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Mathematics: analysis and approaches
Higher level
Paper 1

Monday 1 November 2021 (afternoon)

Candidate session number

2 hours

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Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.





Please **do not** write on this page.

Answers written on this page
will not be marked.



2. [Maximum mark: 9]

The function f is defined by $f(x) = \frac{2x+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$.

(a) Write down the equation of

(i) the vertical asymptote of the graph of f ;

(ii) the horizontal asymptote of the graph of f .

[2]

(b) Find the coordinates where the graph of f crosses

(i) the x -axis;

(ii) the y -axis.

[2]

BRILLIANT LEARNING

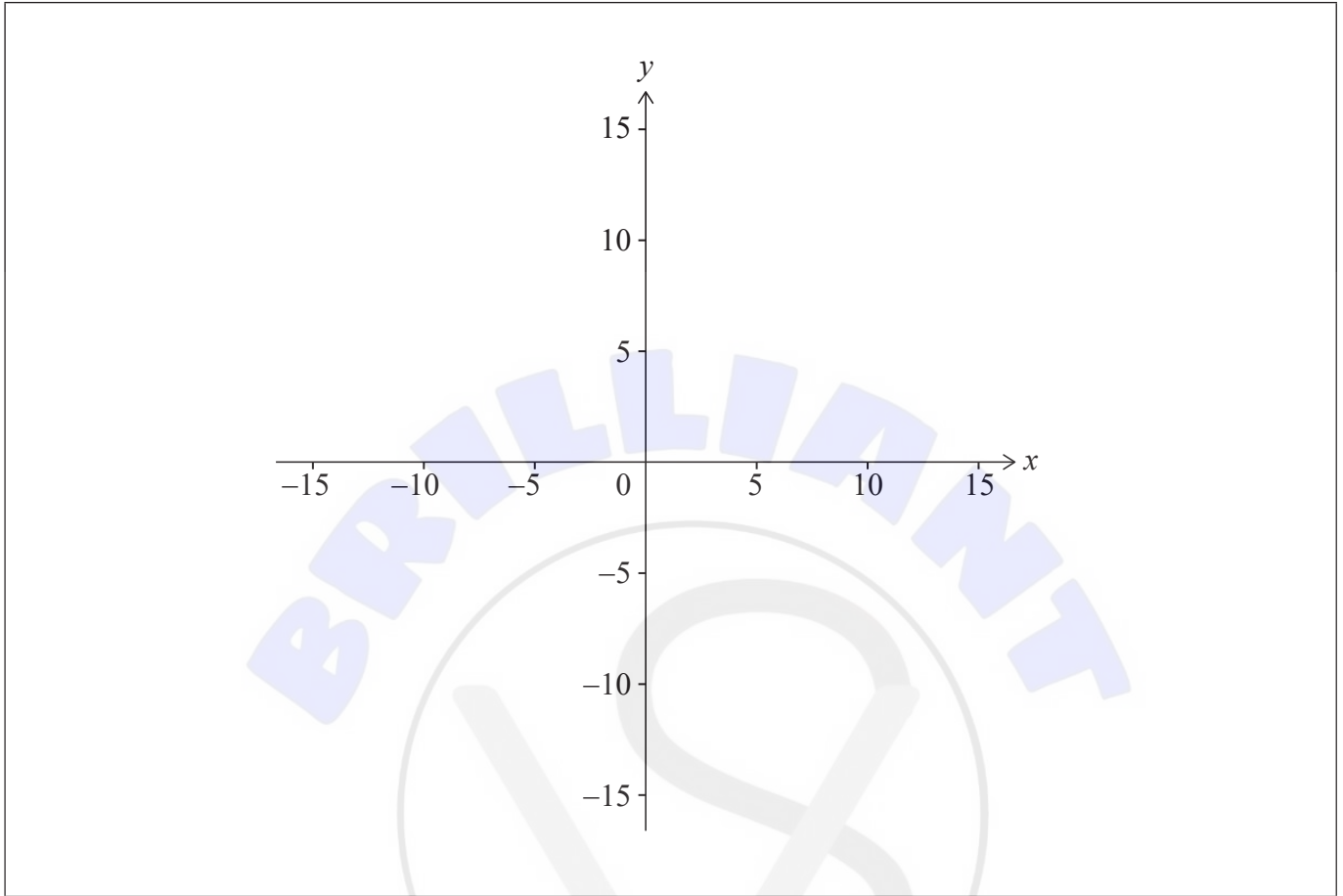
(This question continues on the following page)



(Question 2 continued)

(c) Sketch the graph of f on the axes below.

[1]



The function g is defined by $g(x) = \frac{ax+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$ and $a \in \mathbb{R}$.

(d) Given that $g(x) = g^{-1}(x)$, determine the value of a .

[4]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 16]

A particle P moves along the x -axis. The velocity of P is $v \text{ m s}^{-1}$ at time t seconds, where $v(t) = 4 + 4t - 3t^2$ for $0 \leq t \leq 3$. When $t = 0$, P is at the origin O .

- (a) (i) Find the value of t when P reaches its maximum velocity. [7]
- (ii) Show that the distance of P from O at this time is $\frac{88}{27}$ metres. [7]
- (b) Sketch a graph of v against t , clearly showing any points of intersection with the axes. [4]
- (c) Find the total distance travelled by P . [5]

11. [Maximum mark: 14]

- (a) Prove by mathematical induction that $\frac{d^n}{dx^n}(x^2e^x) = [x^2 + 2nx + n(n-1)]e^x$ for $n \in \mathbb{Z}^+$. [7]
- (b) Hence or otherwise, determine the Maclaurin series of $f(x) = x^2e^x$ in ascending powers of x , up to and including the term in x^4 . [3]
- (c) Hence or otherwise, determine the value of $\lim_{x \rightarrow 0} \left[\frac{(x^2e^x - x^2)^3}{x^9} \right]$. [4]



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12. [Maximum mark: 22]

Consider the equation $(z - 1)^3 = i$, $z \in \mathbb{C}$. The roots of this equation are ω_1 , ω_2 and ω_3 , where $\text{Im}(\omega_2) > 0$ and $\text{Im}(\omega_3) < 0$.

- (a) (i) Verify that $\omega_1 = 1 + e^{i\frac{\pi}{6}}$ is a root of this equation.
- (ii) Find ω_2 and ω_3 , expressing these in the form $a + e^{i\theta}$, where $a \in \mathbb{R}$ and $\theta > 0$. [6]

The roots ω_1 , ω_2 and ω_3 are represented by the points A, B and C respectively on an Argand diagram.

- (b) Plot the points A, B and C on an Argand diagram. [4]
- (c) Find AC. [3]

Consider the equation $(z - 1)^3 = iz^3$, $z \in \mathbb{C}$.

- (d) By using de Moivre's theorem, show that $\alpha = \frac{1}{1 - e^{i\frac{\pi}{6}}}$ is a root of this equation. [3]
- (e) Determine the value of $\text{Re}(\alpha)$. [6]

References:

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