

© International Baccalaureate Organization 2023

All rights reserved. No part of this product may be reproduced in any form or by any electronic or mechanical means, including information storage and retrieval systems, without the prior written permission from the IB. Additionally, the license tied with this product prohibits use of any selected files or extracts from this product. Use by third parties, including but not limited to publishers, private teachers, tutoring or study services, preparatory schools, vendors operating curriculum mapping services or teacher resource digital platforms and app developers, whether fee-covered or not, is prohibited and is a criminal offense.

More information on how to request written permission in the form of a license can be obtained from <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

© Organisation du Baccalauréat International 2023

Tous droits réservés. Aucune partie de ce produit ne peut être reproduite sous quelque forme ni par quelque moyen que ce soit, électronique ou mécanique, y compris des systèmes de stockage et de récupération d'informations, sans l'autorisation écrite préalable de l'IB. De plus, la licence associée à ce produit interdit toute utilisation de tout fichier ou extrait sélectionné dans ce produit. L'utilisation par des tiers, y compris, sans toutefois s'y limiter, des éditeurs, des professeurs particuliers, des services de tutorat ou d'aide aux études, des établissements de préparation à l'enseignement supérieur, des fournisseurs de services de planification des programmes d'études, des gestionnaires de plateformes pédagogiques en ligne, et des développeurs d'applications, moyennant paiement ou non, est interdite et constitue une infraction pénale.

Pour plus d'informations sur la procédure à suivre pour obtenir une autorisation écrite sous la forme d'une licence, rendez-vous à l'adresse <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

© Organización del Bachillerato Internacional, 2023

Todos los derechos reservados. No se podrá reproducir ninguna parte de este producto de ninguna forma ni por ningún medio electrónico o mecánico, incluidos los sistemas de almacenamiento y recuperación de información, sin la previa autorización por escrito del IB. Además, la licencia vinculada a este producto prohíbe el uso de todo archivo o fragmento seleccionado de este producto. El uso por parte de terceros —lo que incluye, a título enunciativo, editoriales, profesores particulares, servicios de apoyo académico o ayuda para el estudio, colegios preparatorios, desarrolladores de aplicaciones y entidades que presten servicios de planificación curricular u ofrezcan recursos para docentes mediante plataformas digitales—, ya sea incluido en tasas o no, está prohibido y constituye un delito.

En este enlace encontrará más información sobre cómo solicitar una autorización por escrito en forma de licencia: <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

Mathematics: analysis and approaches
Standard level
Paper 1

30 October 2023

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

--	--	--	--	--	--	--	--	--	--

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



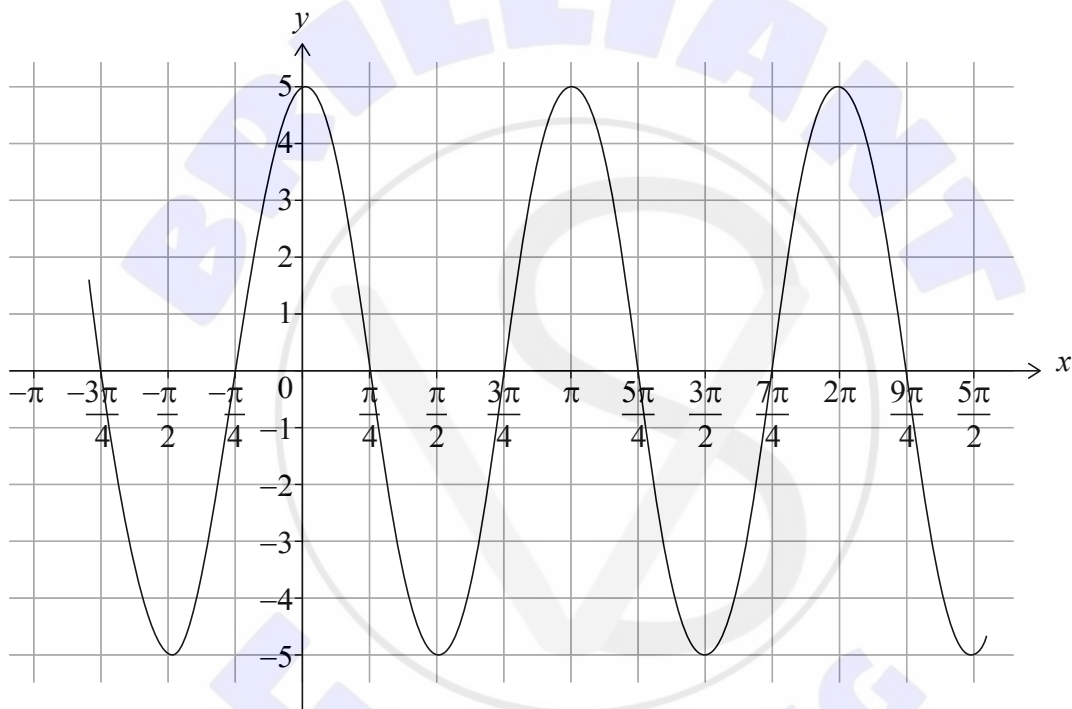
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function $f(x) = a \cos(bx)$, with $a, b \in \mathbb{Z}^+$. The following diagram shows part of the graph of f .



