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Mathematics: analysis and approaches
Higher level
Paper 2

Tuesday 1 November 2022 (morning)

Candidate session number

2 hours

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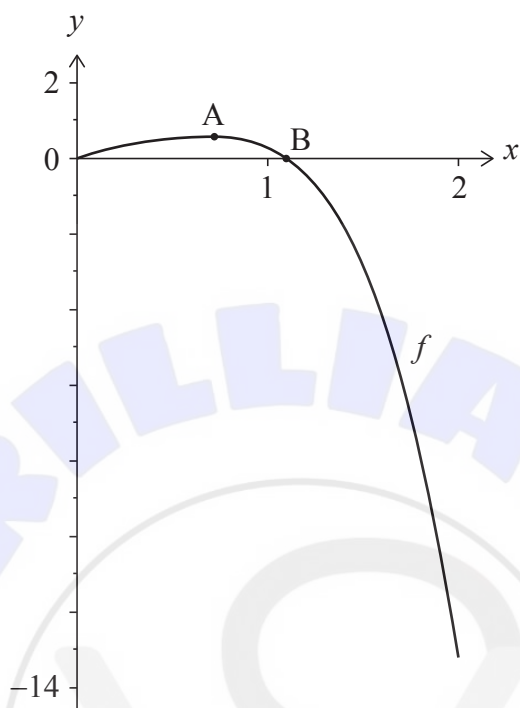
Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.



2. [Maximum mark: 6]

The function f is defined as $f(x) = \ln(xe^x + 1) - x^4$, for $0 \leq x \leq 2$. The graph of f is shown in the following diagram.



The graph of f has a local maximum at point A. The graph intersects the x -axis at the origin and at point B.

- (a) Find the coordinates of A. [2]
- (b) Find the x -coordinate of B. [1]
- (c) Find the total area enclosed by the graph of f , the x -axis and the line $x = 2$. [3]

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9. [Maximum mark: 8]

Consider a quadrilateral ABCD such that $AB = 2$, $BC = 4$, $CD = 6$ and $DA = 8$, as shown in the following diagram. Let $\alpha = \hat{ABC}$ and $\beta = \hat{ADC}$.

diagram not to scale



- (a) (i) Find AC in terms of α .
 - (ii) Find AC in terms of β .
 - (iii) Hence or otherwise, find an expression for α in terms of β . [4]
- (b) Find the maximum area of the quadrilateral ABCD. [4]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 16]

The time worked, T , in hours per week by employees of a large company is normally distributed with a mean of 42 and standard deviation 10.7.

- (a) Find the probability that an employee selected at random works more than 40 hours per week. [2]
- (b) A group of four employees is selected at random. Each employee is asked in turn whether they work more than 40 hours per week. Find the probability that the fourth employee is the only one in the group who works more than 40 hours per week. [3]
- (c) A large group of employees work more than 40 hours per week.
 - (i) An employee is selected at random from this large group.
Find the probability that this employee works less than 55 hours per week.
 - (ii) Ten employees are selected at random from this large group.
Find the probability that exactly five of them work less than 55 hours per week. [7]

It is known that $P(a \leq T \leq b) = 0.904$ and that $P(T > b) = 2P(T < a)$, where a and b are numbers of hours worked per week. An employee who works fewer than a hours per week is considered to be a part-time employee.

- (d) Find the maximum time, in hours per week, that an employee can work and still be considered part-time. [4]



Do **not** write solutions on this page.

11. [Maximum mark: 15]

The function f is defined by $f(x) = e^{2x}(3x - 4)$, where $x \in \mathbb{R}$.

- (a) Find $f'(x)$. [3]
- (b) Hence or otherwise, find the coordinates of the point on the graph of $y = f(x)$ where the tangent is parallel to the line $y = x$. [3]

The region enclosed by the curve $y = f(x)$, the x -axis and the y -axis is rotated through 2π radians about the x -axis to form a solid of revolution.

- (c) Find the volume of this solid. [4]

Consider a function g such that $g(0) = 1$ and $g'(0) = 2$.

- (d) Find the value of
 - (i) $(f \circ g)(0)$;
 - (ii) $(f \circ g)'(0)$. [5]

12. [Maximum mark: 22]

Consider the points $A(1, 2, 3)$, $B(k, -2, 1)$ and $C(5, 0, 2)$, where $k \in \mathbb{R}$.

- (a) Write down \vec{AB} and \vec{AC} . [2]
- (b) Given that the points A , B and C lie on a straight line, show that $k = 9$. [1]
- (c) For $k = 9$, let L_1 be the line passing through A , B and C .
 - (i) Find a vector equation of the line L_1 .
 - (ii) Line L_2 has the equation $\frac{x-1}{2} = \frac{y}{3} = 1-z$. Show that the lines L_1 and L_2 are skew. [10]
- (d) For $k \neq 9$, let Π be the plane containing A , B and C .
 - (i) Find the Cartesian equation of the plane Π .
 - (ii) Find the coordinates of the point on the plane Π which is closest to the origin $(0, 0, 0)$. [9]

References:

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