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Mathematics: applications and interpretation
Higher level
Paper 1

30 October 2023

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

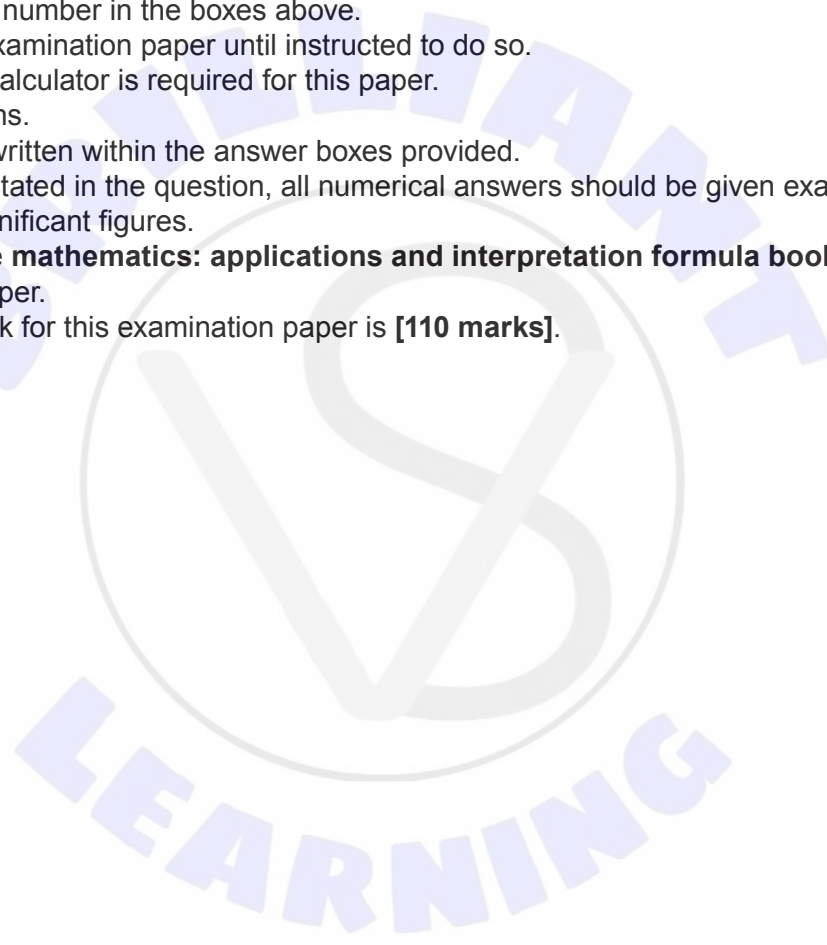
Candidate session number

2 hours

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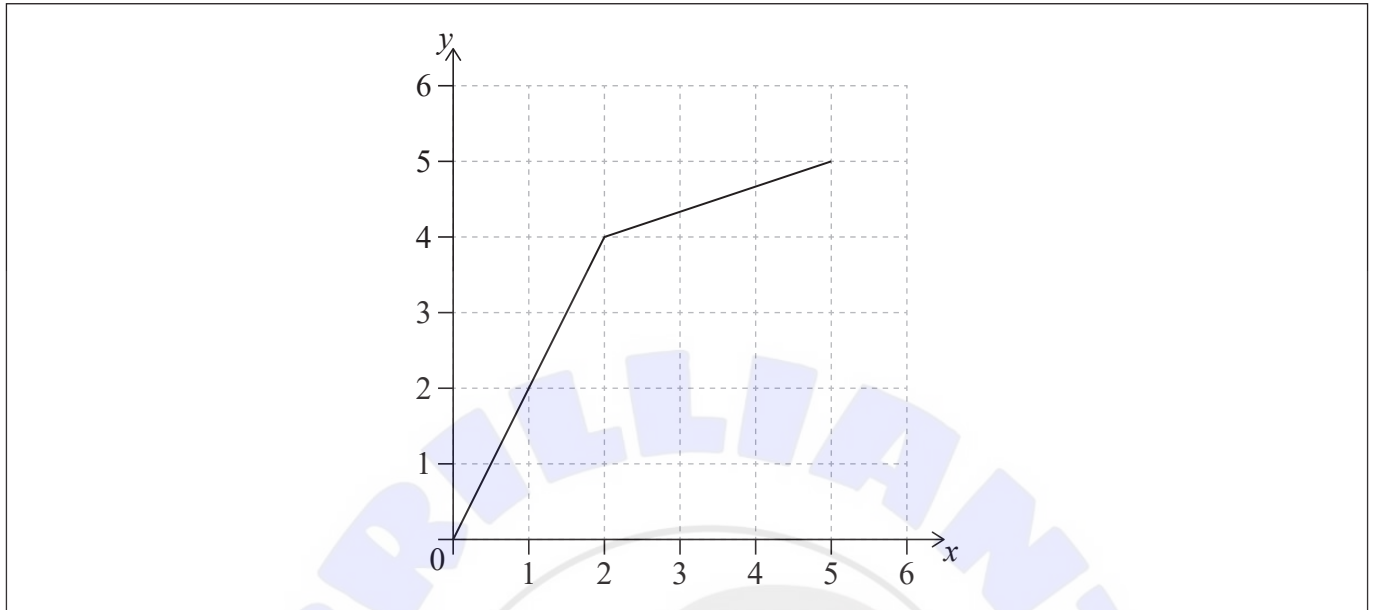
Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.



