

© International Baccalaureate Organization 2023

All rights reserved. No part of this product may be reproduced in any form or by any electronic or mechanical means, including information storage and retrieval systems, without the prior written permission from the IB. Additionally, the license tied with this product prohibits use of any selected files or extracts from this product. Use by third parties, including but not limited to publishers, private teachers, tutoring or study services, preparatory schools, vendors operating curriculum mapping services or teacher resource digital platforms and app developers, whether fee-covered or not, is prohibited and is a criminal offense.

More information on how to request written permission in the form of a license can be obtained from <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

© Organisation du Baccalauréat International 2023

Tous droits réservés. Aucune partie de ce produit ne peut être reproduite sous quelque forme ni par quelque moyen que ce soit, électronique ou mécanique, y compris des systèmes de stockage et de récupération d'informations, sans l'autorisation écrite préalable de l'IB. De plus, la licence associée à ce produit interdit toute utilisation de tout fichier ou extrait sélectionné dans ce produit. L'utilisation par des tiers, y compris, sans toutefois s'y limiter, des éditeurs, des professeurs particuliers, des services de tutorat ou d'aide aux études, des établissements de préparation à l'enseignement supérieur, des fournisseurs de services de planification des programmes d'études, des gestionnaires de plateformes pédagogiques en ligne, et des développeurs d'applications, moyennant paiement ou non, est interdite et constitue une infraction pénale.

Pour plus d'informations sur la procédure à suivre pour obtenir une autorisation écrite sous la forme d'une licence, rendez-vous à l'adresse <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

© Organización del Bachillerato Internacional, 2023

Todos los derechos reservados. No se podrá reproducir ninguna parte de este producto de ninguna forma ni por ningún medio electrónico o mecánico, incluidos los sistemas de almacenamiento y recuperación de información, sin la previa autorización por escrito del IB. Además, la licencia vinculada a este producto prohíbe el uso de todo archivo o fragmento seleccionado de este producto. El uso por parte de terceros —lo que incluye, a título enunciativo, editoriales, profesores particulares, servicios de apoyo académico o ayuda para el estudio, colegios preparatorios, desarrolladores de aplicaciones y entidades que presten servicios de planificación curricular u ofrezcan recursos para docentes mediante plataformas digitales—, ya sea incluido en tasas o no, está prohibido y constituye un delito.

En este enlace encontrará más información sobre cómo solicitar una autorización por escrito en forma de licencia: <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.



Mathematics: analysis and approaches

Higher level

Paper 1

8 May 2023

Zone A afternoon | Zone B morning | Zone C afternoon

Candidate session number

2 hours

--	--	--	--	--	--	--	--	--	--

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

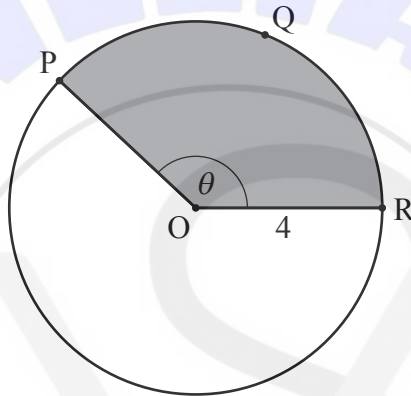
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius 4 cm.

diagram not to scale



The points P , Q and R lie on the circumference of the circle and $\widehat{POR} = \theta$, where θ is measured in radians.

The length of arc PQR is 10 cm.

- (a) Find the perimeter of the shaded sector. [2]
- (b) Find θ . [2]
- (c) Find the area of the shaded sector. [2]

(This question continues on the following page)



2. [Maximum mark: 5]

A function f is defined by $f(x) = 1 - \frac{1}{x-2}$, where $x \in \mathbb{R}$, $x \neq 2$.

