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**Mathematics: analysis and approaches**  
**Higher level**  
**Paper 1**

Friday 6 May 2022 (afternoon)

Candidate session number

2 hours

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**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.





















Do **not** write solutions on this page.

### Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 18]

Consider the series  $\ln x + p \ln x + \frac{1}{3} \ln x + \dots$ , where  $x \in \mathbb{R}, x > 1$  and  $p \in \mathbb{R}, p \neq 0$ .

(a) Consider the case where the series is geometric.

(i) Show that  $p = \pm \frac{1}{\sqrt{3}}$ .

(ii) Hence or otherwise, show that the series is convergent.

(iii) Given that  $p > 0$  and  $S_{\infty} = 3 + \sqrt{3}$ , find the value of  $x$ . [6]

(b) Now consider the case where the series is arithmetic with common difference  $d$ .

(i) Show that  $p = \frac{2}{3}$ .

(ii) Write down  $d$  in the form  $k \ln x$ , where  $k \in \mathbb{Q}$ .

(iii) The sum of the first  $n$  terms of the series is  $\ln\left(\frac{1}{x^3}\right)$ .  
Find the value of  $n$ . [12]

11. [Maximum mark: 15]

Consider the three planes

$$\Pi_1 : 2x - y + z = 4$$

$$\Pi_2 : x - 2y + 3z = 5$$

$$\Pi_3 : -9x + 3y - 2z = 32$$

(a) Show that the three planes do not intersect. [4]

(b) (i) Verify that the point  $P(1, -2, 0)$  lies on both  $\Pi_1$  and  $\Pi_2$ .

(ii) Find a vector equation of  $L$ , the line of intersection of  $\Pi_1$  and  $\Pi_2$ . [5]

(c) Find the distance between  $L$  and  $\Pi_3$ . [6]



Do **not** write solutions on this page.

**12.** [Maximum mark: 21]

The function  $f$  is defined by  $f(x) = e^x \sin x$ , where  $x \in \mathbb{R}$ .

(a) Find the Maclaurin series for  $f(x)$  up to and including the  $x^3$  term. [4]

(b) Hence, find an approximate value for  $\int_0^1 e^{x^2} \sin(x^2) dx$ . [4]

The function  $g$  is defined by  $g(x) = e^x \cos x$ , where  $x \in \mathbb{R}$ .

(c) (i) Show that  $g(x)$  satisfies the equation  $g''(x) = 2(g'(x) - g(x))$ .

(ii) Hence, deduce that  $g^{(4)}(x) = 2(g'''(x) - g''(x))$ . [5]

(d) Using the result from part (c), find the Maclaurin series for  $g(x)$  up to and including the  $x^4$  term. [5]

(e) Hence, or otherwise, determine the value of  $\lim_{x \rightarrow 0} \frac{e^x \cos x - 1 - x}{x^3}$ . [3]

**References:**

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