2. [Maximum mark: 7]

The graph of the function f is given in the following diagram.



(a) Write down $f(2)$. [1]

(b) On the axes, sketch $y = f^{-1}(x)$. [2]

The function g is defined as $g(x) = 3x - 1$.

(c) Find an expression for $g^{-1}(x)$. [2]

(d) Find a value of x where $f^{-1}(x) = g^{-1}(x)$. [2]

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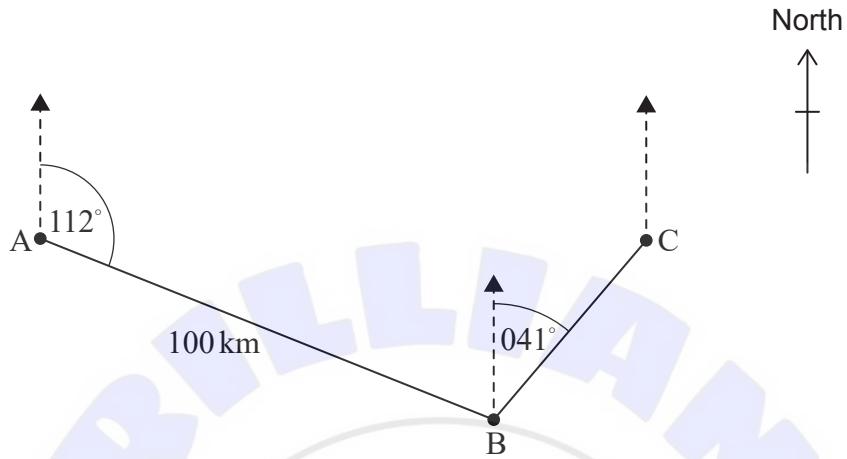
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4. [Maximum mark: 6]

Jason sails his boat from point A for a distance of 100 km, on a bearing of 112° , to arrive at point B. He then sails on a bearing of 041° to point C. Jason's journey is shown in the diagram.

diagram not to scale



(a) Find $\hat{A}BC$. [2]

Point C is directly east of point A.

(b) Calculate the distance that Jason sails to return directly from point C to point A. [4]

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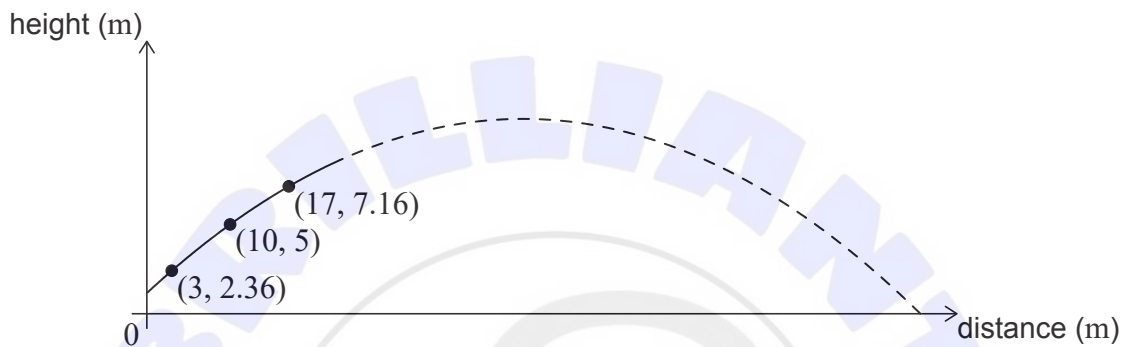
5. [Maximum mark: 9]

A sports player on a horizontal athletic field hits a ball. The height of the ball above the field, in metres, after it is hit can be modelled using a quadratic function of the form $f(x) = ax^2 + bx + c$, where x represents the horizontal distance, in metres, that the ball has travelled from the player.