- (a) Write down the value of a . [1]
- (b) (i) Write down the period of f . [3]
 (ii) Hence, find the value of b . [3]
- (c) Find the value of $f\left(\frac{\pi}{6}\right)$. [3]

(This question continues on the following page)





Please **do not** write on this page.

Answers written on this page
will not be marked.



Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 17]

A ballet company performs *The Nutcracker* every year. Last year they gave a total of 60 performances at their theatre which has a maximum capacity of 800. The number of tickets sold, n , at each performance is shown in the following frequency table.

Number of tickets sold, n	Number of performances
$0 < n \leq 200$	3
$200 < n \leq 400$	p
$400 < n \leq 600$	18
$600 < n \leq 800$	30

- (a) (i) Find the value of p .
 (ii) Write down the modal class.

[2]

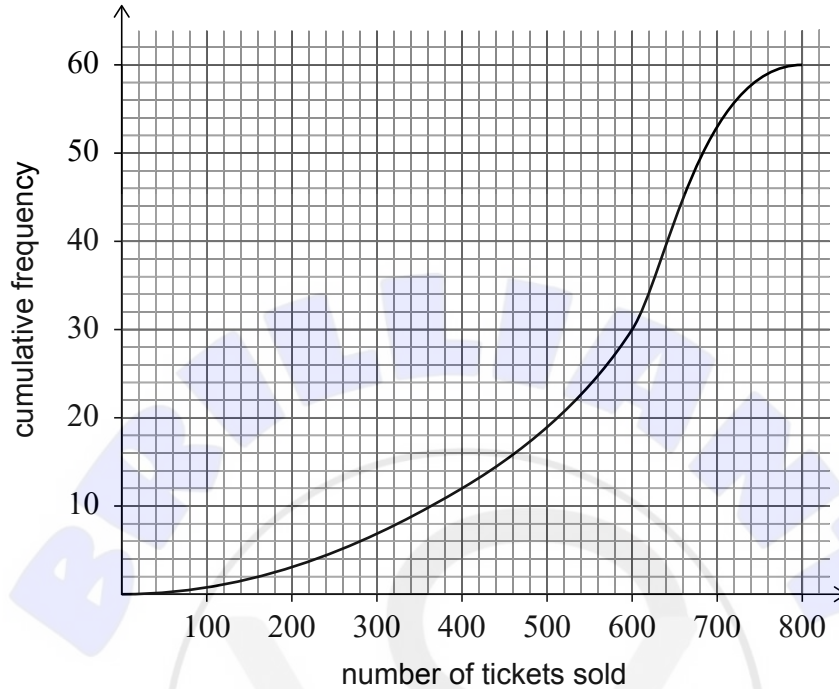
(This question continues on the following page)



Do **not** write solutions on this page.

(Question 7 continued)

The following cumulative frequency diagram also displays these data.



(b) Use the cumulative frequency curve to estimate

- (i) the median number of tickets sold;
- (ii) the number of performances where at least 80% of the tickets were sold.

[4]

(This question continues on page 13)





Please **do not** write on this page.

Answers written on this page
will not be marked.



Do **not** write solutions on this page.

