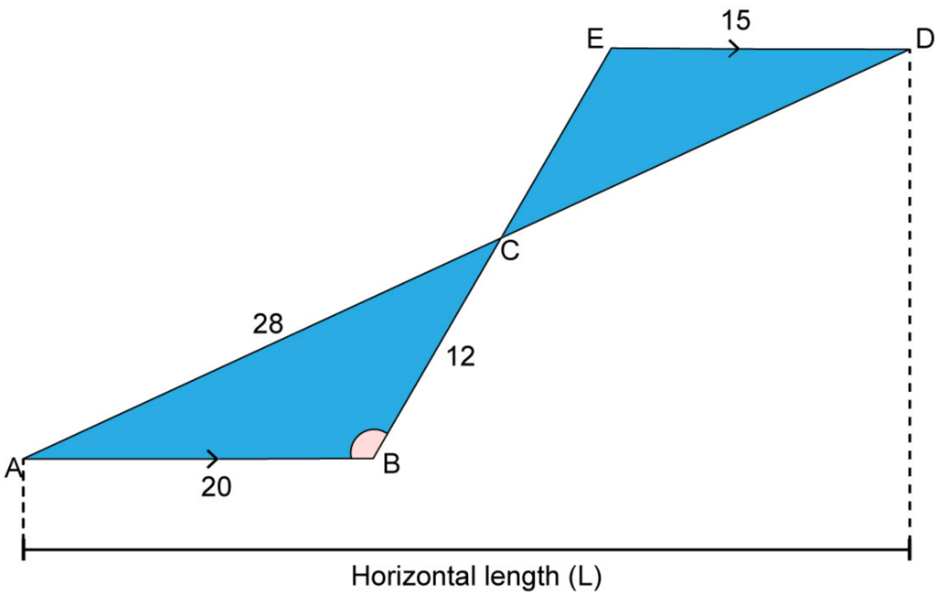
Determine EC.



Question 5c (5 marks)

Find the value of L.