A specialized camera tracks the initial path of the ball after it is hit by the player. The camera records that the ball travels through the three points (3, 2.36), (10, 5) and (17, 7.16), as shown in **Diagram 1**.

diagram not to scale

Diagram 1

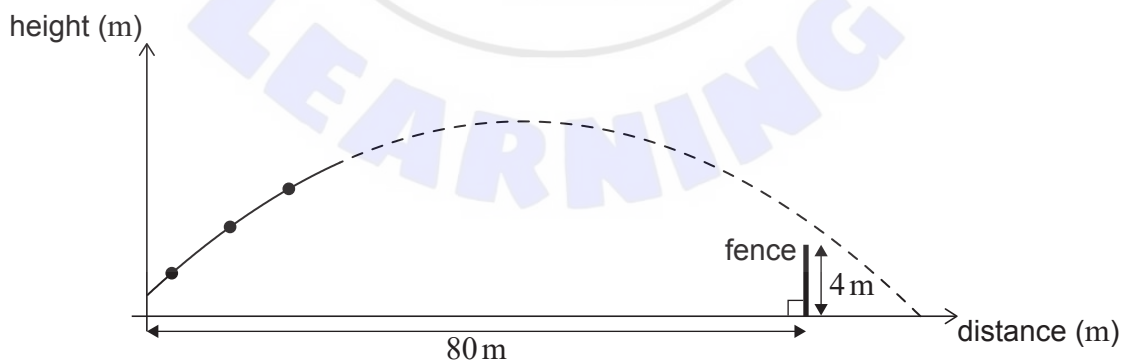


- (a) Use the coordinates (3, 2.36) to write down an equation in terms of a , b , and c . [1]
- (b) Use your answer to part (a) and two similar equations to find the equation of the quadratic model for the height of the ball. [3]

A 4-metre-high fence is 80 metres from where the player hit the ball, as shown in **Diagram 2**.

diagram not to scale

Diagram 2



- (c) Show that the model predicts that the ball will go over the fence. [3]
- (d) Find the horizontal distance that the ball will travel, from the player until it first hits the ground, according to this model. [2]

(This question continues on the following page)



6. [Maximum mark: 8]

On the following Voronoi diagram, the coordinates of three farmhouses are $A(0, 3)$, $B(8, 3)$ and $C(8, 13)$, where distances are measured in kilometres. Each farmhouse owns the land that is closest to it, and their boundaries are defined by the dotted lines on **Diagram 1**.