(a) The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

Write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

(b) Find the coordinates of the point where the graph of $y = f(x)$ intersects

(i) the y -axis;

(ii) the x -axis. [2]



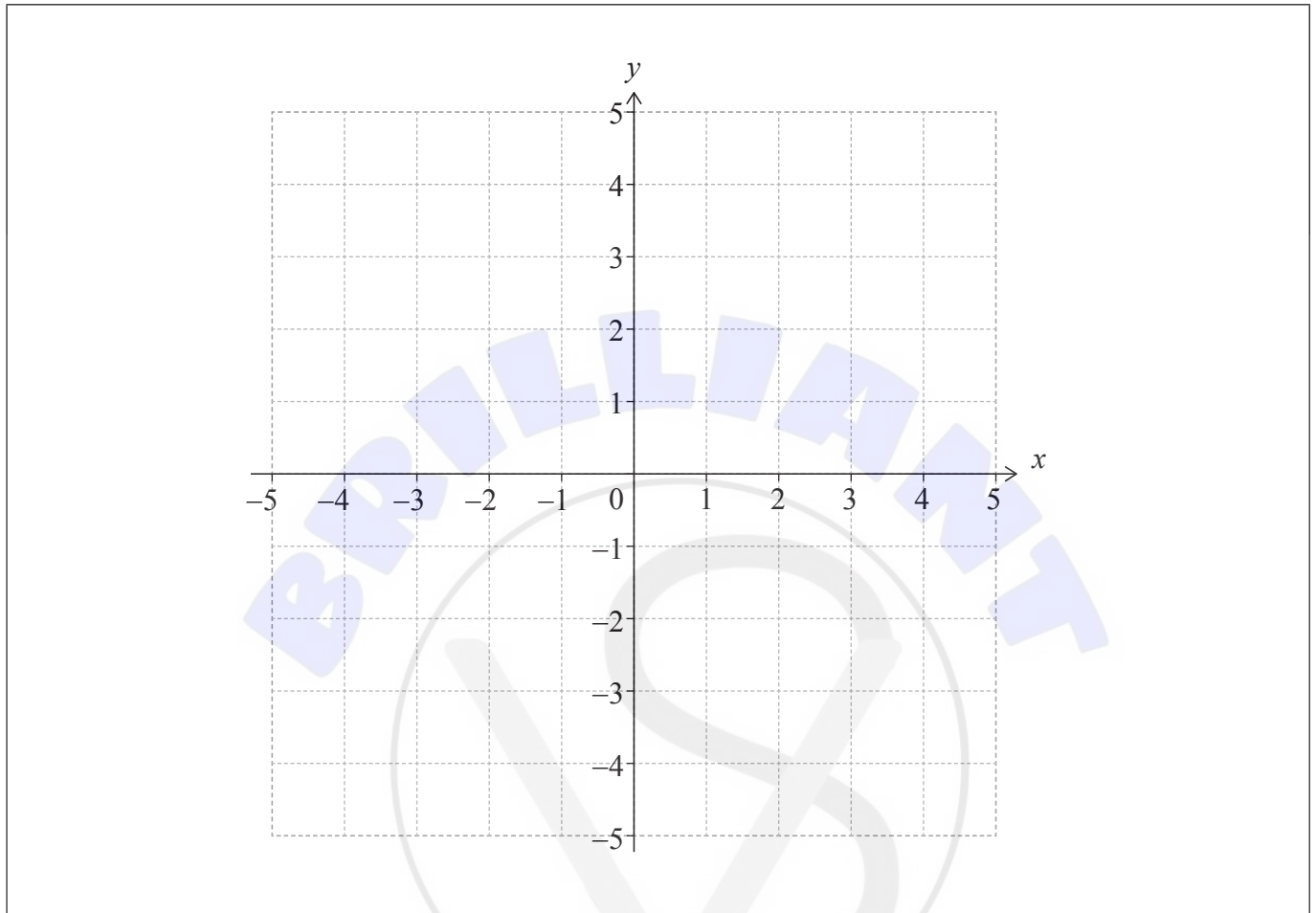
(This question continues on the following page)



(Question 2 continued)

- (c) On the following set of axes, sketch the graph of $y = f(x)$, showing all the features found in parts (a) and (b).

[1]



Do **not** write solutions on this page.