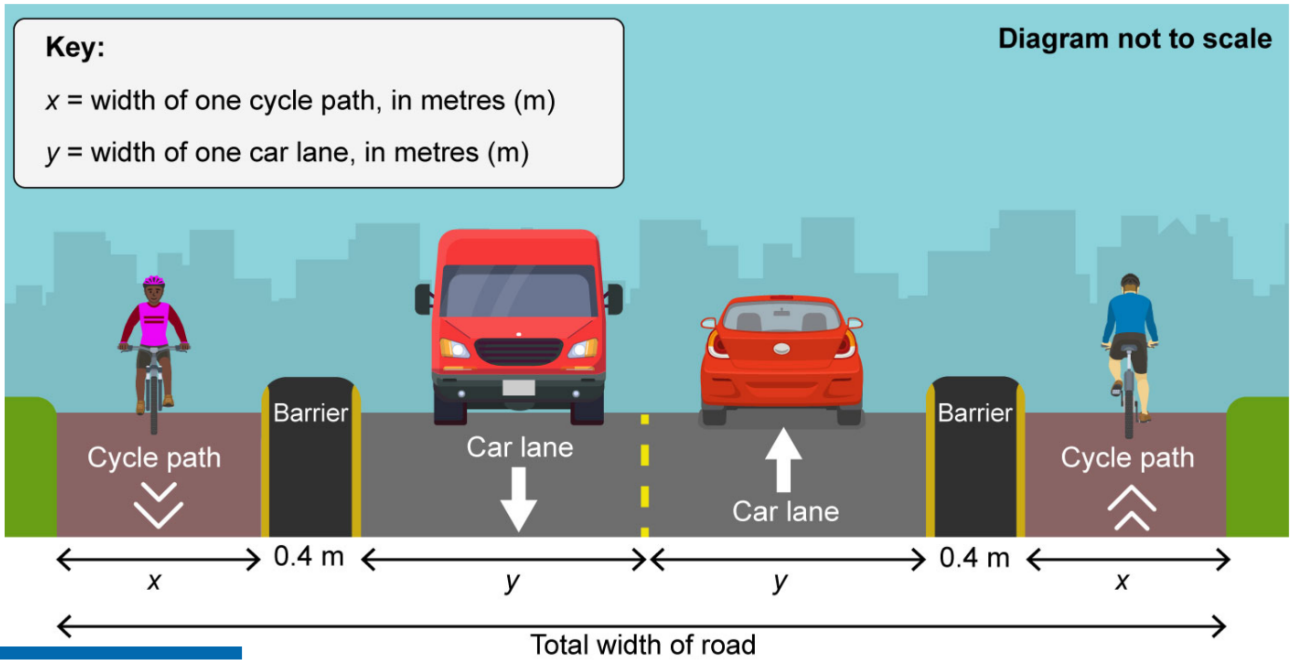
Diagram not to scale



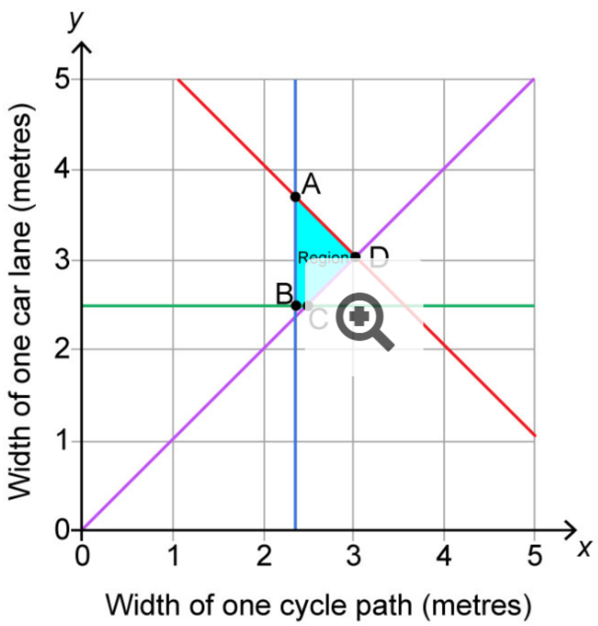
Scroll down to continue

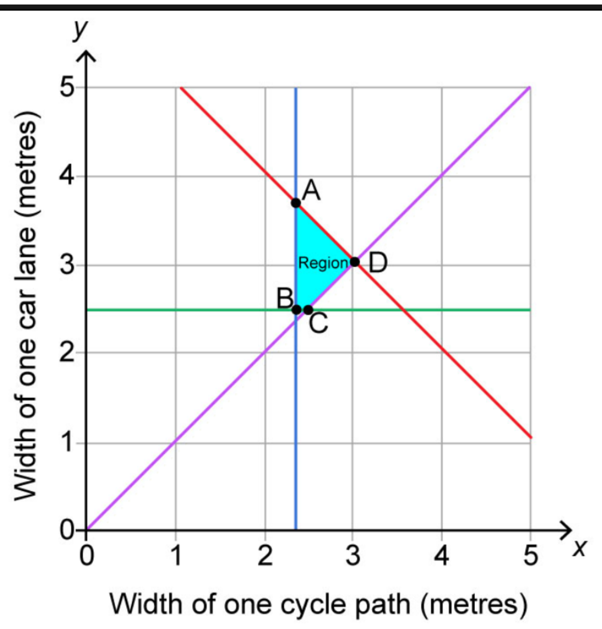
Question 6 (9 marks)

The city council has decided to improve road safety for cyclists. The city council will design a road with separated cycle paths.



The widths of the cycle path and the car lanes are represented by the shaded region in the graph below.




y
x
Width of one car lane (metres)
Width of one cycle path (metres)
A
B
C
D
Region
0
1
2
3
4
5

Question 6a (4 marks)

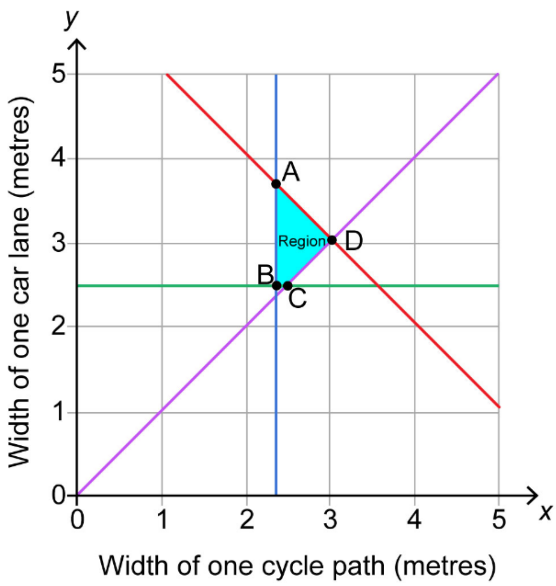
Using the information provided in the graph:

- **Identify** the shaded region by completing the inequalities below.
- There are four constraints. The first constraint is given. **State** the other three constraints in the spaces provided.

Draggable inequalities	Inequalities	Constraints in context
≥	$2(x + y + 0.4) \leq 12.8$	The total width of the road cannot exceed 12.8 m
>	x ☐ 2.3	The width of one cycle path is ☐
≤	y ☐ 2.5	The width of one car lane is ☐
<	y ☐ x	☐

Question 6b (2 marks)

The table below shows the coordinates of the vertices A, B, C and D.



Identify the vertices that maximize the total width of the road.

Question 6c (3 marks)

The plan is to construct a **5 km long** road with separated cycle paths.The cost to construct each part of the road is listed in the table below.