Diagram 1

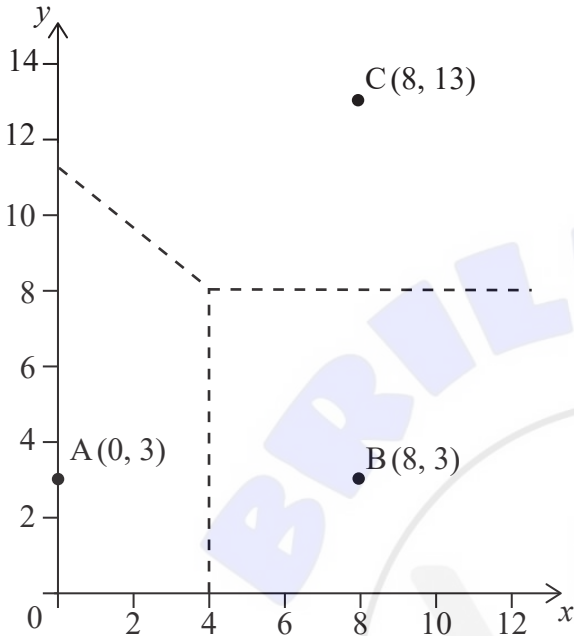
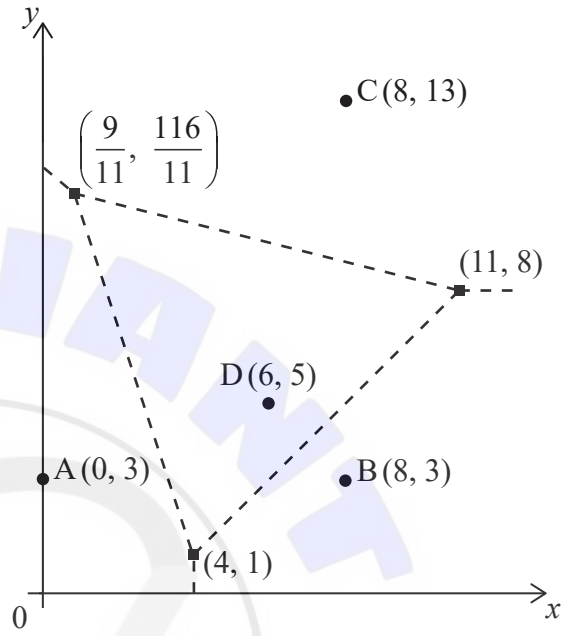


Diagram 2



To provide water to the farms it is decided to construct a well at the point where the boundaries meet on **Diagram 1**.

- (a) Write down the coordinates of this point. [1]
- (b) Find the equation of the perpendicular bisector of $[AC]$. [3]

An additional farmhouse $D(6, 5)$ is built on the land. The Voronoi diagram has been redrawn to show the new boundaries. The coordinates of the vertices of these boundaries are indicated on **Diagram 2**.

A wind turbine is to be built at one of the vertices.

- (c) The wind turbine should be as far from the nearest farmhouses as possible.
 - (i) By calculating appropriate distances, find the location of the wind turbine.
 - (ii) Hence, write down the distance of the wind turbine to the nearest farmhouse. [4]

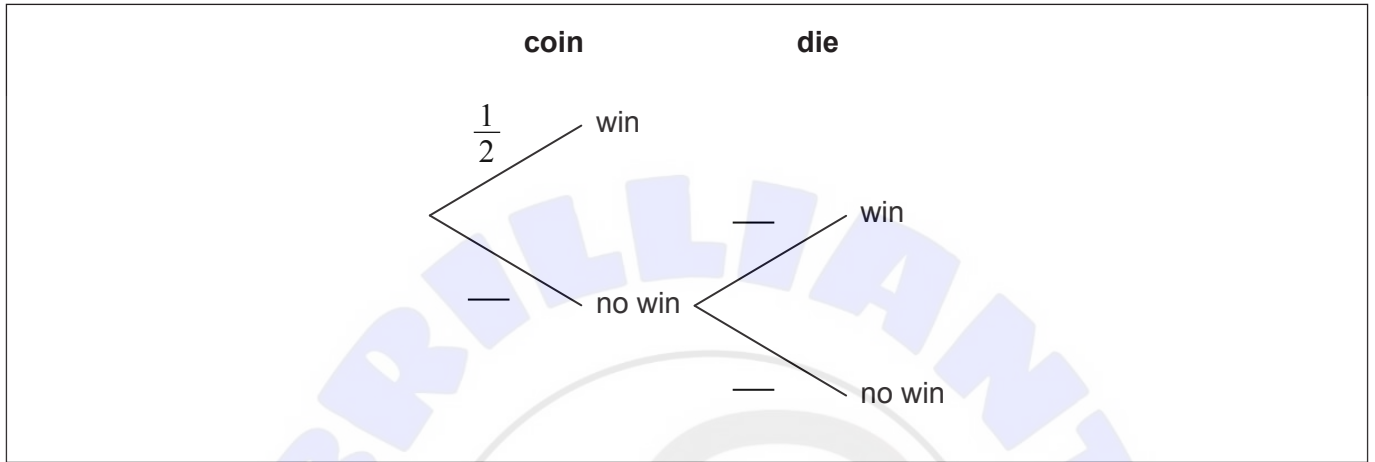
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7. [Maximum mark: 7]

Michèle is playing a game. In the game, she must first flip a fair coin which will result in the coin landing on heads or tails. If the coin lands on heads, then she wins a prize. If it lands on tails, then she has another chance but this time she must roll a fair six-sided dice and get a five or six in order to win a prize.

