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Mathematics: applications and interpretation
Higher level
Paper 1

Monday 1 November 2021 (afternoon)

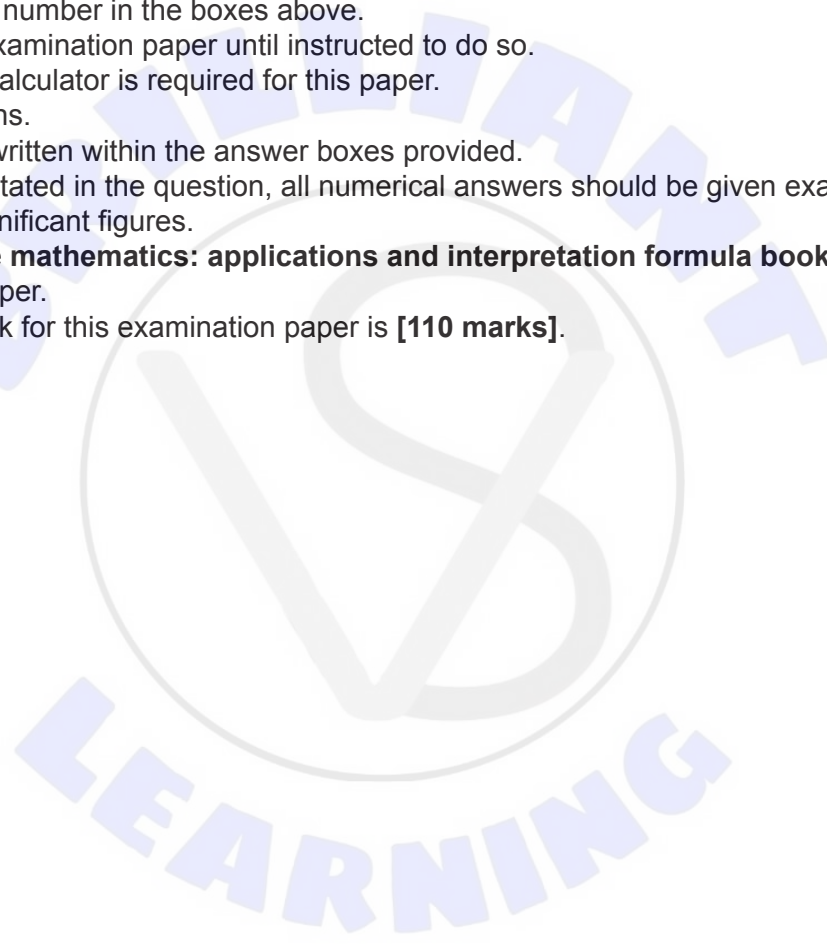
Candidate session number

2 hours

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Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.



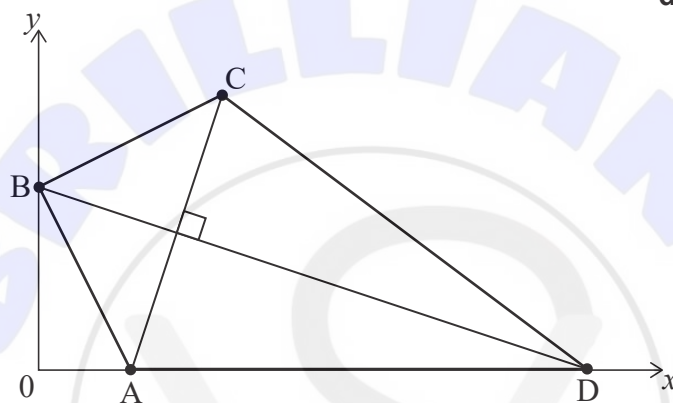
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

Dilara is designing a kite ABCD on a set of coordinate axes in which one unit represents 10 cm.