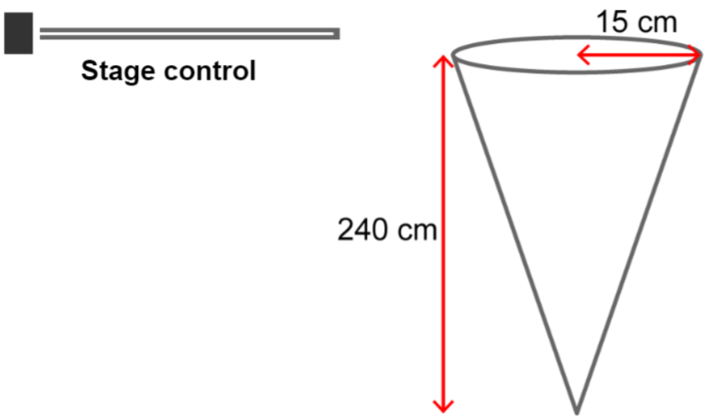
Item	Cost (Correct to one decimal place)
All cycle paths and barriers	$1.2 million per kilometre (km)
All car lanes	$3.1 million per kilometre (km)

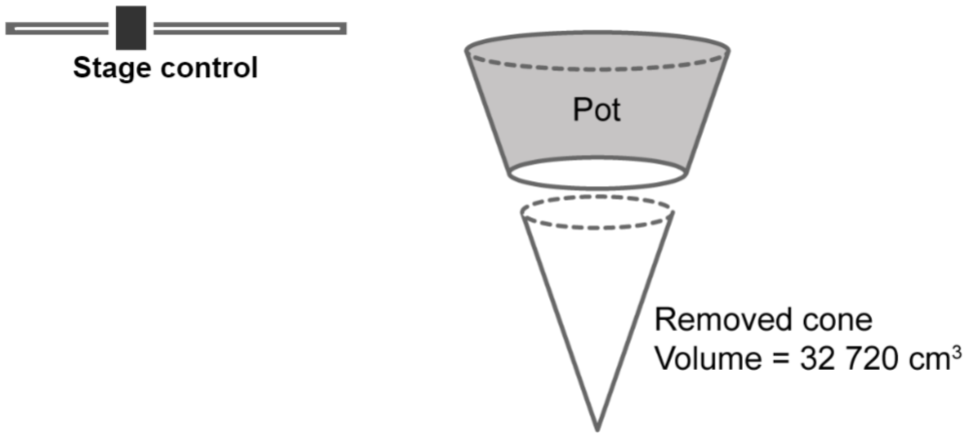
Calculate the upper bound for the total cost to construct the road.

Styles

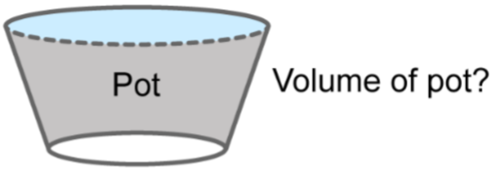
Question 7 (22 marks)

In this question you will make calculations to help a community design a rooftop garden to grow tomato plants.
Tomato plants are grown in a pot. A pot is a truncated cone. Interact with the simulation to reveal how a pot is formed.




Stage control
Pot
Removed cone
Volume = 32 720 cm³


Stage control


Pot
Volume of pot?

Question 7a (4 marks)

Calculate the volume of the pot. Give your answer to the nearest 10 cm^3.

Cost of items	
1 tomato plant	$6.00
1 Litre (L) soil	$0.40
1 empty pot	$1.50

$1\ cm^3 = 1\ mL$

Question 7b (2 marks)

Pots will be filled with soil up to 90 % of maximum capacity. **Determine** the volume in litres (L) of soil needed for one pot.

Styles

Cost of items	
1 tomato plant	$6.00
1 Litre (L) soil	$0.40
1 empty pot	$1.50

$1\ cm^3 = 1\ mL$

Question 7c (2 marks)

Determine the total cost to grow one tomato plant in a pot.

Video Script

This community will grow their tomato plants in pots in a rooftop garden.

Each plant should have space around the pot for the plant to grow and for people to walk in between.