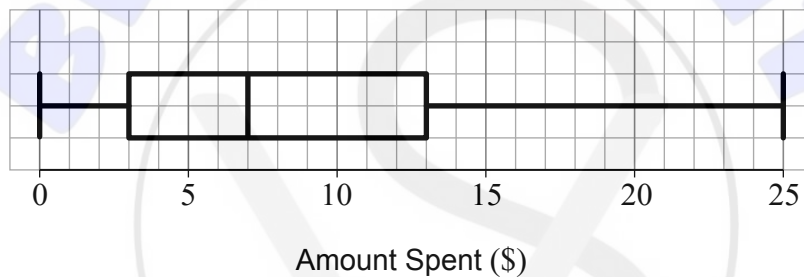
(Question 7 continued)

After a performance, the company decides to conduct a survey to obtain feedback from the audience.

- (c) (i) State one disadvantage of the company surveying only the first 5% of the audience as they leave the theatre.
- (ii) Describe briefly how the company could collect feedback from 5% of the audience using the systematic sampling method.
- (iii) State the sampling method which should be used if the survey is to be representative of the number of children and the number of adults in the audience. [4]

Last year 36 000 tickets were sold to *The Nutcracker*.

- (d) The following box and whisker diagram displays the amount spent by the audience at the souvenir shop when they attended the performance.



- (i) Estimate the number of people who spent between \$3 and \$25.
 - (ii) Half the audience spent less than \$ a . Estimate the value of a . [3]
- (e) This year the company will again give 60 performances and expects to sell 17 additional tickets for each performance.
- (i) Calculate the mean number of tickets the company expects to sell this year for each performance.
 - (ii) State what effect, if any, this increase in ticket sales would have on the variance of the number of tickets sold for each performance. [4]



Do **not** write solutions on this page.

8. [Maximum mark: 15]

The functions f and g are defined by

$$f(x) = \ln(2x - 9), \text{ where } x > \frac{9}{2}$$

$$g(x) = 2 \ln x - \ln d, \text{ where } x > 0, d \in \mathbb{R}^+.$$

(a) State the equation of the vertical asymptote to the graph of $y = g(x)$. [1]

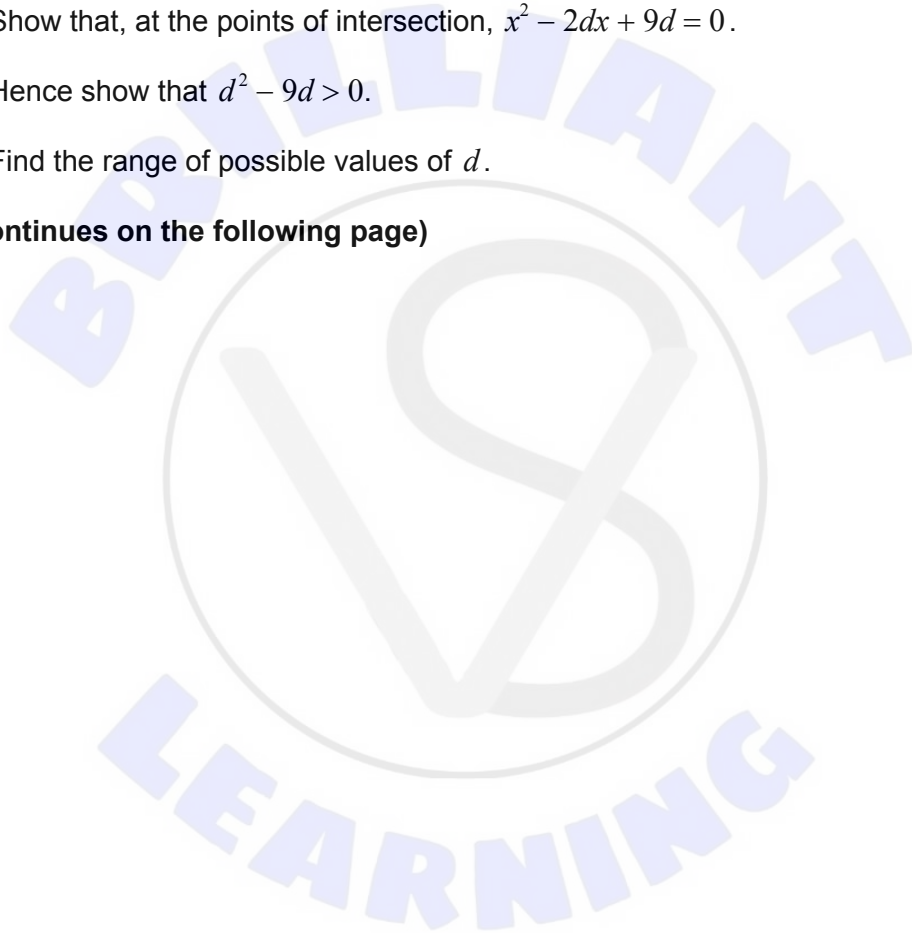
The graphs of $y = f(x)$ and $y = g(x)$ intersect at two distinct points.