- (a) Complete the tree diagram by writing in the three missing probabilities. [2]



- (b) Find the probability that Michèle does **not** win a prize. [2]

- (c) Given that Michèle won a prize, find the probability that the coin landed on heads. [3]

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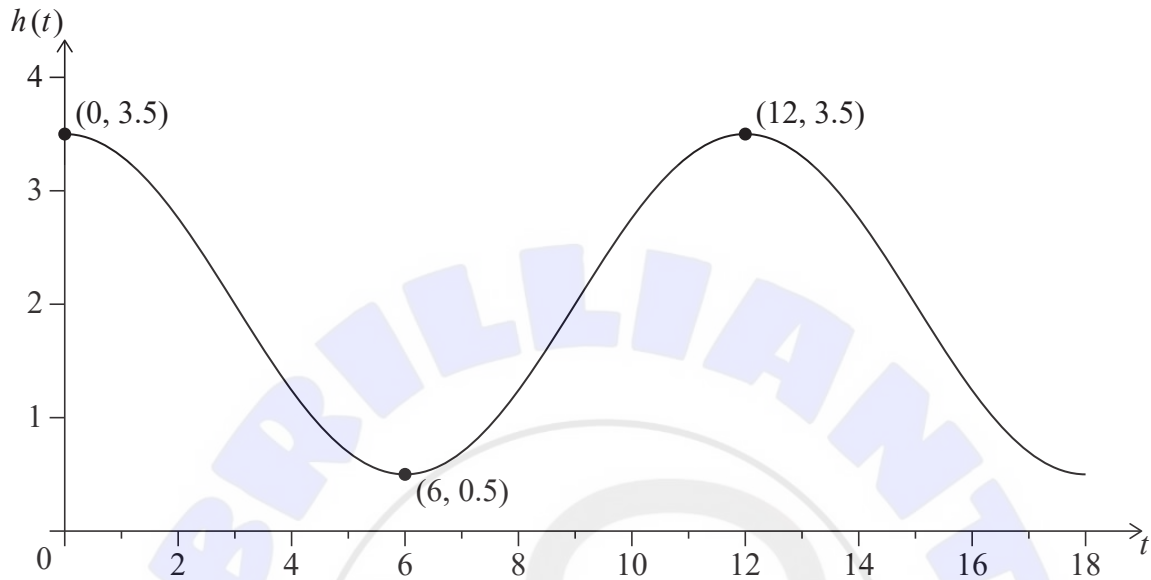
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9. [Maximum mark: 8]

Joon is a keen surfer and wants to model waves passing a particular point P, which is off the shore of his favourite beach. Joon sets up a model of the waves in terms of $h(t)$, the height of the water in metres, and t , the time in seconds from when he begins recording the height of the water at point P.



The function has the form $h(t) = p \cos\left(\frac{\pi}{6}t\right) + q, t \geq 0$.

(a) Find the values of p and q . [2]

(b) Find

(i) $h'(t)$.

(ii) $h''(t)$. [3]

Joon will begin to surf the wave when the rate of change of h with respect to t , at P, is at its maximum. This will first occur when $t = k$.

(c) (i) Find the value of k .

(ii) Find the height of the water at this time. [3]

(This question continues on the following page)



10. [Maximum mark: 7]

The decay of a chemical isotope over five years is recorded in **Table 1**. The mass of the chemical M is measured to the nearest gram at the beginning of each year t of the experiment.

Table 1

Time t (years)	1	2	3	4	5
Mass M (grams)	1000	660	517	435	381

It is believed that the decay of the isotope can be modelled by an equation of the form $M = a \times t^b$.

- (a) Use power regression on your graphic display calculator to find the value of a and the value of b . [2]

The values of t and M can be transformed such that $x = \ln t$ and $y = \ln M$. **Table 2** shows data for x and y to three decimal places.

