Diploma Programme
Programme du diplôme
Programa del Diploma

# Mathematics: analysis and approaches
# Standard level
# Paper 2

Friday 7 May 2021 (morning)

Candidate session number

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1 hour 30 minutes

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**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.





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Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

## Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

**1.** [Maximum mark: 6]

(a) Find $\int (6x+7)\mathrm{d}x$. [3]

(b) Given $f'(x) = 6x + 7$ and $f(1.2) = 7.32$, find $f(x)$. [3]

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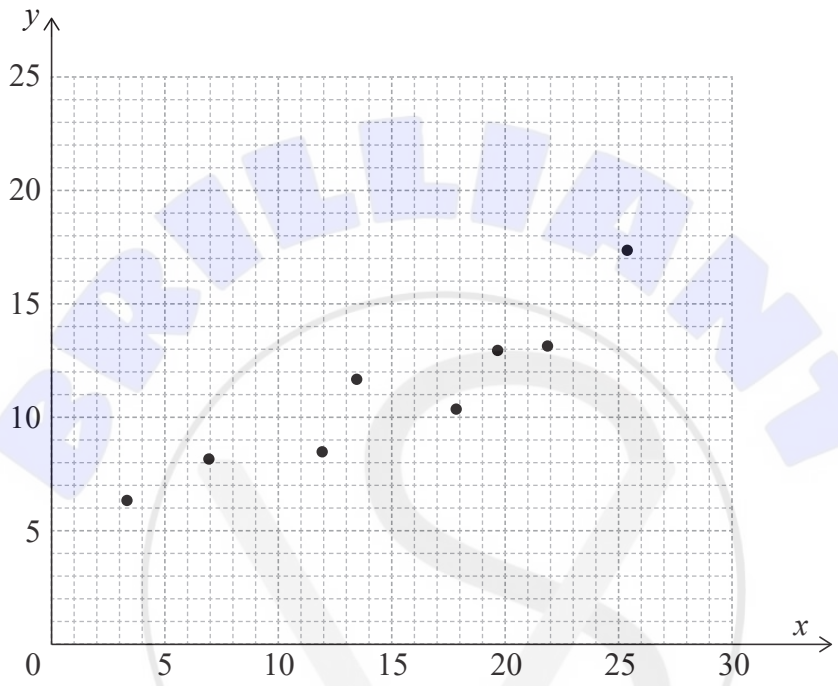
**2.** [Maximum mark: 7]

The following table shows the data collected from an experiment.

| $x$ | 3.3 | 6.9 | 11.9 | 13.4 | 17.8 | 19.6 | 21.8 | 25.3 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 6.3 | 8.1 | 8.4 | 11.6 | 10.3 | 12.9 | 13.1 | 17.3 |

The data is also represented on the following scatter diagram.




The relationship between $x$ and $y$ can be modelled by the regression line of $y$ on $x$ with equation $y = ax + b$, where $a, b \in \mathbb{R}$.

(a) Write down the value of $a$ and the value of $b$. [2]

(b) Use this model to predict the value of $y$ when $x = 18$. [2]

(c) Write down the value of $\overline{x}$ and the value of $\overline{y}$. [1]

(d) Draw the line of best fit on the scatter diagram. [2]

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**(Question 2 continued)**

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**3.** [Maximum mark: 6]

A company produces bags of sugar whose masses, in grams, can be modelled by a normal distribution with mean $1000$ and standard deviation $3.5$. A bag of sugar is rejected for sale if its mass is less than $995$ grams.