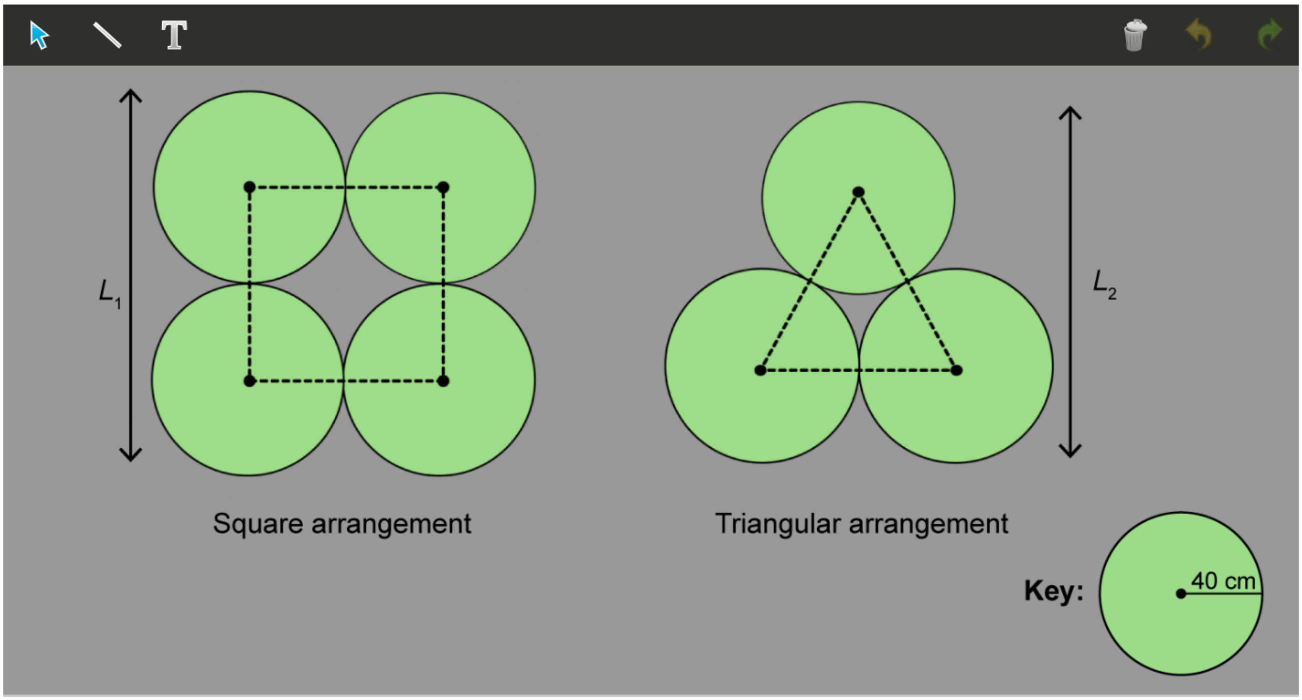
This circular shape represents the space required for each plant.

The circular shapes can be arranged in different ways to make best use of the space available in the garden.

Here is a square arrangement and here is a triangular arrangement.

In the following questions, you will make calculations to make the best use of the space available.

Question 7d (1 mark)



Question 7d (1 mark)

Determine the length of L_1 for the square arrangement.

Question 7e (3 marks)

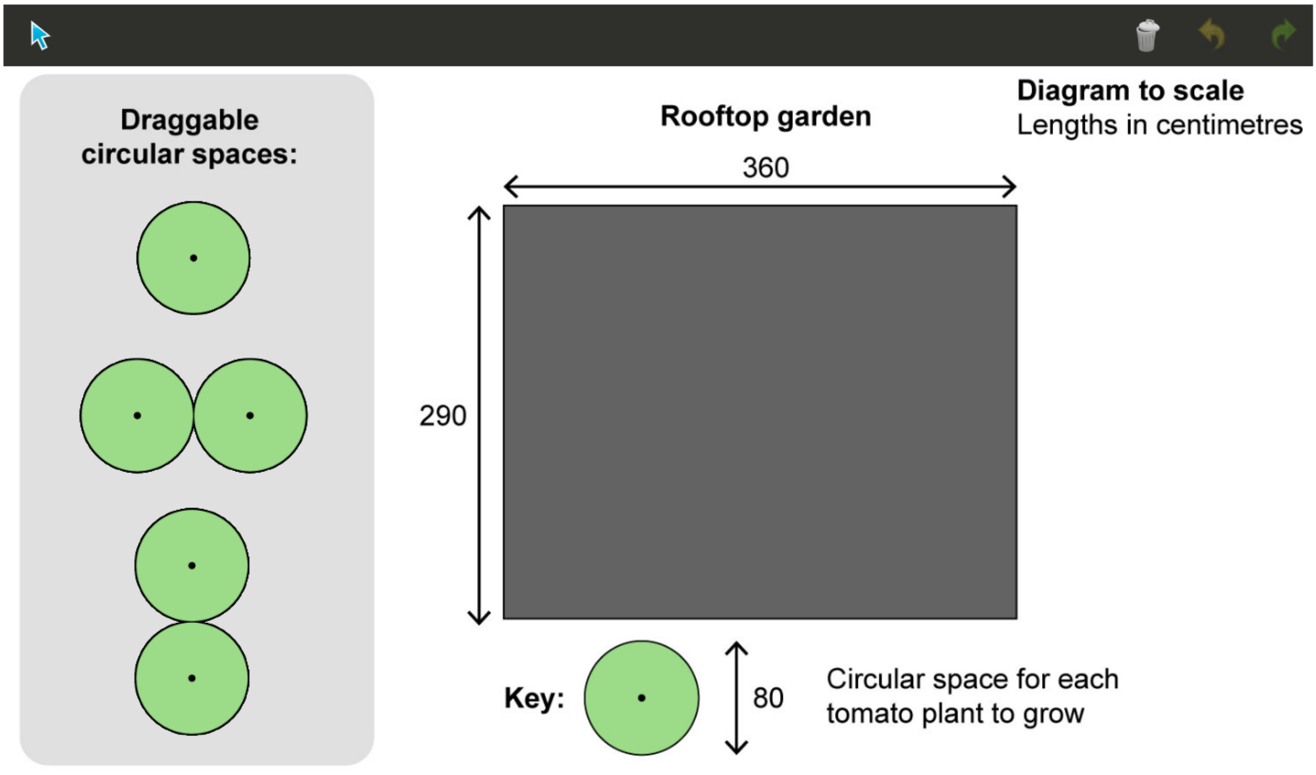
Find the length of L_2 for the triangular arrangement.