The coordinates of A, B and C are (2, 0), (0, 4) and (4, 6) respectively. Point D lies on the x -axis. [AC] is perpendicular to [BD]. This information is shown in the following diagram.

diagram not to scale



- (a) Find the gradient of the line through A and C. [2]
- (b) Write down the gradient of the line through B and D. [1]
- (c) Find the equation of the line through B and D. Give your answer in the form $ax + by + d = 0$, where a , b and d are integers. [2]
- (d) Write down the x -coordinate of point D. [1]

(This question continues on the following page)

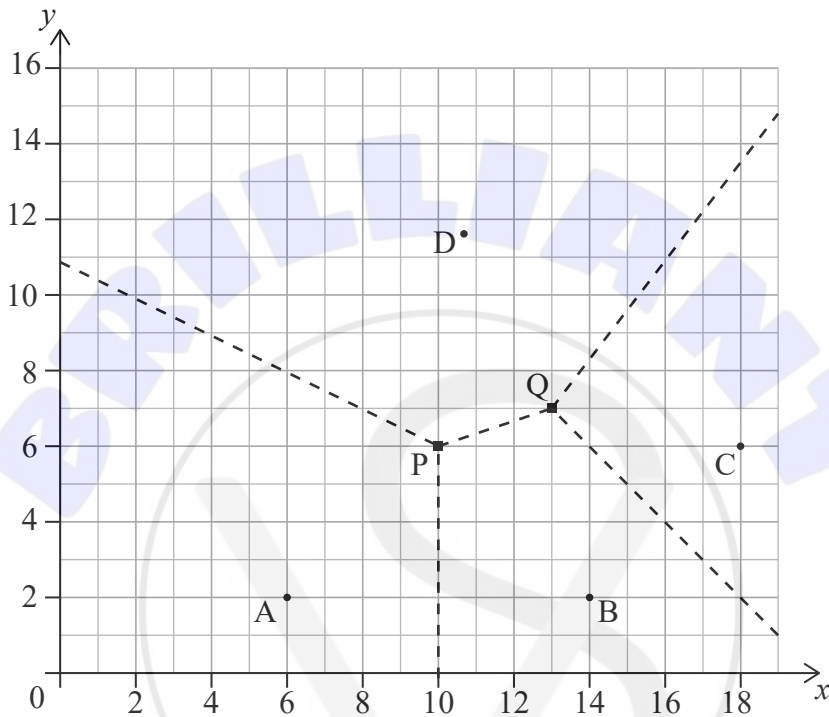


4. [Maximum mark: 6]

There are four stations used by the fire wardens in a national forest.

On the following Voronoi diagram, the coordinates of the stations are $A(6, 2)$, $B(14, 2)$, $C(18, 6)$ and $D(10.8, 11.6)$ where distances are measured in kilometres.

The dotted lines represent the boundaries of the regions patrolled by the fire warden at each station. The boundaries meet at $P(10, 6)$ and $Q(13, 7)$.



To reduce the areas of the regions that the fire wardens patrol, a new station is to be built within the quadrilateral ABCD. The new station will be located so that it is as far as possible from the nearest existing station.

- (a) Show that the new station should be built at P. [3]

The Voronoi diagram is to be updated to include the region around the new station at P. The edges defined by the perpendicular bisectors of $[AP]$ and $[BP]$ have been added to the following diagram.