(a) Find the probability that a bag selected at random is rejected. [2]

(b) Estimate the number of bags which will be rejected from a random sample of $100$ bags. [1]

(c) Given that a bag is not rejected, find the probability that it has a mass greater than $1005$ grams. [3]

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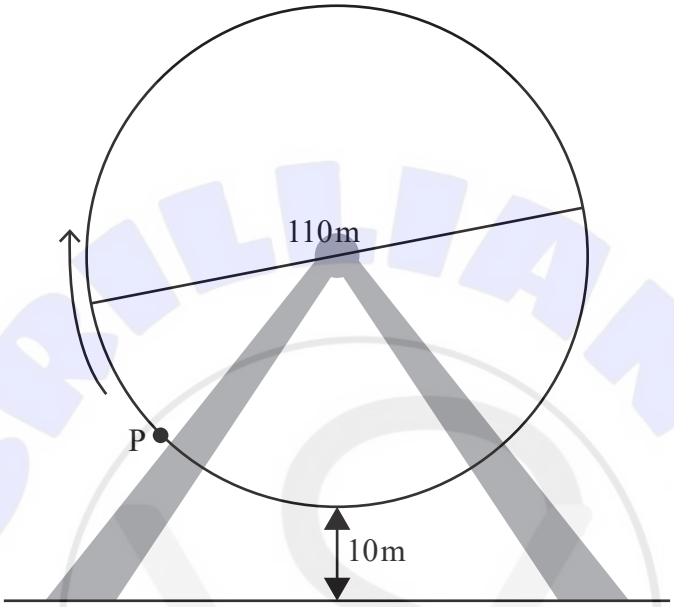
**4.** [Maximum mark: 5]

A Ferris wheel with diameter $110$ metres rotates at a constant speed. The lowest point on the wheel is $10$ metres above the ground, as shown on the following diagram. P is a point on the wheel. The wheel starts moving with P at the lowest point and completes one revolution in $20$ minutes.

**diagram not to scale**




The height, $h$ metres, of P above the ground after $t$ minutes is given by $h(t) = a\cos(bt) + c$, where $a$, $b$, $c \in \mathbb{R}$.

Find the values of $a$, $b$ and $c$.

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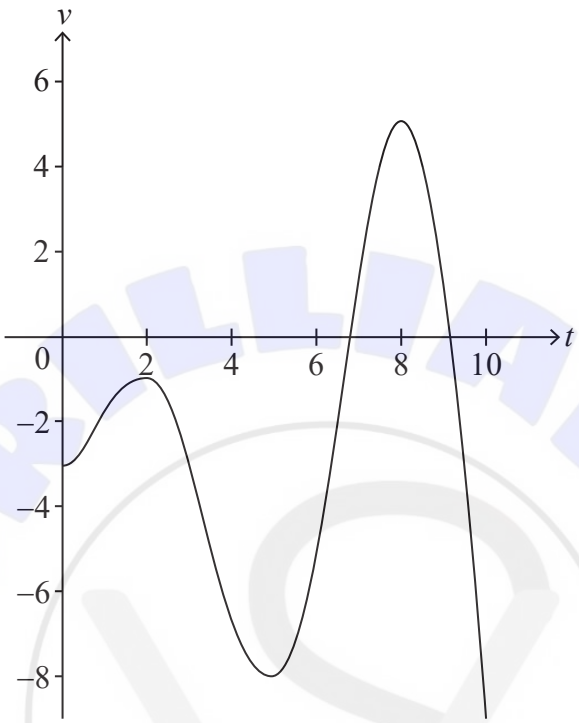
**5.** [Maximum mark: 6]

A particle moves in a straight line. The velocity, $v\,\text{ms}^{-1}$, of the particle at time $t$ seconds is given by $v(t) = t\sin t - 3$, for $0 \le t \le 10$.

The following diagram shows the graph of $v$.




(a) Find the smallest value of $t$ for which the particle is at rest. [2]

(b) Find the total distance travelled by the particle. [2]

(c) Find the acceleration of the particle when $t = 7$. [2]

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**6.** [Maximum mark: 5]

Consider the expansion of $(3 + x^2)^{n+1}$, where $n \in \mathbb{Z}^+$.

Given that the coefficient of $x^4$ is $20\,412$, find the value of $n$.

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will not be marked.

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## Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

**7.** [Maximum mark: 16]

Two friends Amelia and Bill, each set themselves a target of saving $\$20\,000$. They each have $\$9000$ to invest.

(a) Amelia invests her $\$9000$ in an account that offers an interest rate of $7\%$ per annum compounded **annually**.

(i) Find the value of Amelia's investment after $5$ years to the nearest hundred dollars.