Question 7f (10 marks)

Design a rooftop garden to grow the tomato plants. Your design should make best use of all the rooftop garden space to grow as many tomato plants as possible.

In your answer you should:

- identify **three** relevant factors for your design
- show calculations for your design
- show your design on the canvas
- calculate the cost to a sensible degree of accuracy (using your result in part (c))
- justify whether your results make sense in the context of the question.


Draggable
circular spaces:
Rooftop garden
Diagram to scale
Lengths in centimetres
360
290
Key:
80
Circular space for each
tomato plant to grow

Relevant factors:

Calculations and justification:

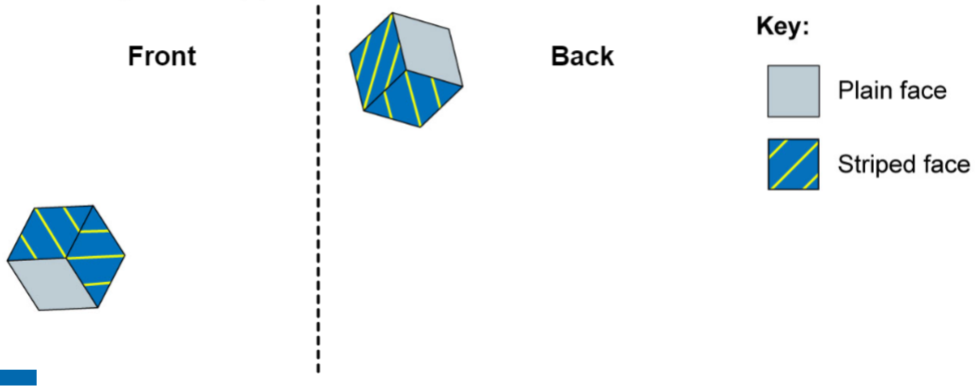
Question 8 (31 marks)

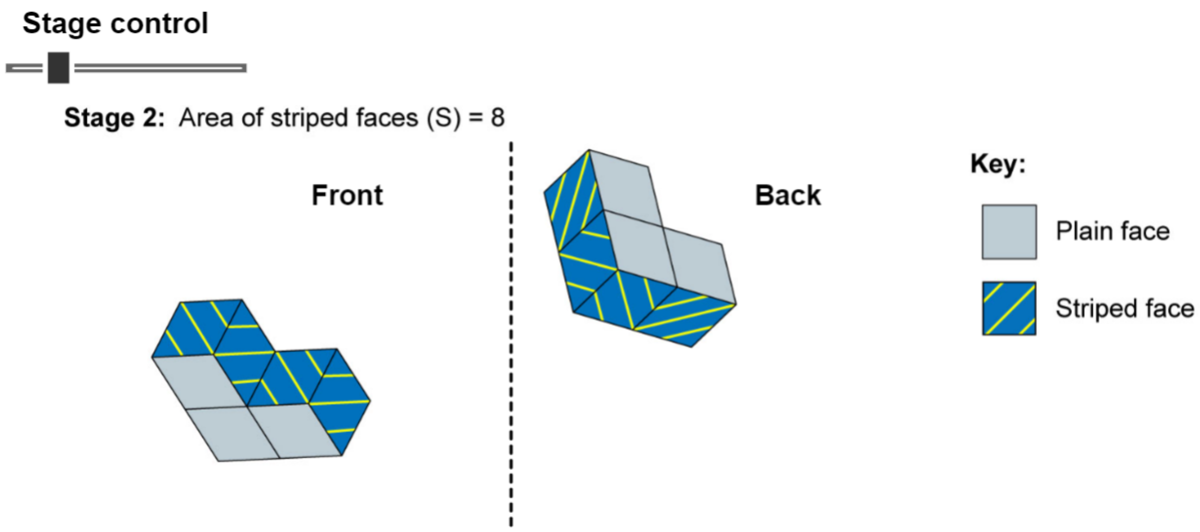
A sequence of three-dimensional (3D) shapes are created using cube blocks. In this question, you will investigate the area of faces in 3D shapes.