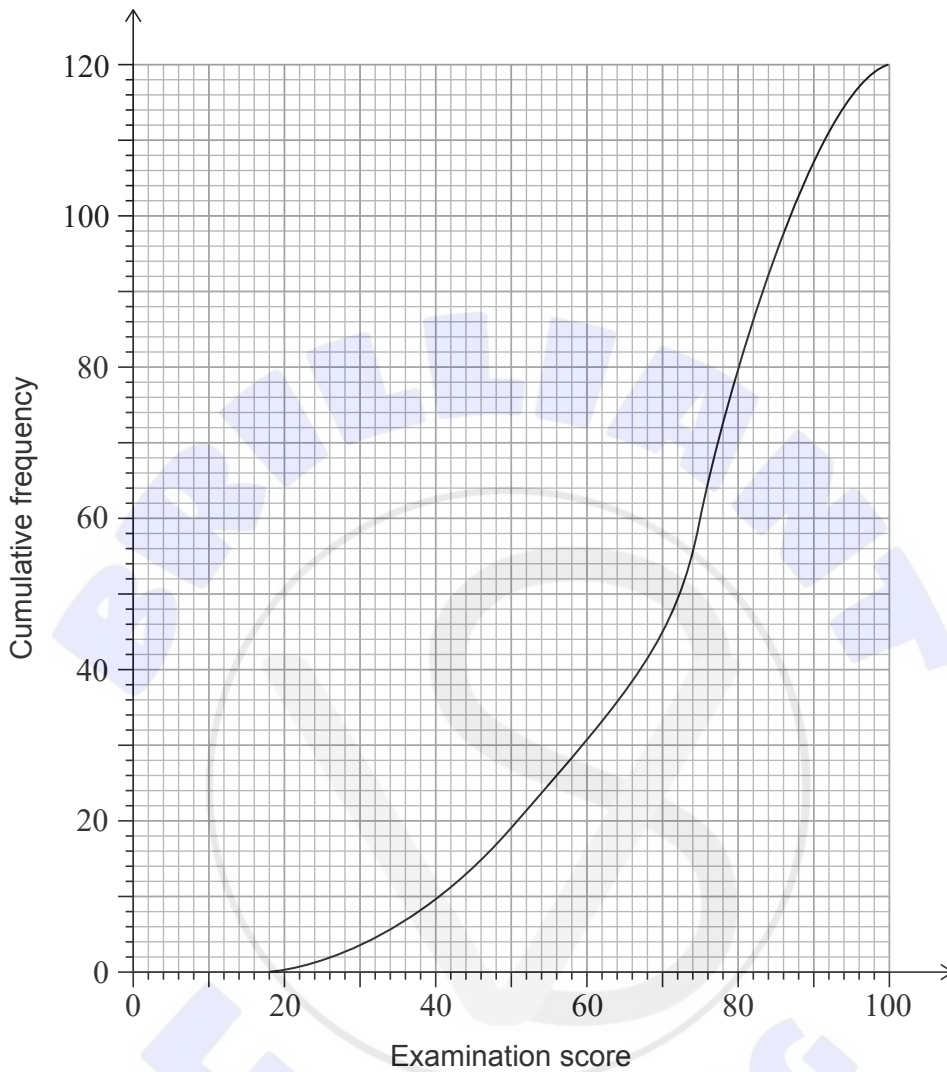
- (b) (i) Write down the equation of the perpendicular bisector of $[PC]$.
 (ii) Hence draw the missing boundaries of the region around P on the following diagram. [3]

(This question continues on the following page)



7. [Maximum mark: 8]

A group of 120 students sat a history exam. The cumulative frequency graph shows the scores obtained by the students.



(a) Find the median of the scores obtained. [1]

The students were awarded a grade from 1 to 5, depending on the score obtained in the exam. The number of students receiving each grade is shown in the following table.

Grade	1	2	3	4	5
Number of students	6	13	26	a	b

(b) Find an expression for a in terms of b . [2]

(This question continues on the following page)



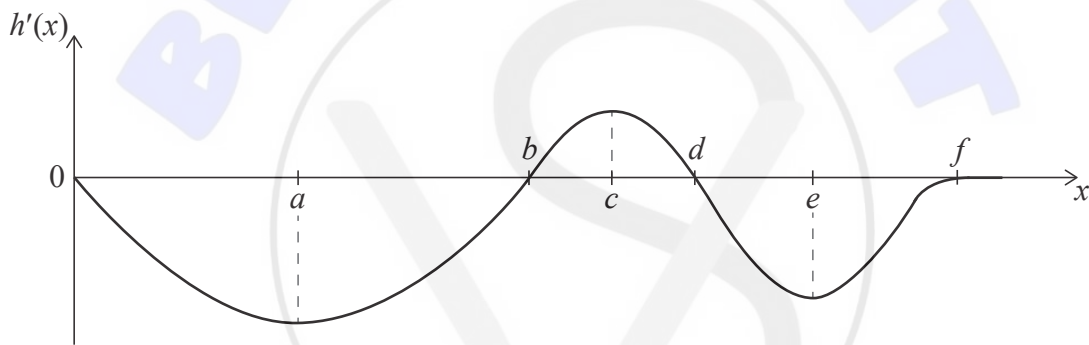
8. [Maximum mark: 5]

Juri skis from the top of a hill to a finishing point at the bottom of the hill. She takes the shortest route, heading directly to the finishing point (F).



Let $h(x)$ define the height of the hill above F at a horizontal distance x from the starting point at the top of the hill.

The graph of the **derivative** of $h(x)$ is shown below. The graph of $h'(x)$ has local minima and maxima when x is equal to a , c and e . The graph of $h'(x)$ intersects the x -axis when x is equal to b , d , and f .



- (a) (i) Identify the x value of the point where $|h'(x)|$ has its maximum value.
- (ii) Interpret this point in the given context.

