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Mathematics: analysis and approaches
Higher level
Paper 1

Monday 31 October 2022 (afternoon)

Candidate session number

2 hours

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Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.



2. [Maximum mark: 7]

Consider a circle with a diameter AB , where A has coordinates $(1, 4, 0)$ and B has coordinates $(-3, 2, -4)$.

(a) Find

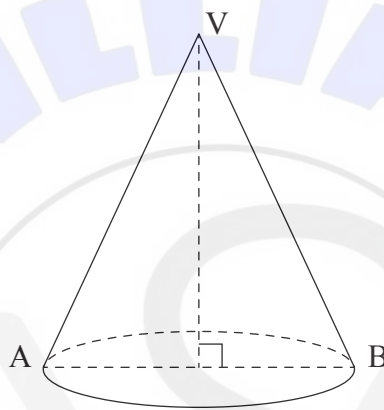
(i) the coordinates of the centre of the circle;

(ii) the radius of the circle.

[4]

The circle forms the base of a right cone whose vertex V has coordinates $(-1, -1, 0)$.

diagram not to scale



(b) Find the exact volume of the cone.

[3]

Area with horizontal dotted lines for writing the answer to part (b).



Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 20]

The function f is defined by $f(x) = \cos^2 x - 3 \sin^2 x$, $0 \leq x \leq \pi$.

- (a) Find the roots of the equation $f(x) = 0$. [5]
- (b) (i) Find $f'(x)$.
- (ii) Hence find the coordinates of the points on the graph of $y = f(x)$ where $f'(x) = 0$. [7]
- (c) Sketch the graph of $y = |f(x)|$, clearly showing the coordinates of any points where $f'(x) = 0$ and any points where the graph meets the coordinate axes. [4]
- (d) Hence or otherwise, solve the inequality $|f(x)| > 1$. [4]

11. [Maximum mark: 16]

Consider a three-digit code abc , where each of a , b and c is assigned one of the values 1, 2, 3, 4 or 5.

- (a) Find the total number of possible codes
- (i) assuming that each value can be repeated (for example, 121 or 444);
- (ii) assuming that no value is repeated. [4]

Let $P(x) = x^3 + ax^2 + bx + c$, where each of a , b and c is assigned one of the values 1, 2, 3, 4 or 5. Assume that no value is repeated.

Consider the case where $P(x)$ has a factor of $(x^2 + 3x + 2)$.

- (b) (i) Find an expression for b in terms of a .
- (ii) Hence show that the only way to assign the values is $a = 4$, $b = 5$ and $c = 2$.
- (iii) Express $P(x)$ as a product of linear factors.
- (iv) Hence or otherwise, sketch the graph of $y = P(x)$, clearly showing the coordinates of any intercepts with the axes. [12]



Do **not** write solutions on this page.

12. [Maximum mark: 18]

Let z_n be the complex number defined as $z_n = (n^2 + n + 1) + i$ for $n \in \mathbb{N}$.

- (a) (i) Find $\arg(z_0)$.
- (ii) Write down an expression for $\arg(z_n)$ in terms of n . [3]

Let $w_n = z_0 z_1 z_2 z_3 \dots z_{n-1} z_n$ for $n \in \mathbb{N}$.

- (b) (i) Show that $\arctan(a) + \arctan(b) = \arctan\left(\frac{a+b}{1-ab}\right)$ for $a, b \in \mathbb{R}^+$, $ab < 1$.
- (ii) Hence or otherwise, show that $\arg(w_1) = \arctan(2)$. [5]
- (c) Prove by mathematical induction that $\arg(w_n) = \arctan(n+1)$ for $n \in \mathbb{N}$. [10]

References:

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