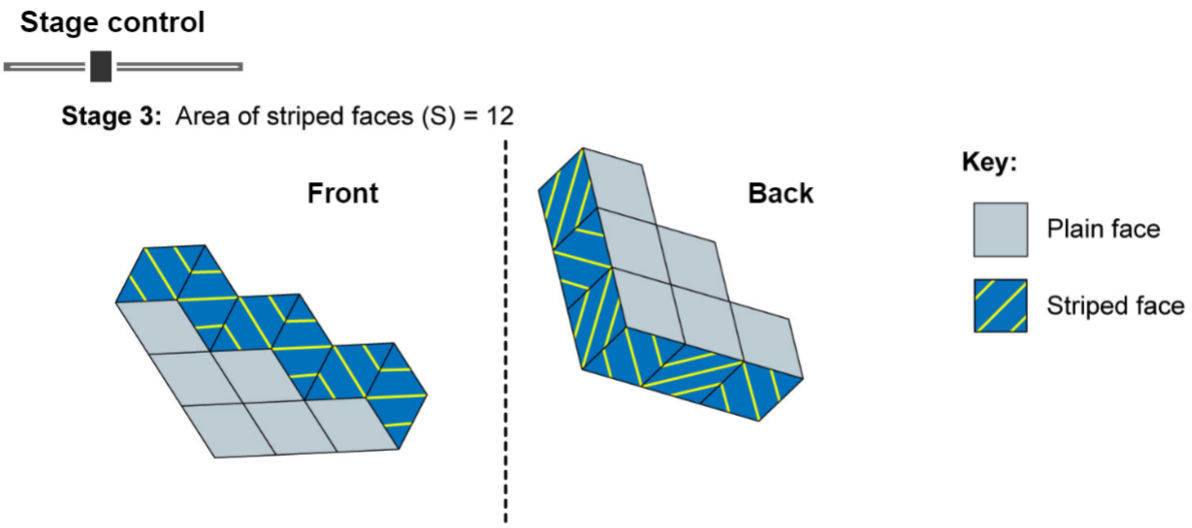
In the simulation below, some faces on the 3D shapes are striped and some are plain.
Interact with the stage control to see how the area of striped faces (S) increases.
All cubes are of side = 1 unit.

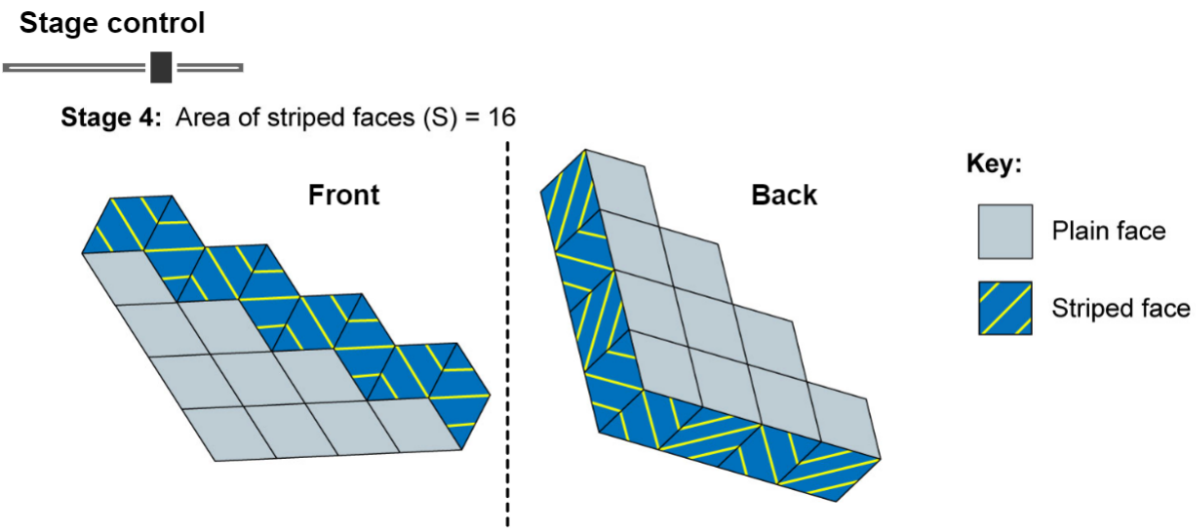
Stage control

Stage 1: Area of striped faces (S) = 4




Stage control
Stage 2: Area of striped faces (S) = 8
Front
Back
Key:
Plain face
Striped face


Stage control
Stage 3: Area of striped faces (S) = 12
Front
Back
Key:
Plain face
Striped face


Stage control
Stage 4: Area of striped faces (S) = 16
Front
Back
Key:
Plain face
Striped face

Question 8a (1 mark)

Write down the missing values in the table up to row 6.

Stage number (n)	Area of striped faces (S)
1	4
2	8
3	12
4	16
5	
6	

Reset

Question 8b (2 marks)

Describe, in words, two patterns in the table for the area of striped faces (S).

Styles

Question 8a (1 mark)

Write down the missing values in the table up to row 6.

Stage number (n)	Area of striped faces (S)
1	4
2	8
3	12
4	16
5	
6	

Reset

Question 8c (2 marks)

Write down, in simplest form, a general rule for S in terms of n.

Styles

Question 8a (1 mark)

Write down the missing values in the table up to row 6.

Stage number (n)	Area of striped faces (S)
1	4
2	8
3	12
4	16
5	
6	

Reset

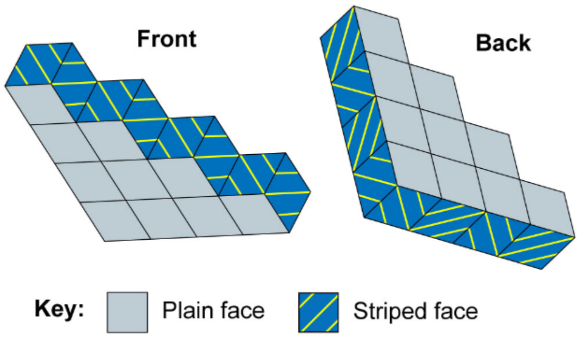
Question 8d (3 marks)

Verify your general rule for S.