[2]

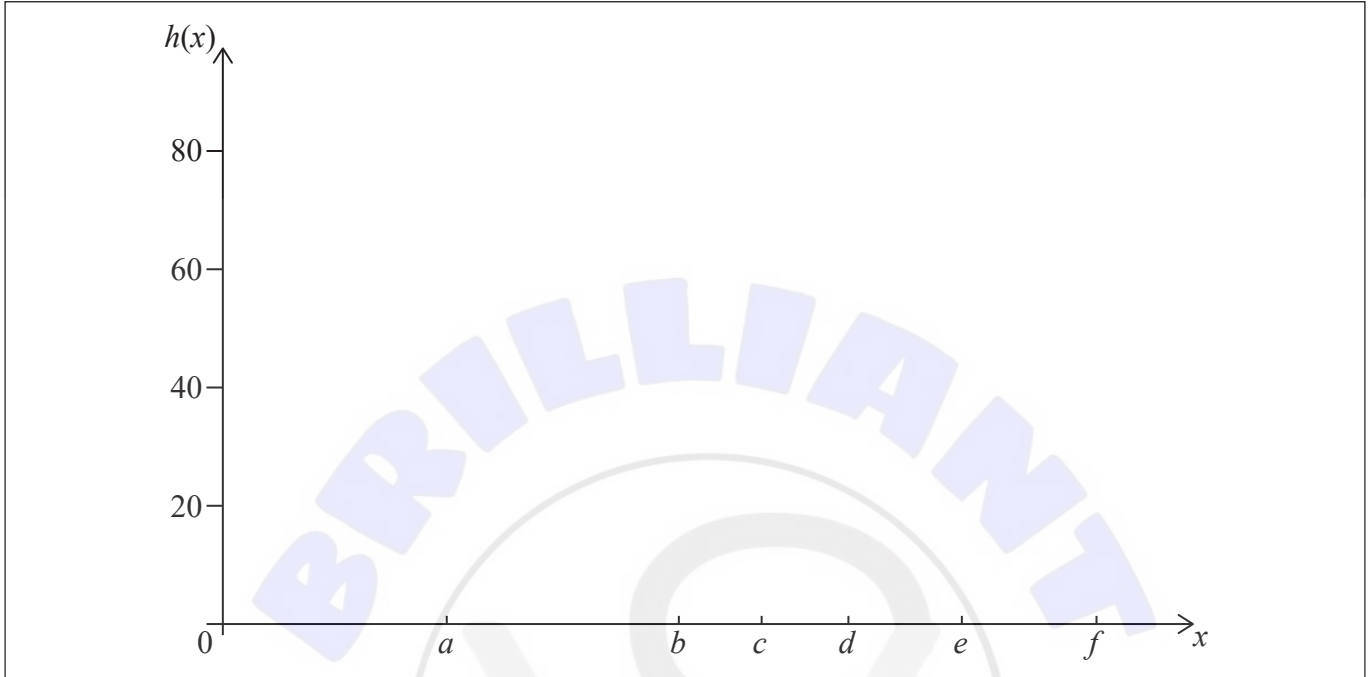
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(Question 8 continued)

Juri starts at a height of 60 metres and finishes at F, where $x = f$.

- (b) Sketch a possible diagram of the hill on the following pair of coordinate axes. [3]



A large rectangular area containing horizontal dotted lines for sketching the hill.