Table 2

x	0	0.693	1.099	1.386	1.609
y	6.908	6.492	6.248	6.075	5.943

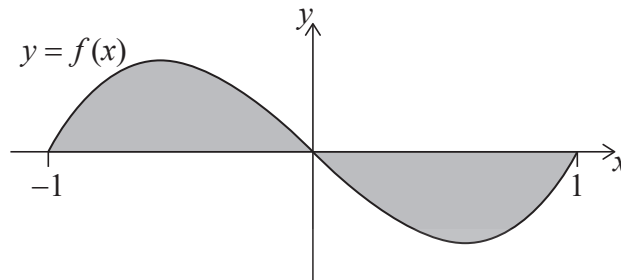
- (b) Find the linear regression equation of y on x , in the form $y = cx + d$. Give the values of c and d to three decimal places. [2]
- (c) Hence, show that this linear regression is equivalent to the power regression found in part (a). [3]

(This question continues on the following page)



11. [Maximum mark: 7]

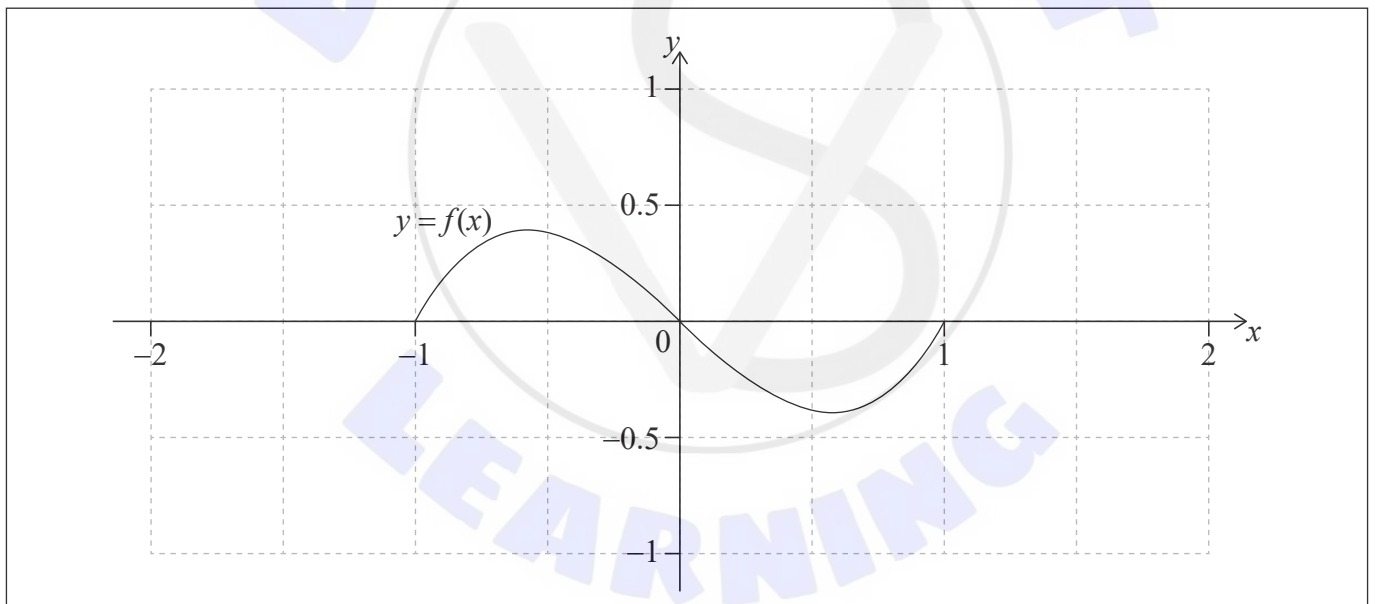
Consider the function $f(x) = x^3 - x$, for $-1 \leq x \leq 1$. The shaded region, R , is bounded by the graph of $y = f(x)$ and the x -axis.



- (a) (i) Write down an integral that represents the area of R .
- (ii) Find the area of R . [2]

Another function, g , is defined such that $g(x) = 2f(x - 1)$.

- (b) On the following set of axes, the graph of $y = f(x)$ has been drawn. On the same set of axes, sketch the graph of $y = g(x)$. [2]



The region R from the original graph $y = f(x)$ is rotated through 2π radians about the x -axis to form a solid.

- (c) Find the volume of the solid. [3]

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12. [Maximum mark: 4]

Consider the differential equation $\frac{dy}{dx} = 3x - y + 1$.