B I U x_2 x^2 Ω Σ
Styles

Question 8e (1 mark)

The area of plain faces at the front and at the back of the 3D shapes is *P*.

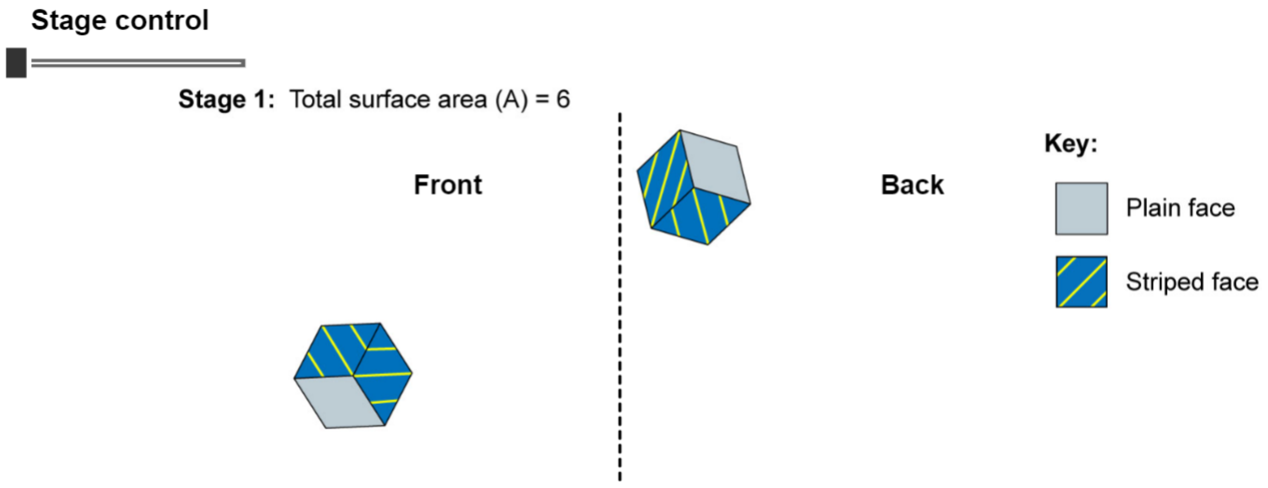


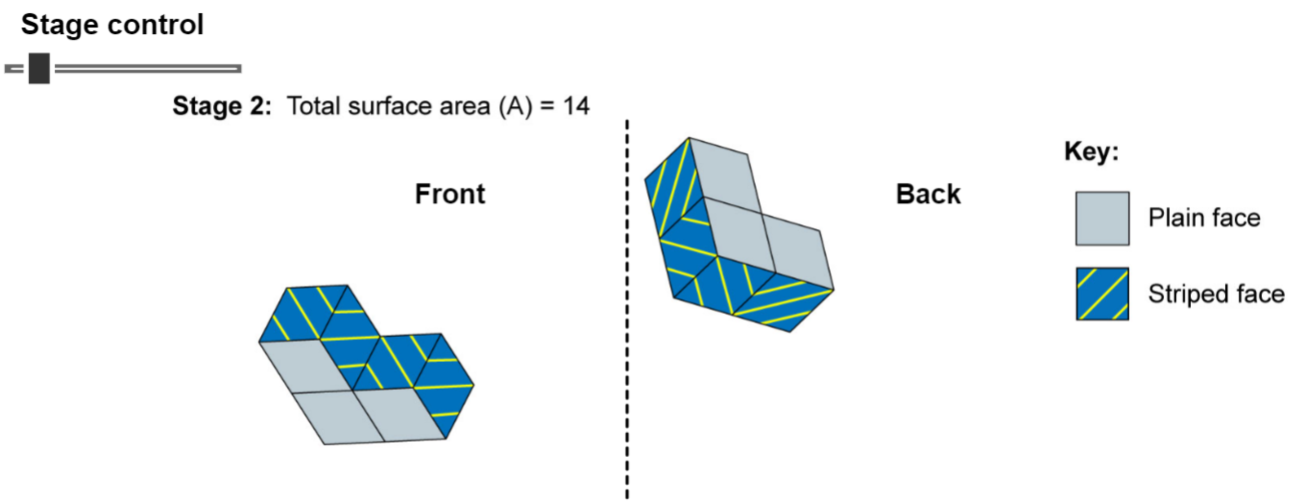
Show that *P* in stage 4 is 20.