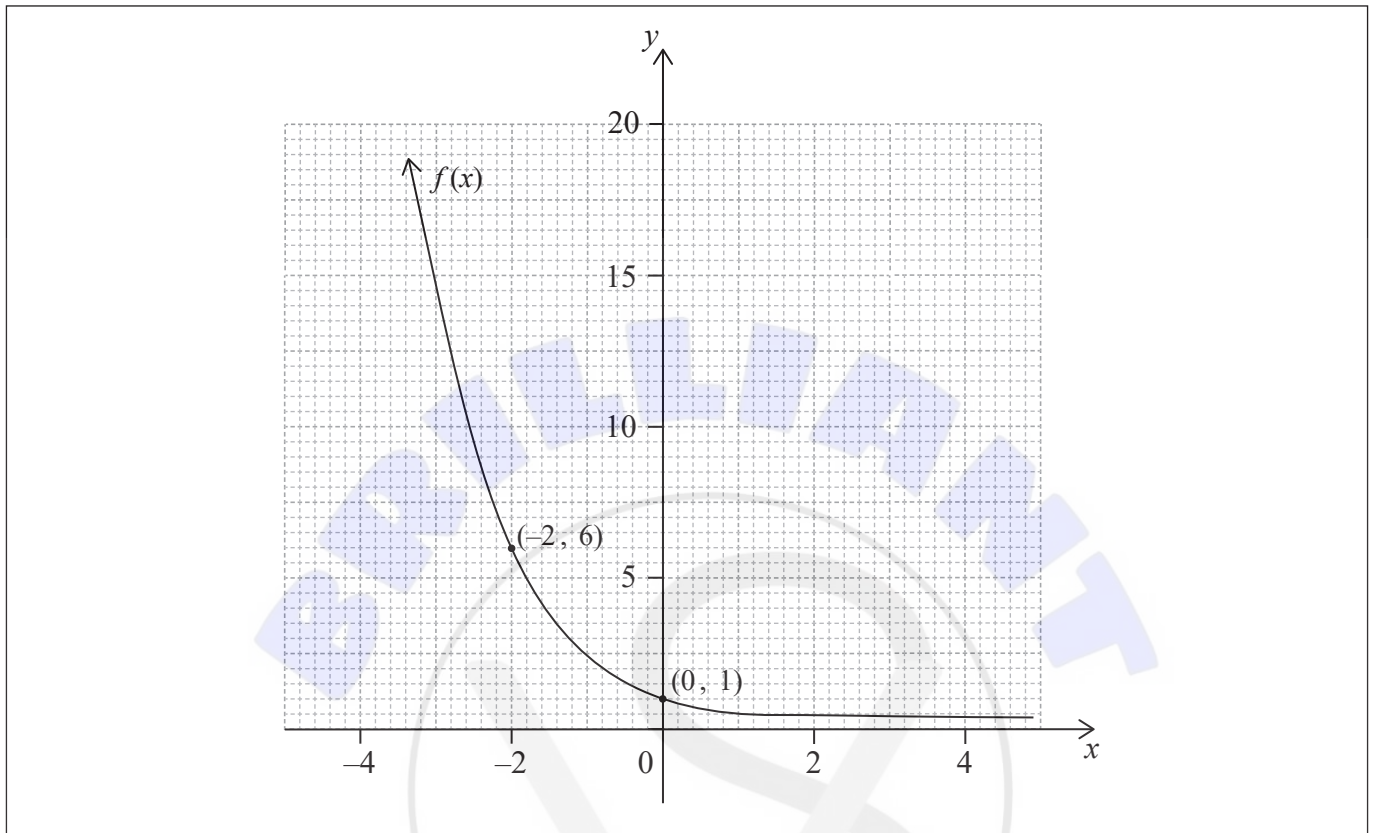
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10. [Maximum mark: 4]

The graph of $y = f(x)$ is given on the following set of axes. The graph passes through the points $(-2, 6)$ and $(0, 1)$, and has a horizontal asymptote at $y = 0$.



Let $g(x) = 2f(x - 2) + 4$.

- (a) Find $g(0)$. [2]
- (b) On the same set of axes draw the graph of $y = g(x)$, showing any intercepts and asymptotes. [2]