(b) (i) Show that, at the points of intersection, $x^2 - 2dx + 9d = 0$.

(ii) Hence show that $d^2 - 9d > 0$.

(iii) Find the range of possible values of d . [9]

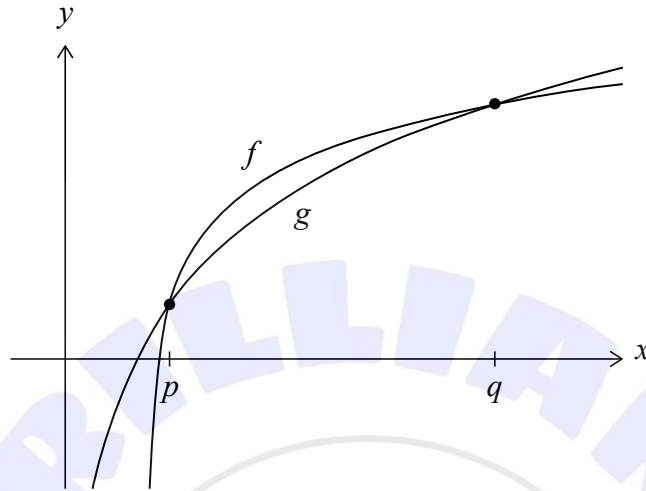
(This question continues on the following page)



Do **not** write solutions on this page.

(Question 8 continued)

The following diagram shows part of the graphs of $y = f(x)$ and $y = g(x)$.



The graphs intersect at $x = p$ and $x = q$, where $p < q$.

- (c) In the case where $d = 10$, find the value of $q - p$. Express your answer in the form $a\sqrt{b}$, where $a, b \in \mathbb{Z}^+$.

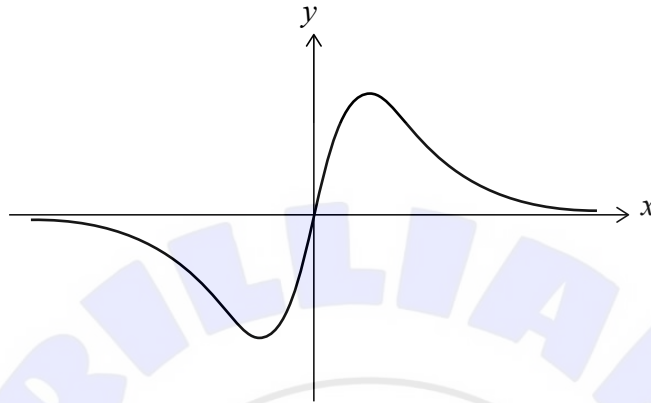
[5]



Do **not** write solutions on this page.

9. [Maximum mark: 13]

Consider the function f defined by $f(x) = \frac{8x}{(x^2+1)^3}$, where $x \in \mathbb{R}$. The graph of f is shown in the following diagram.



(a) Show that $f'(x) = \frac{8(1-5x^2)}{(x^2+1)^4}$. [4]

(b) Find $\int f(x)dx$. [4]

Consider a function $g(x)$ defined for $x \in \mathbb{R}$. The derivative of g is such that $g'(x) = f'(x)$, for all $x \in \mathbb{R}$.

Let R be the region enclosed by the graph of f , the graph of g , the line $x = 0$ and the line $x = 3$. The area of R is $\frac{27}{2}$.

(c) Find the two possible expressions for $g(x)$. [5]