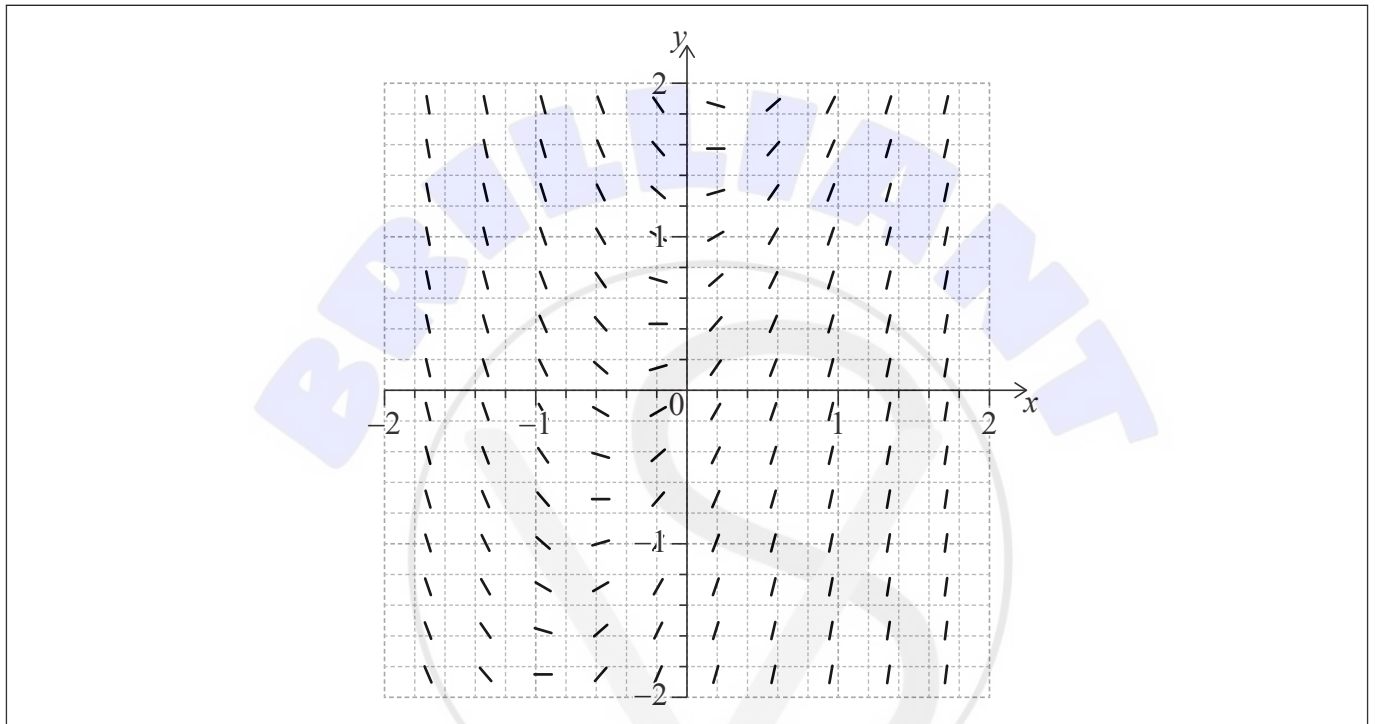
- (a) Find the equation of the tangent to the solution curve at the point $(-1, -1)$ in the form $ax + by + c = 0$.

[2]

The slope field for this differential equation is shown in the following diagram.

- (b) Sketch the solution curve that passes through the point $(-1, -1)$.

[2]



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15. [Maximum mark: 7]

The eating habits of students in a school are studied over a number of months. The focus of the study is whether non-vegetarians become vegetarians, and whether vegetarians remain vegetarians.

Each month, students choose between the vegetarian or non-vegetarian lunch options. Once they have chosen for the month, they cannot change the option until the next month.

In any month during the study, it is noticed that the probability of a non-vegetarian becoming vegetarian the following month is 0.1, and that the probability of a vegetarian remaining a vegetarian the following month is 0.8.

This situation can be represented by the transition matrix

$$T = \begin{pmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{pmatrix}.$$

- (a) Interpret the value 0.9 in T in terms of the changes in the eating habits of the students in the school. [1]
- (b) Find the eigenvalues of matrix T . [3]

One of the eigenvectors of T is $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

- (c) Find another, non-parallel, eigenvector and interpret it in context. [3]

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References:

3. Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 <https://creativecommons.org/licenses/by/2.5/#>. Image adapted.

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will not be marked.