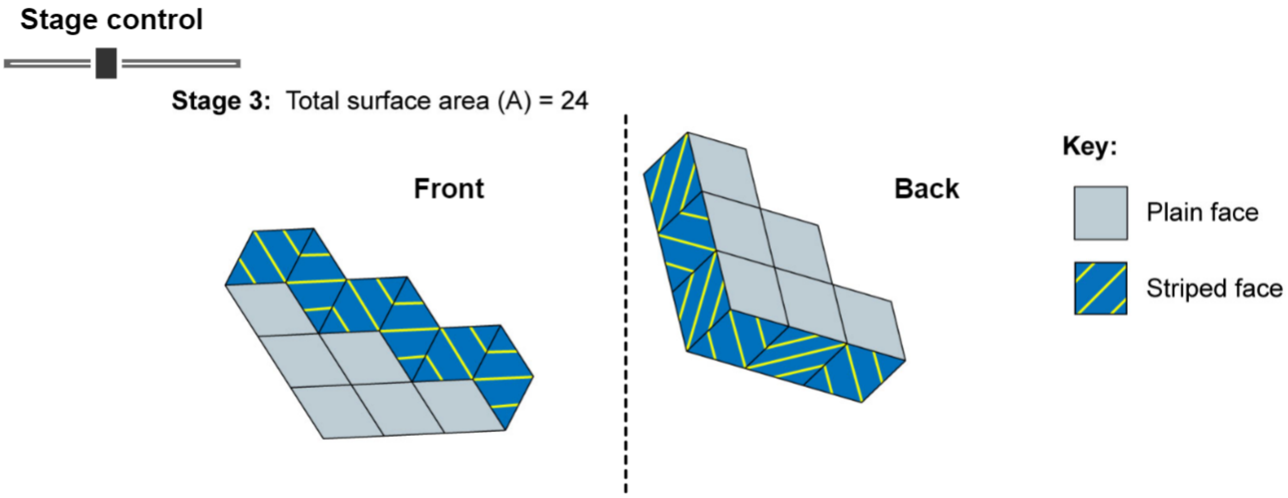
Question 8f (22 marks)

You will now investigate the total surface area (*A*) of the 3D shapes.

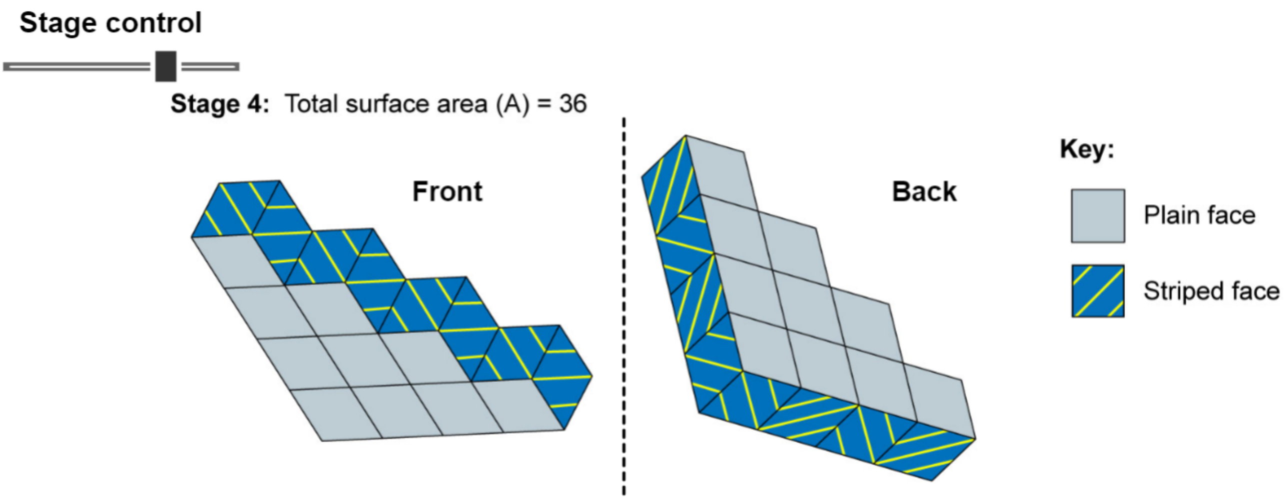
Interact with the stage control to see how *A* increases.




Stage control
Stage 2: Total surface area (A) = 14
Front
Back
Key:
Plain face
Striped face



Stage control
Stage 3: Total surface area (A) = 24
Front
Back
Key:
Plain face
Striped face


Stage control
Stage 4: Total surface area (A) = 36
Front
Back
Key:
Plain face
Striped face

Stage number (*n*)	Area of striped faces (*S*)	Area of plain faces (*P*)	Total surface area (*A*)	
1	4	2	6	
2	8	6	14	
3	12	12	24	
4	16	20	36	
5				
6				

Investigate the values in the table to find a relationship for the total surface area (*A*) in terms of *n*. In your answer, you should communicate the following in an organized and coherent manner:

- predict more values and record these in the table
- describe in words a pattern in the table for total surface area (*A*)
- write down, in simplest form, a general rule for *A* in terms of *n*
- test and verify your general rule for *A*
- justify your general rule for *A*.

Styles