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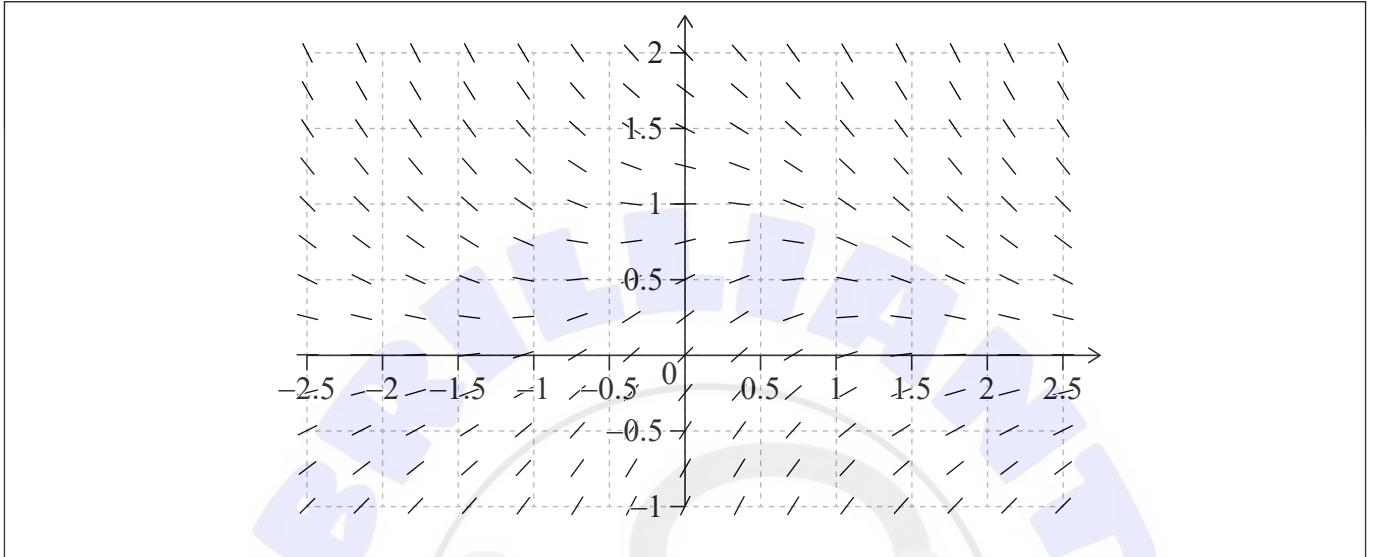


13. [Maximum mark: 7]

The slope field for the differential equation $\frac{dy}{dx} = e^{-x^2} - y$ is shown in the following two graphs.

(a) Calculate the value of $\frac{dy}{dx}$ at the point (0, 1). [1]

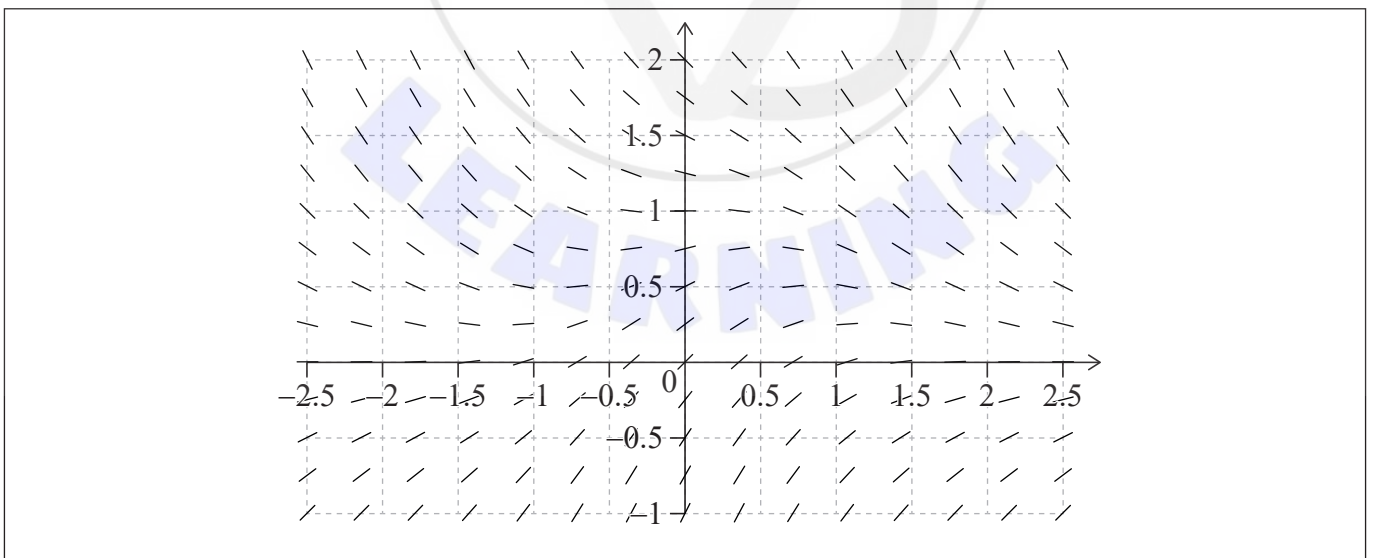
(b) Sketch, on the first graph, a curve that represents the points where $\frac{dy}{dx} = 0$. [2]



(c) On the second graph,

(i) sketch the solution curve that passes through the point (0, 0).

(ii) sketch the solution curve that passes through the point (0, 0.75). [4]



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