(ii) Determine the number of years required for Amelia's investment to reach the target. [5]

(b) Bill invests his $\$9000$ in an account that offers an interest rate of $r\%$ per annum compounded **monthly**, where $r$ is set to two decimal places.

Find the minimum value of $r$ needed for Bill to reach the target after $10$ years. [3]

(c) A third friend Chris also wants to reach the $\$20\,000$ target. He puts his money in a safe where he does not earn any interest. His system is to add more money to this safe each year. Each year he will add half the amount added in the previous year.

(i) Show that Chris will never reach the target if his initial deposit is $\$9000$.

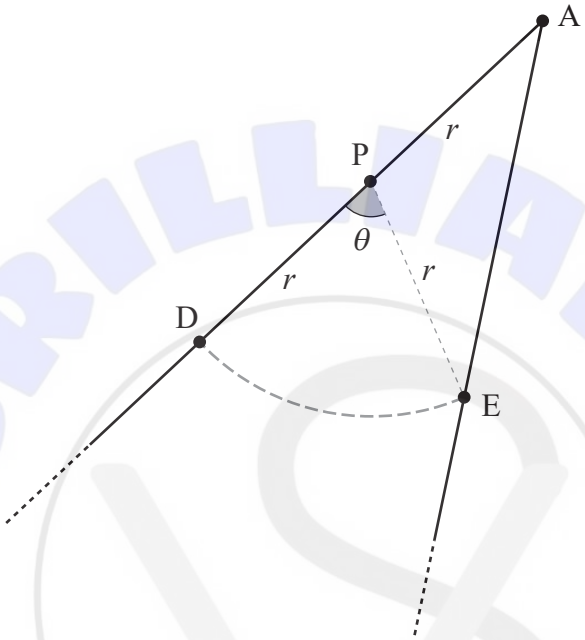
(ii) Find the amount Chris needs to deposit initially in order to reach the target after $5$ years. Give your answer to the nearest dollar. [8]

Do **not** write solutions on this page.

**8.** [Maximum mark: 14]

Two straight fences meet at point $\mathrm{A}$ and a field lies between them.

A horse is tied to a post, $\mathrm{P}$, by a rope of length $r$ metres. Point $\mathrm{D}$ is on one fence and point $\mathrm{E}$ is on the other, such that $\mathrm{PD} = \mathrm{PE} = \mathrm{PA} = r$ and $\mathrm{D\hat{P}E} = \theta$ radians. This is shown in the following diagram.




The length of the arc $\mathrm{DE}$ shown in the diagram is $28\,\mathrm{m}$.

(a) Write down an expression for $r$ in terms of $\theta$. [1]

(b) Show that the area of the field that the horse can reach is $\frac{392}{\theta^2}(\theta + \sin\theta)$. [4]

(c) The area of field that the horse can reach is $460\,\mathrm{m}^2$. Find the value of $\theta$. [2]

(d) Hence, find the size of $\mathrm{D\hat{A}E}$. [2]

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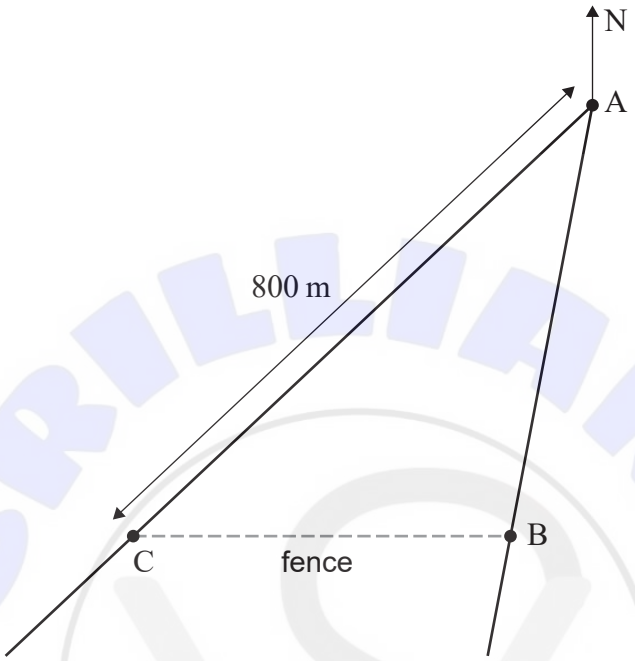
**(Question 8 continued)**

A new fence is to be constructed between points $B$ and $C$ which will enclose the field, as shown in the following diagram.