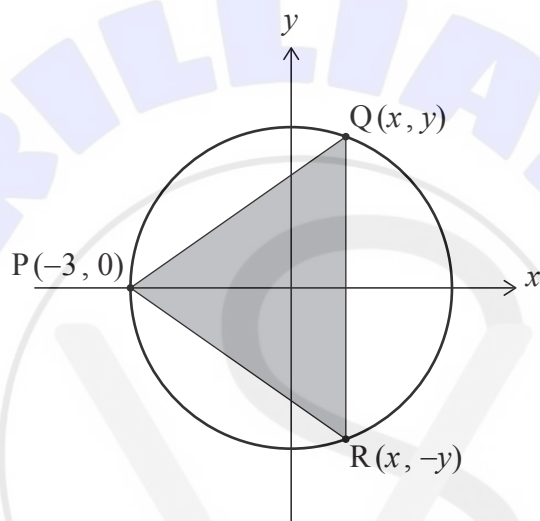
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 14]

A circle with equation $x^2 + y^2 = 9$ has centre $(0, 0)$ and radius 3.

A triangle, PQR, is inscribed in the circle with its vertices at $P(-3, 0)$, $Q(x, y)$ and $R(x, -y)$, where Q and R are variable points in the first and fourth quadrants respectively. This is shown in the following diagram.



- (a) For point Q, show that $y = \sqrt{9 - x^2}$. [1]
- (b) Hence, find an expression for A , the area of triangle PQR, in terms of x . [3]
- (c) Show that $\frac{dA}{dx} = \frac{9 - 3x - 2x^2}{\sqrt{9 - x^2}}$. [4]
- (d) Hence or otherwise, find the y -coordinate of R such that A is a maximum. [6]



Do **not** write solutions on this page.

11. [Maximum mark: 22]

Consider the complex number $u = -1 + \sqrt{3}i$.

(a) By finding the modulus and argument of u , show that $u = 2e^{i\frac{2\pi}{3}}$. [3]

(b) (i) Find the smallest positive integer n such that u^n is a real number.

(ii) Find the value of u^n when n takes the value found in part (b)(i). [5]

(c) Consider the equation $z^3 + 5z^2 + 10z + 12 = 0$, where $z \in \mathbb{C}$.

(i) Given that u is a root of $z^3 + 5z^2 + 10z + 12 = 0$, find the other roots.

(ii) By using a suitable transformation from z to w , or otherwise, find the roots of the equation $1 + 5w + 10w^2 + 12w^3 = 0$, where $w \in \mathbb{C}$. [9]

(d) Consider the equation $z^2 = 2z^*$, where $z \in \mathbb{C}$, $z \neq 0$.

By expressing z in the form $a + bi$, find the roots of the equation. [5]

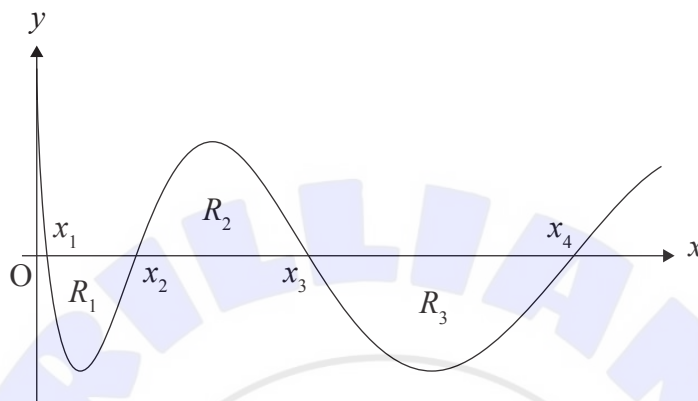


Do **not** write solutions on this page.

12. [Maximum mark: 17]

(a) By using an appropriate substitution, show that $\int \cos \sqrt{x} \, dx = 2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} + C$. [6]

The following diagram shows part of the curve $y = \cos \sqrt{x}$ for $x \geq 0$.



The curve intersects the x -axis at $x_1, x_2, x_3, x_4, \dots$

The n th x -intercept of the curve, x_n , is given by $x_n = \frac{(2n-1)^2 \pi^2}{4}$, where $n \in \mathbb{Z}^+$.

(b) Write down a similar expression for x_{n+1} . [1]

The regions bounded by the curve and the x -axis are denoted by R_1, R_2, R_3, \dots , as shown on the above diagram.

(c) Calculate the area of region R_n .

Give your answer in the form $kn\pi$, where $k \in \mathbb{Z}^+$. [7]

(d) Hence, show that the areas of the regions bounded by the curve and the x -axis, R_1, R_2, R_3, \dots , form an arithmetic sequence. [3]

References:

© International Baccalaureate Organization 2023





Please **do not** write on this page.

Answers written on this page
will not be marked.