Point $C$ is due west of $B$ and $AC = 800\,m$. The bearing of $B$ from $A$ is $195°$.

(e) (i) Find the size of $A\hat{B}C$.

(ii) Find the length of new fence required. [5]

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**9.** [Maximum mark: 15]

Consider the function $f$ defined by $f(x) = 90\,e^{-0.5x}$ for $x \in \mathbb{R}^+$.

The graph of $f$ and the line $y = x$ intersect at point P.

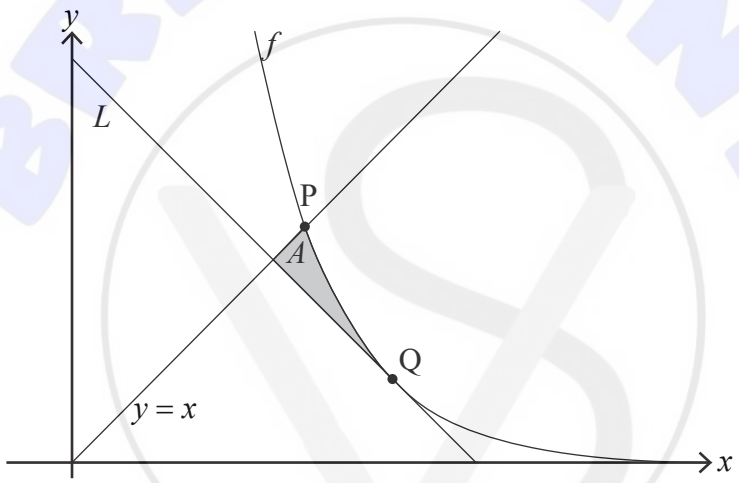
(a) Find the $x$-coordinate of P. [2]

The line $L$ has a gradient of $-1$ and is a tangent to the graph of $f$ at the point Q.

(b) Find the exact coordinates of Q. [4]

(c) Show that the equation of $L$ is $y = -x + 2\ln 45 + 2$. [2]

The shaded region $A$ is enclosed by the graph of $f$ and the lines $y = x$ and $L$.




(d) (i) Find the $x$-coordinate of the point where $L$ intersects the line $y = x$.

(ii) Hence, find the area of $A$. [5]

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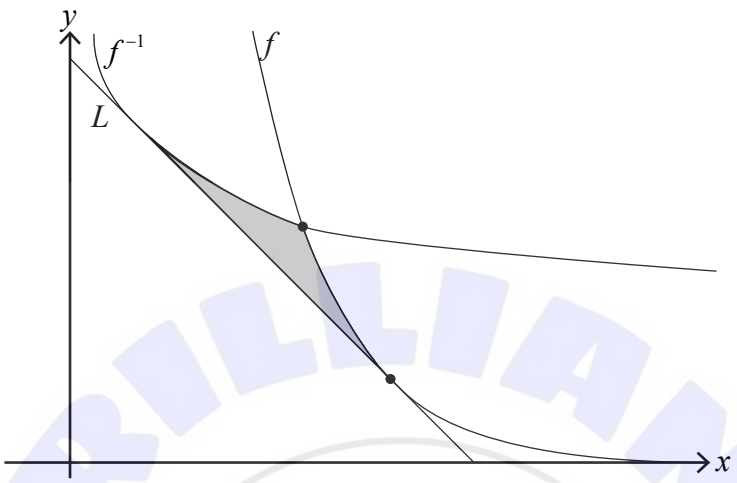
**(Question 9 continued)**

The line $L$ is tangent to the graphs of both $f$ and the inverse function $f^{-1}$.




(e) Find the shaded area enclosed by the graphs of $f$ and $f^{-1}$ and the line $L$. [2]



**References:**

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